(171) $\bar{u} = \langle 4, -3 \rangle$

$\bar{v} = \langle 3, 2 \rangle$

$\bar{v} = \bar{w} + \bar{q}$

$\bar{v} - \bar{w} = \bar{q}$

$$\text{comp}_{\bar{u}}(\bar{v}) = \bar{v} \cdot \frac{\bar{u}}{|\bar{u}|} \quad \left(= |\bar{v}| \cos\theta \right)$$

$$= \langle 3, 2 \rangle \cdot \frac{\langle 4, -3 \rangle}{5}$$

$|\bar{w}| = \text{comp}_{\bar{u}}(\bar{v})$

$$= \frac{12 - 6}{5}$$

$$= \frac{6}{5}$$

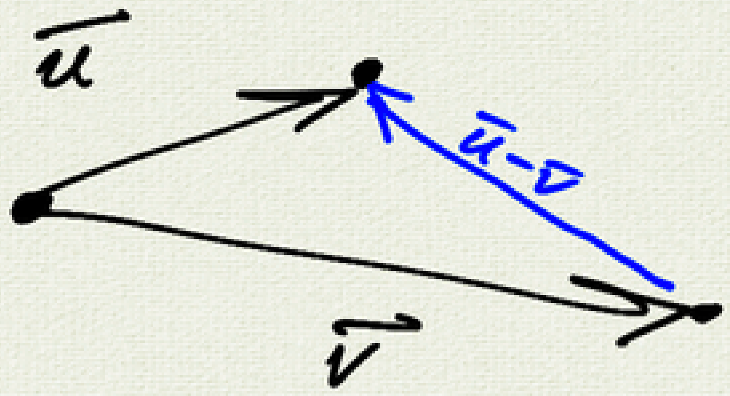
$$\bar{w} = \text{proj}_{\bar{u}} \bar{v} = \left(\frac{6}{5}\right) \frac{\bar{u}}{|\bar{u}|}$$

$$= \left(\frac{6}{5}\right) \frac{\langle 4, -3 \rangle}{5}$$

$$= \frac{1}{25} \langle 24, -18 \rangle$$

$\longrightarrow \bar{q} = \bar{v} - \bar{w}$

$$= \langle 3, 2 \rangle - \left\langle \frac{24}{25}, -\frac{18}{25} \right\rangle$$



$\bar{v} + (\bar{u} - \bar{v}) = \bar{u}$

(101) $\bar{v} = \langle 2\sin t, 2\cos t, 1\rangle$

$\|\bar{u}\| = 2$

$|v|^2 = 2^2\sin^2 t + 2^2\cos^2 t + 1$

$= 2^2 + 1$

$= 5$

$|v| = \sqrt{5}$

$\sin^2 t + \cos^2 t = 1$

$\frac{\bar{v}}{\sqrt{5}}$ unit vector

$\bar{u} = \boxed{-2\frac{\bar{v}}{\sqrt{5}}}$ (2x long) opposite direction

$= -\frac{2}{\sqrt{5}}\langle 2\sin t, 2\cos t, 1\rangle$

---

(141) $\bar{a} = \langle x, y\rangle$

$\bar{b} = \langle -y, x\rangle$

$\bar{a}\cdot\bar{b} = 0 \iff$ <u>orthogonal</u> definition

$\bar{a}\cdot\bar{b} = 0$ always

$\bar{a}\cdot\bar{b} = |\bar{a}|\,|\bar{b}|\cos\theta$

$\bar{a}\cdot\bar{b} = 0 \iff |\bar{a}| = 0$

or $|\bar{b}| = 0$

or $\cos\theta = 0$

$\theta = \pi/2$

1.4 Matrices and determinants

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} e \\ f \end{pmatrix}$$

$$\begin{aligned} ax + cy &= e \\ bx + dy &= f \end{aligned} \quad \longleftrightarrow \quad \left(\begin{array}{cc|c} a & c & e \\ b & d & f \end{array}\right)$$

$$A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} e \\ f \end{pmatrix}$$

Matrix $\quad A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$ $\quad a_{ij}$

row column

2×3 matrix

rows columns

---

operations: addition

$$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix} = \begin{pmatrix} 6 & 10 \\ 8 & 12 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \times$$

