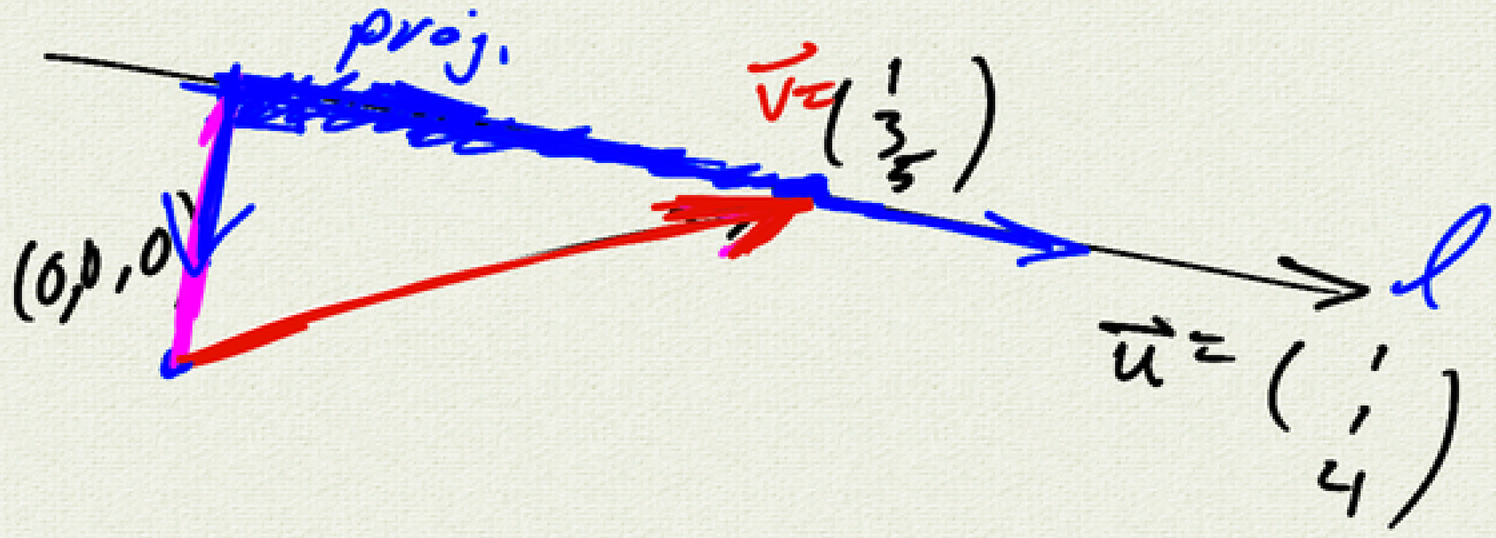
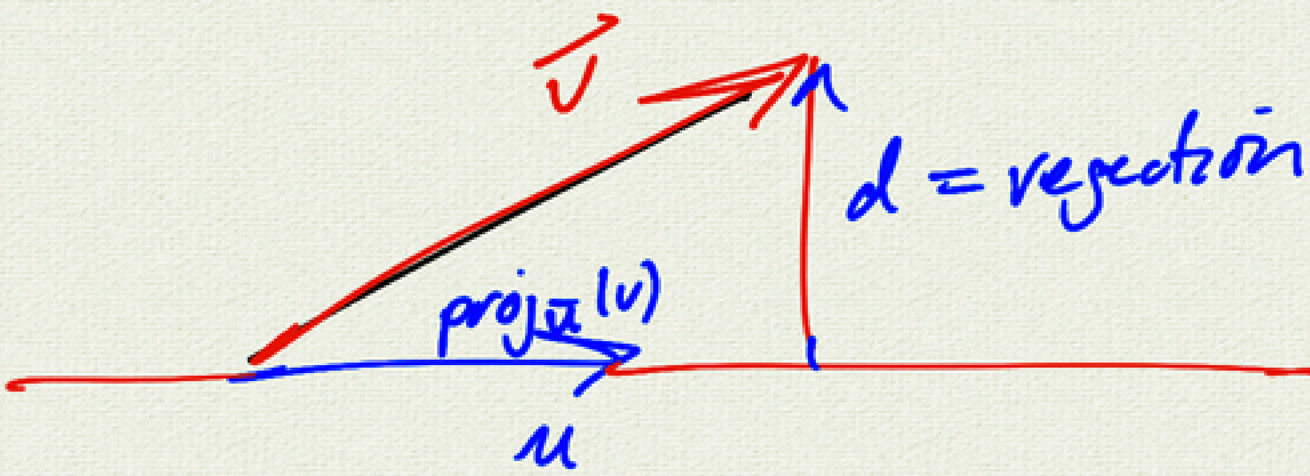
(257)