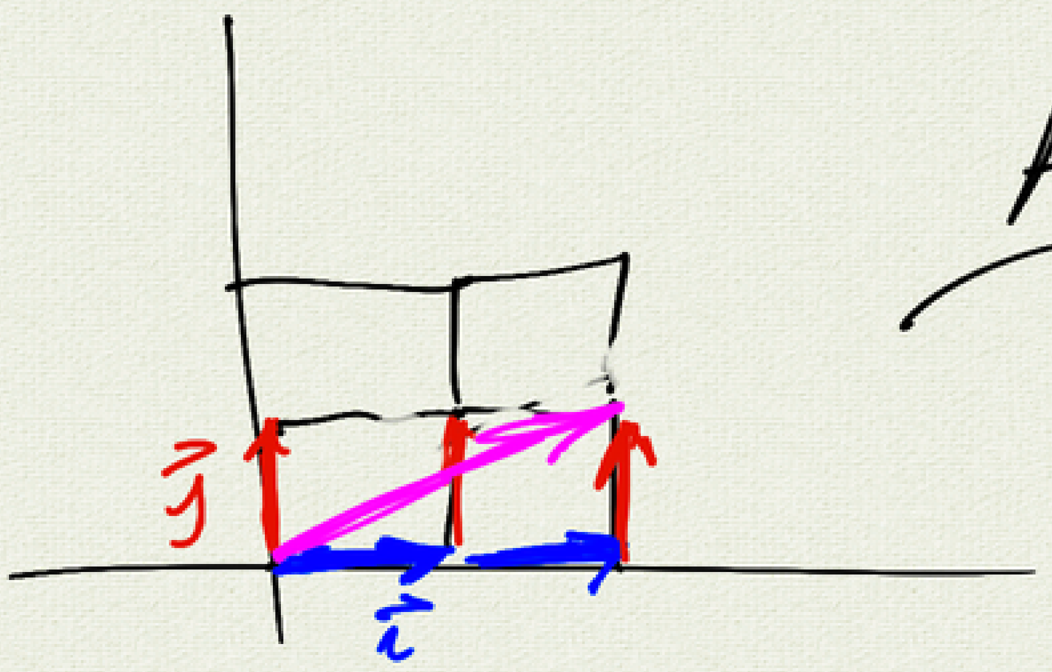
scalar multiplication

$$2\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 6 \\ 4 & 8 \end{pmatrix}$$