$$x = 1+t$$
$$y = 3+t$$
$$z = 5+4t$$

$$\vec r(t) = \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\3\\5\end{pmatrix} + t \underbrace{\begin{pmatrix}1\\1\\4\end{pmatrix}}_{\vec u}$$





length of projection

$$|proj_u(v)| = \bar v \cdot \frac{\bar u}{|\bar u|}$$
$$= \begin{pmatrix}1\\3\\5\end{pmatrix} \cdot \begin{pmatrix}1\\1\\4\end{pmatrix} \frac{1}{\sqrt{18}}$$
$$= \frac{1}{3\sqrt2}\, 24$$
$$= \frac{8}{\sqrt2} = 4\sqrt2$$

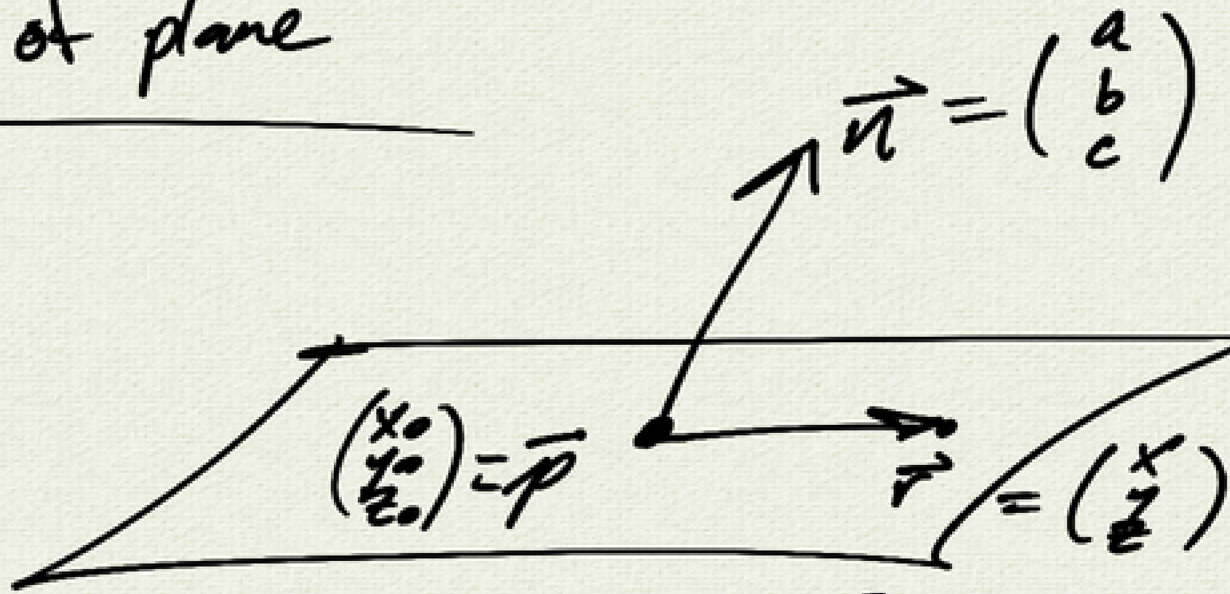
$$proj_u(v) = 4\sqrt2 \frac{\bar u}{|\bar u|}$$
$$= 4\sqrt2 \underbrace{\frac{1}{\sqrt{18}} \begin{pmatrix}1\\1\\4\end{pmatrix}}_{\frac{\bar u}{|\bar u|}}$$
$$= \frac{4}{3}\begin{pmatrix}1\\1\\4\end{pmatrix}$$