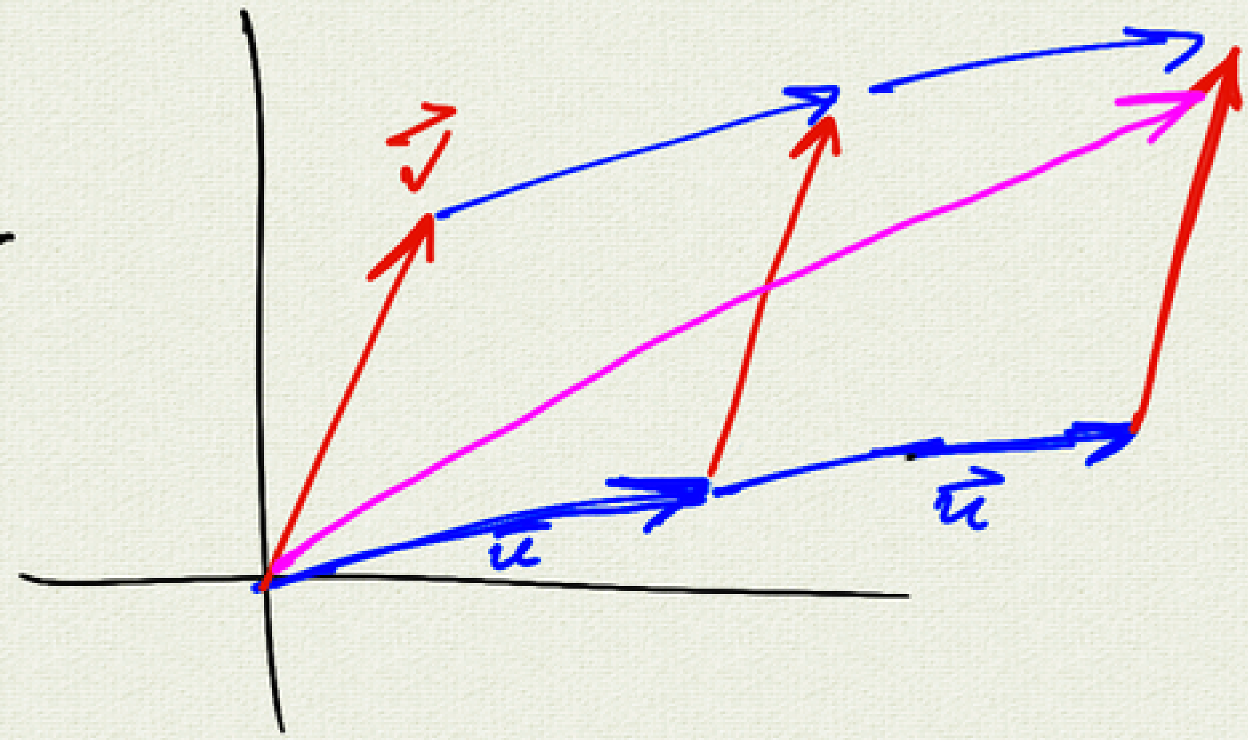
---

vector $\langle 2, 3 \rangle \longleftrightarrow \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ column vector

linear transformation

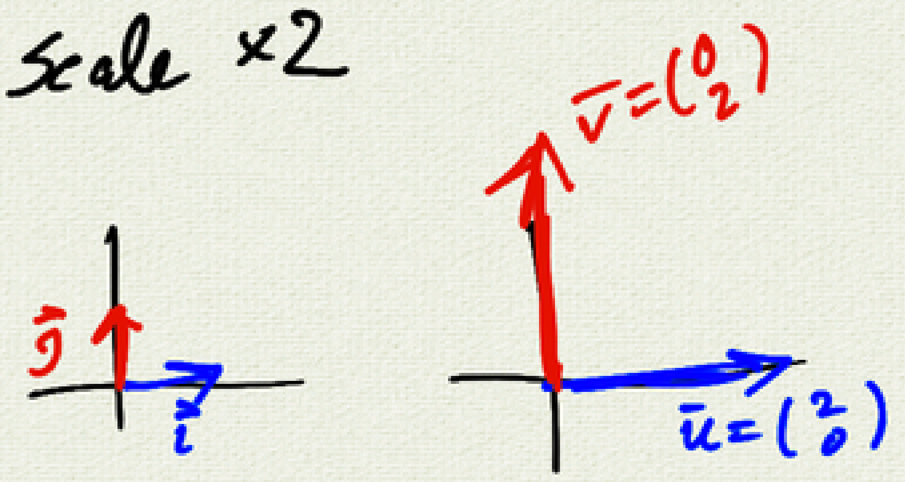


$\xrightarrow{A}$

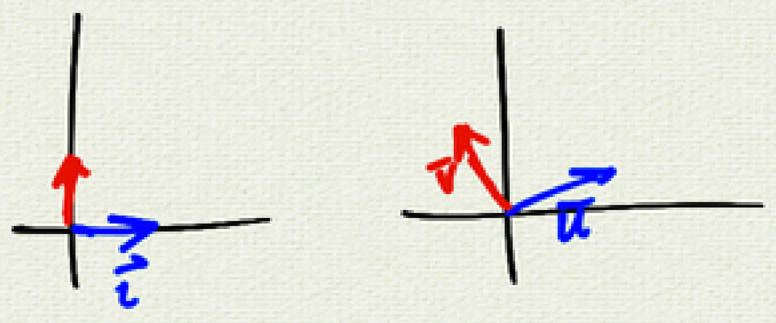


examples:

scale ×2



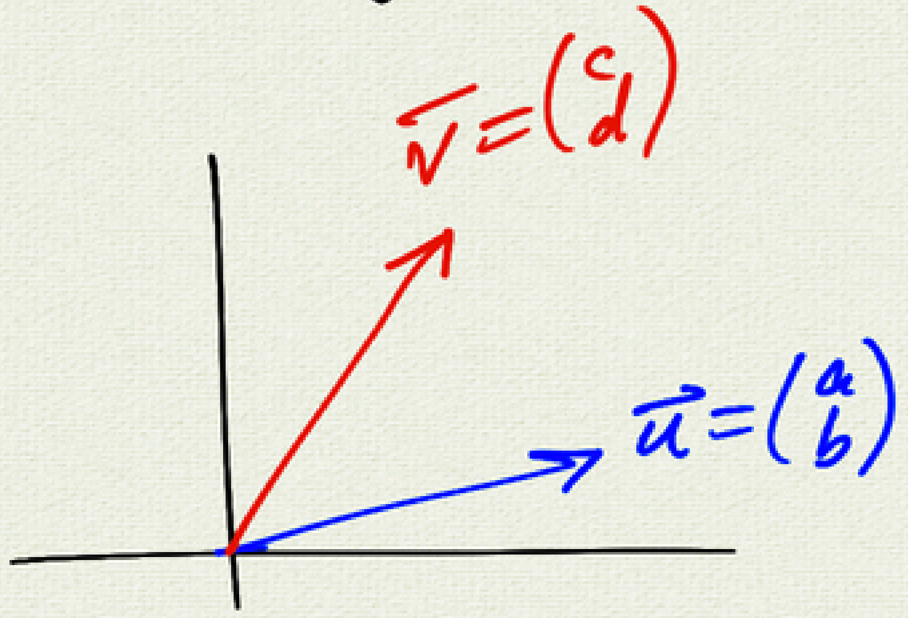
rotate 30°



---

vector $\begin{pmatrix} x \\ y \end{pmatrix} \xrightarrow{A} x\vec{u} + y\vec{v}$

$= x\vec{i} + y\vec{j}$



$$A\begin{pmatrix} x \\ y \end{pmatrix} = x\vec{u} + y\vec{v}$$
$$= x\begin{pmatrix} a \\ b \end{pmatrix} + y\begin{pmatrix} c \\ d \end{pmatrix}$$
$$= \begin{pmatrix} xa + yc \\ xb + yd \end{pmatrix}$$
$$= \underbrace{\begin{pmatrix} a & c \\ b & d \end{pmatrix}}_{A}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix} = \begin{pmatrix} \boxed{17} & \boxed{23} \\ & \end{pmatrix}$$

$1 \cdot 5 + 2 \cdot 6 = 17$

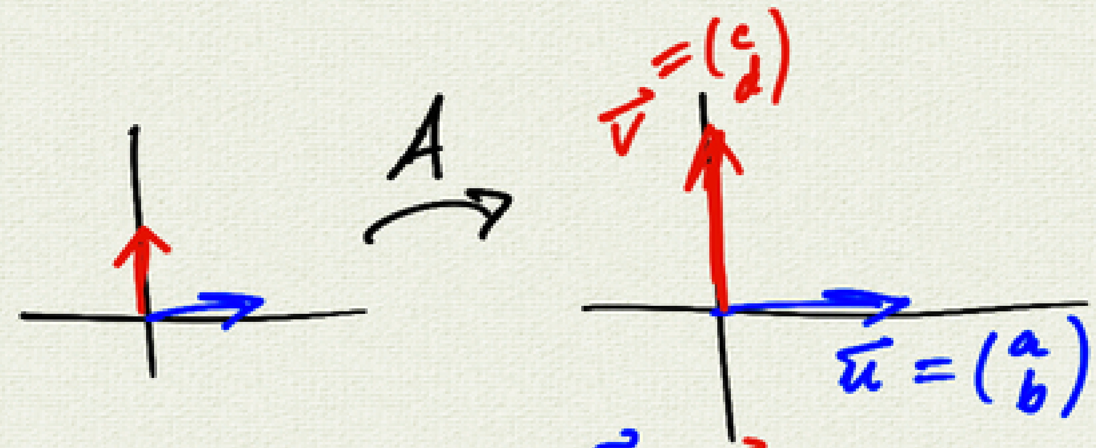
$1 \cdot 7 + 2 \cdot 8 = 23$

examples

Scale $\times 2$

$$\underset{\vec{\imath}}{\begin{pmatrix}1\\0\end{pmatrix}} \to \underset{\vec{u}}{\begin{pmatrix}2\\0\end{pmatrix}}$$

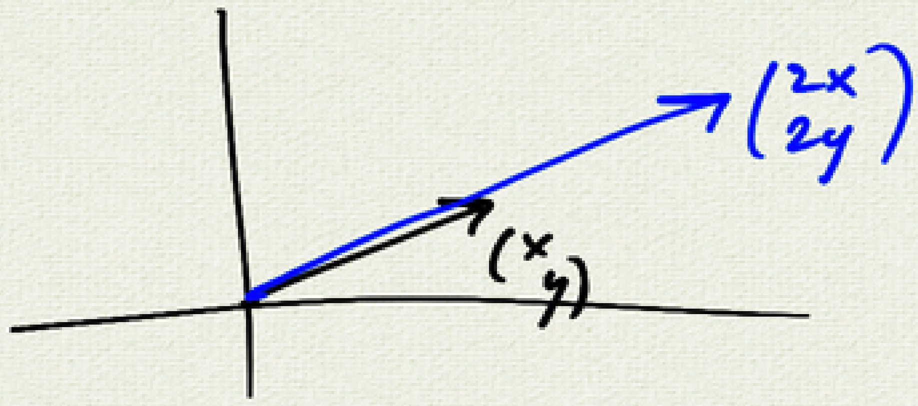
$$\underset{\vec{\jmath}}{\begin{pmatrix}0\\1\end{pmatrix}} \to \begin{pmatrix}0\\2\end{pmatrix}$$



$$A = \begin{array}{c} \quad \vec{u} \;\; \vec{v} \\ \begin{pmatrix}a & c\\ b & d\end{pmatrix}\end{array}$$

$$A = \begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}$$

$$A\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}2x\\2y\end{pmatrix}$$



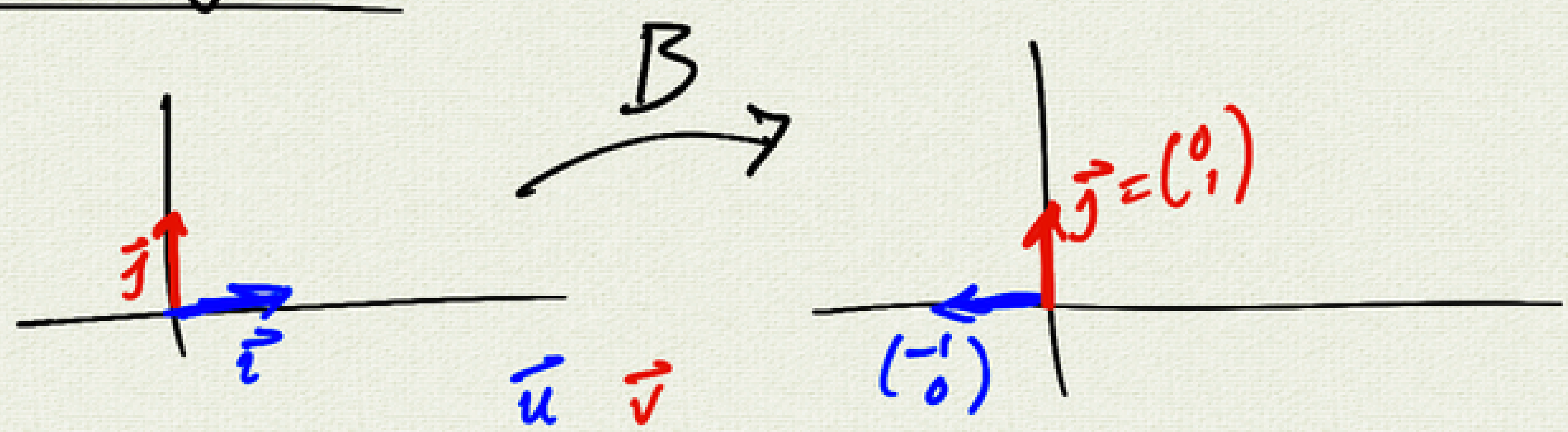
$$I = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

$$A = 2I$$

$$A\begin{pmatrix}x\\y\end{pmatrix} = 2I\begin{pmatrix}x\\y\end{pmatrix} = 2\begin{pmatrix}x\\y\end{pmatrix}$$