rejection $\bar d = \bar v - proj_u(\bar v)$

$$= \begin{pmatrix}1\\3\\5\end{pmatrix} - \frac{4}{3}\begin{pmatrix}1\\1\\4\end{pmatrix}$$
$$= \begin{pmatrix}-1/3\\5/3\\-1/3\end{pmatrix}$$
$$= -\frac{1}{3}\begin{pmatrix}1\\-5\\1\end{pmatrix}$$

$$d = |\bar d| = \frac{1}{3}\sqrt{27} = \sqrt3$$

equation of plane



$\vec{n} \cdot (\vec{r} - \vec{p}) = 0$ point-normal

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix} = 0$$

"scalar"

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

"general"

$ax + by + cz = d$

$d = ax_0 + by_0 + cz_0$

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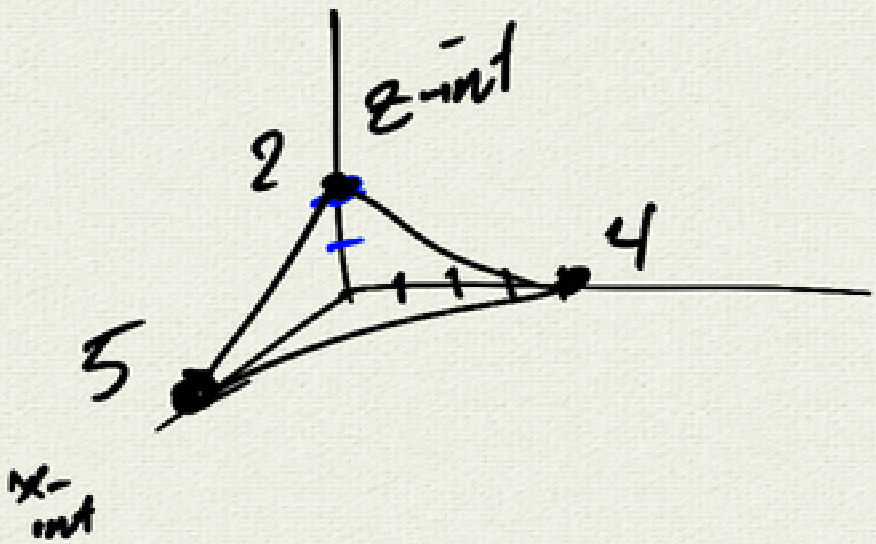
$$\overset{a}{4}x + \overset{b}{5}y + \overset{c}{10}z - 20 = 0$$