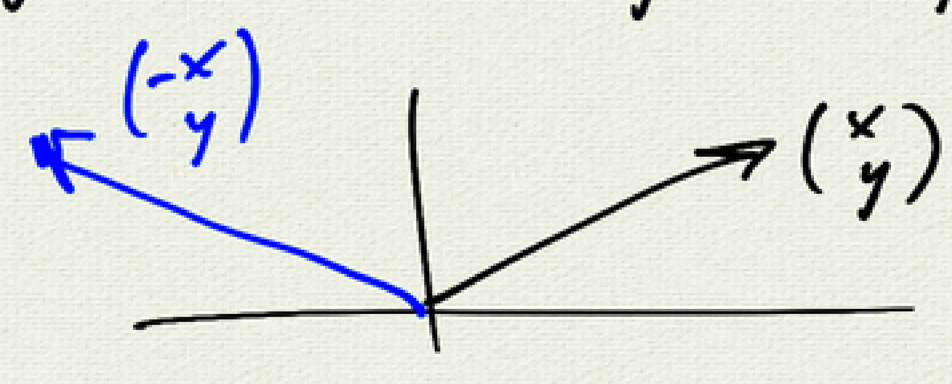
---

reflection in y-axis

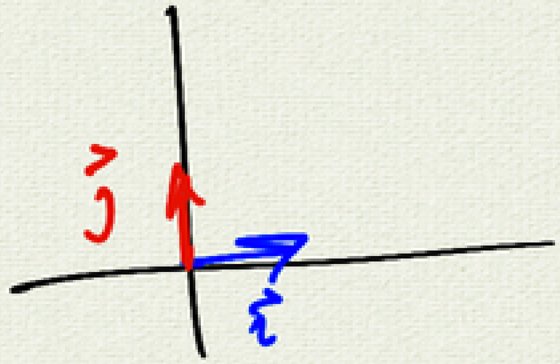


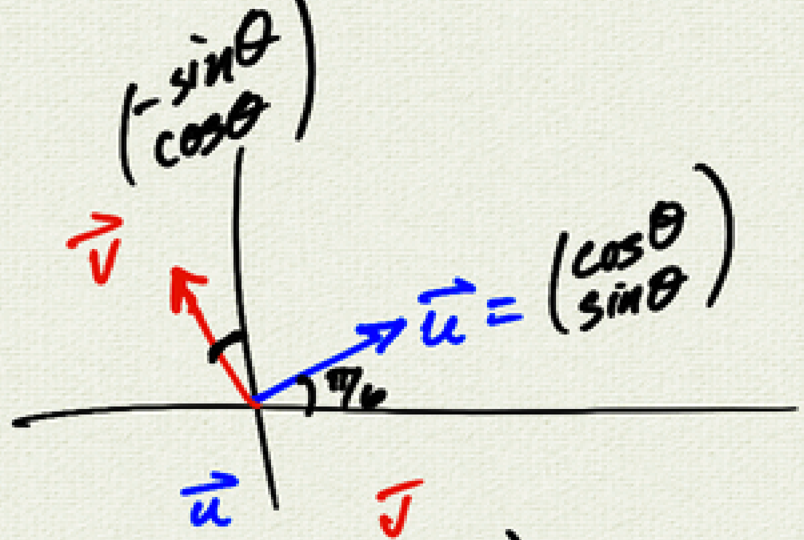
$$B = \begin{array}{c} \quad \vec{u} \;\; \vec{v} \\ \begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}\end{array}$$

$$B\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-x\\y\end{pmatrix}$$



rotation by $\frac{\pi/6}{\theta}$



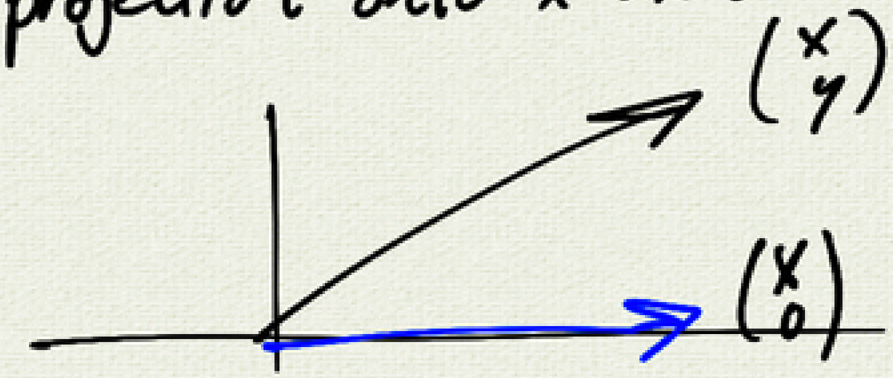


$\left(= \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix} \right)$

$$C = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

---

projection onto x-axis

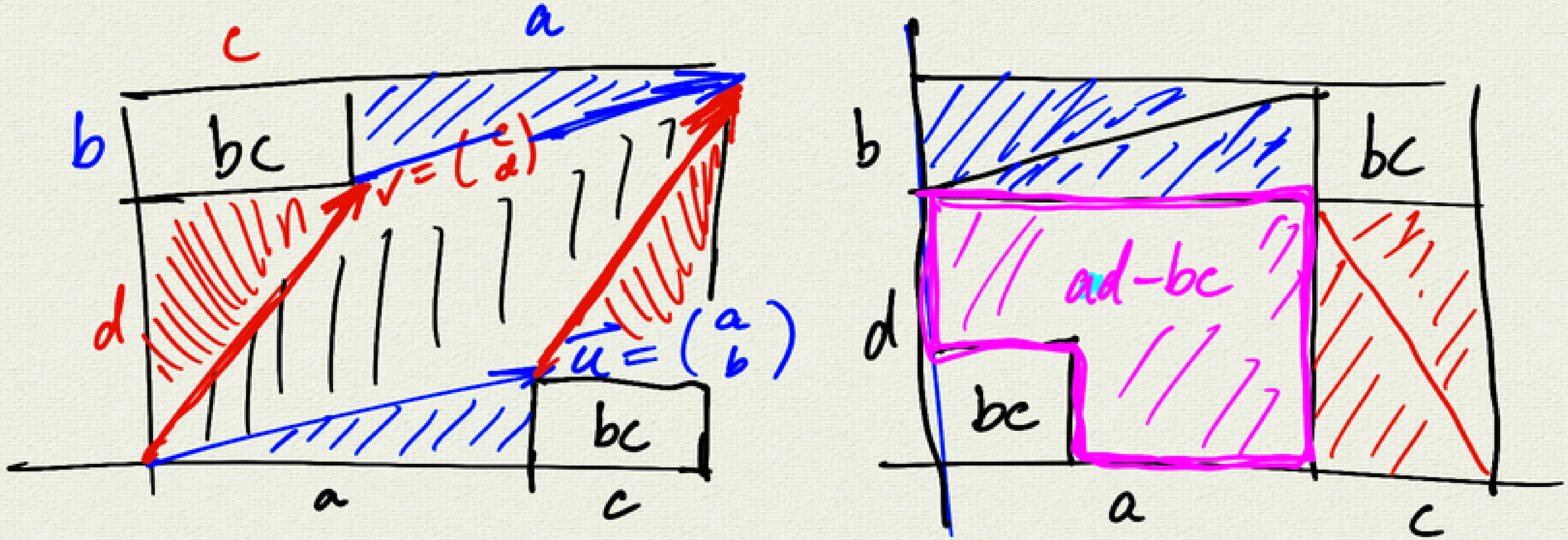


$\vec{i} \to \vec{i}$

$\vec{j} \to \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$D = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$D\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix}$$



$\Longrightarrow$ area ▱ $= ad - bc$

$$A = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \Longrightarrow \det A = ad - bc$$
$$= \text{area } ▱$$