$$\bar{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ 10 \end{pmatrix}$$

$$ax + by + cz = d$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$



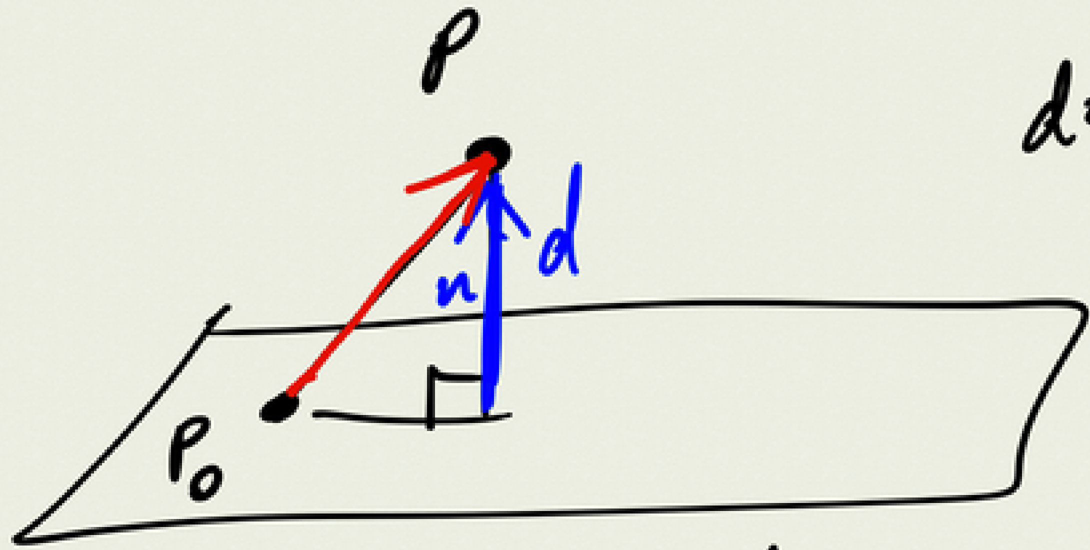
$$\boxed{4x + 5y} + 10z = 20$$

289 $P(1,5,-4)$

plane $3x - y + 2z - 6 = 0$

x-intercept

$\boxed{\begin{array}{c}(2,0,0) \\ \text{use as } P_0\end{array}}$

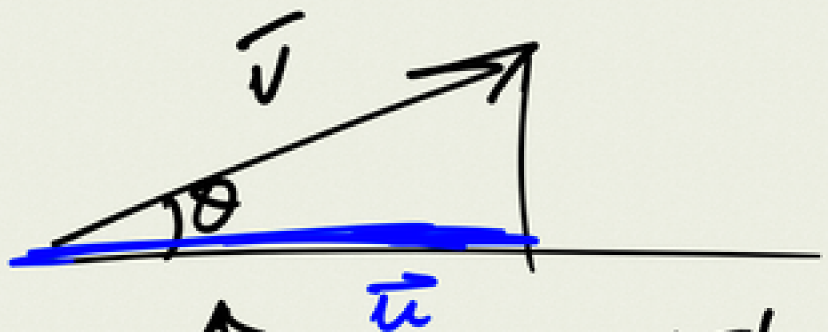


$$d = \left| (\bar{p} - \bar{p}_0) \cdot \frac{\bar{n}}{|\bar{n}|} \right|$$

$$\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \frac{1}{\sqrt{14}}$$

$$\bar{p} - \bar{p}_0 = \left| \begin{pmatrix} -1 \\ 5 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \frac{1}{\sqrt{14}} \right|$$

$$= \frac{16}{\sqrt{14}}$$



$$d = |\bar{v}| \cos\theta$$

$$= \frac{\boxed{\bar{u} \cdot \bar{v}}}{|\bar{u}|} \qquad \frac{|u||v|\cos\theta}{|u|}$$

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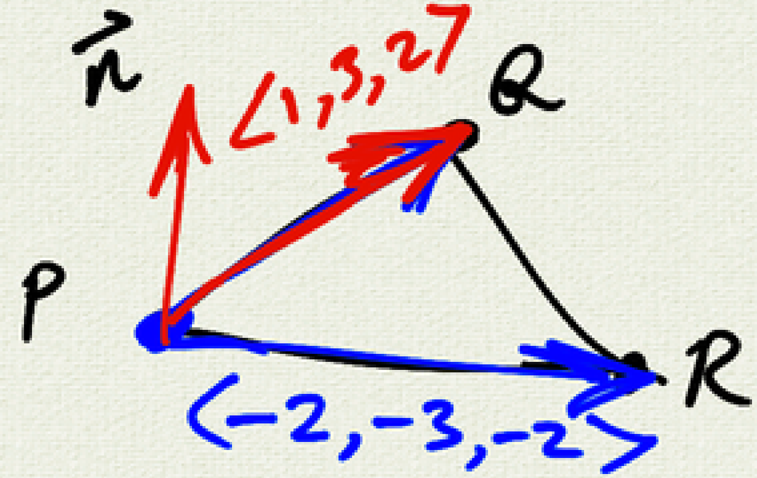
$P(1,1,1)$