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} d & -c \\ -b & a \end{pmatrix} = \begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix}$$

$$\Longrightarrow A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$$

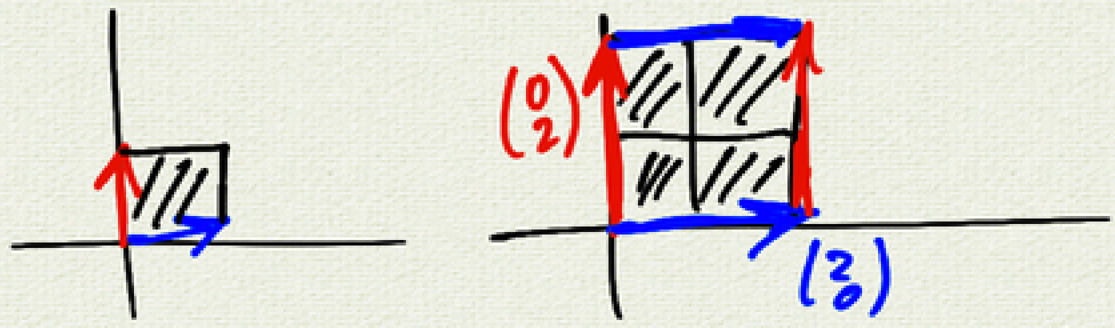
A^{-1} exists $\Longleftrightarrow$ $ad - bc \neq 0$

$\det A \neq 0$

area ▱ $\neq 0$

examples

scale ×2 $\quad A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \quad \det A = 4$

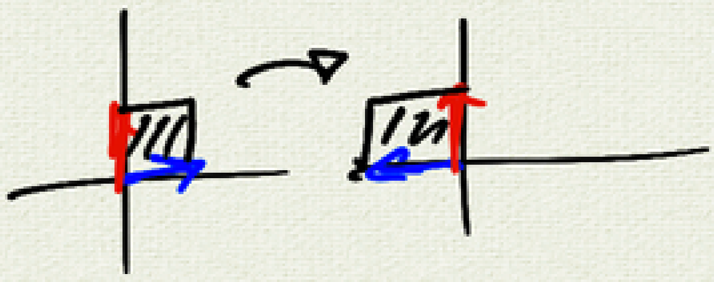


$$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$$
$$= \frac{1}{4}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

check: $AA^{-1} = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

---

reflection $\quad B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$



$\det B = -1$

$B^{-1} = B$

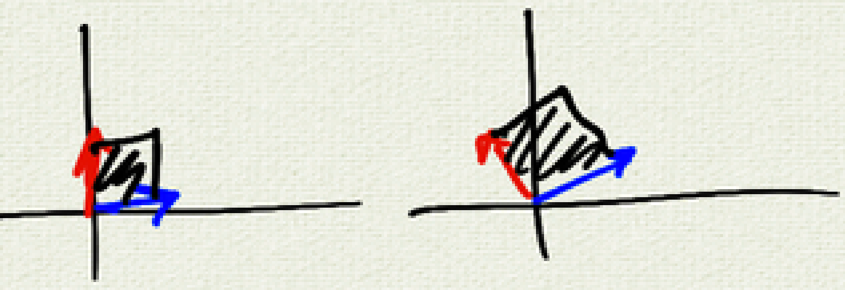
check: $B^2 = I$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

---

rotation $\quad C = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$\det C = 1$

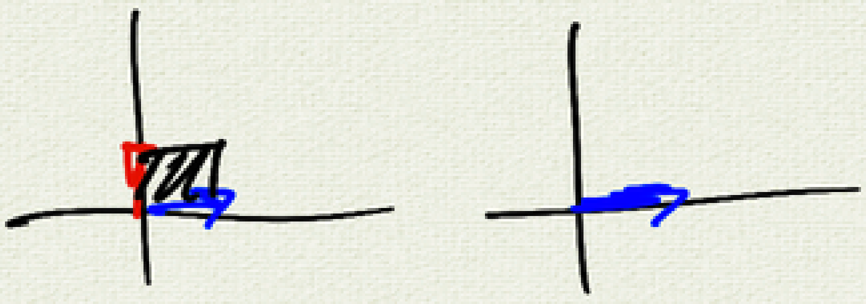


$$C^{-1} = \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix}$$
$$= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

---

projection $\quad D = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$\det D = 0$



D^{-1} does not exist