$Q(2,4,3)$

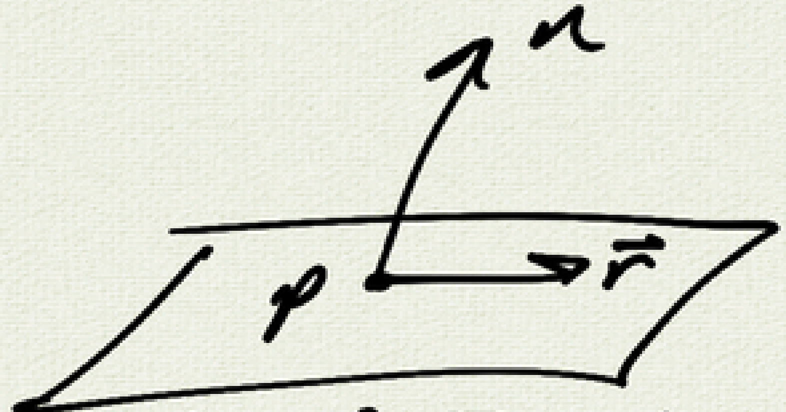
$R(-1,-2,-1)$



$$\vec{n} = \langle 1,3,2\rangle \times \langle -2,-3,-2\rangle$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & 2 \\ -2 & -3 & -2 \end{vmatrix}$$

$$= -2\vec{j} + 3\vec{k}$$



$$\bar{p} = \langle 1,1,1\rangle$$

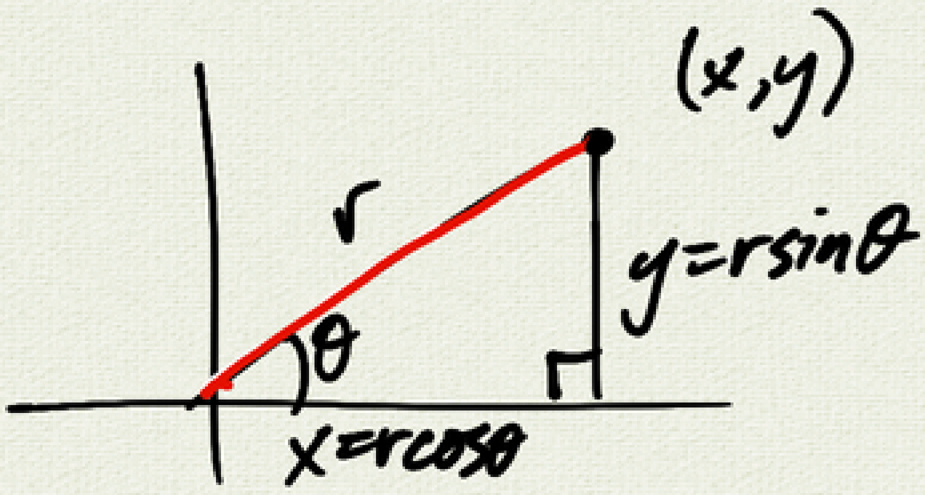
$$0(x-1) - 2(y-1) + 3(z-1) = 0 \qquad (\bar{r}-\bar{p})\cdot\bar{n} = 0$$

$$\begin{pmatrix} 0 \\ -2 \\ 3 \end{pmatrix}$$

$$-2y + 2 + 3z - 3 = 0$$

$$\boxed{-2y + 3z = 1}$$

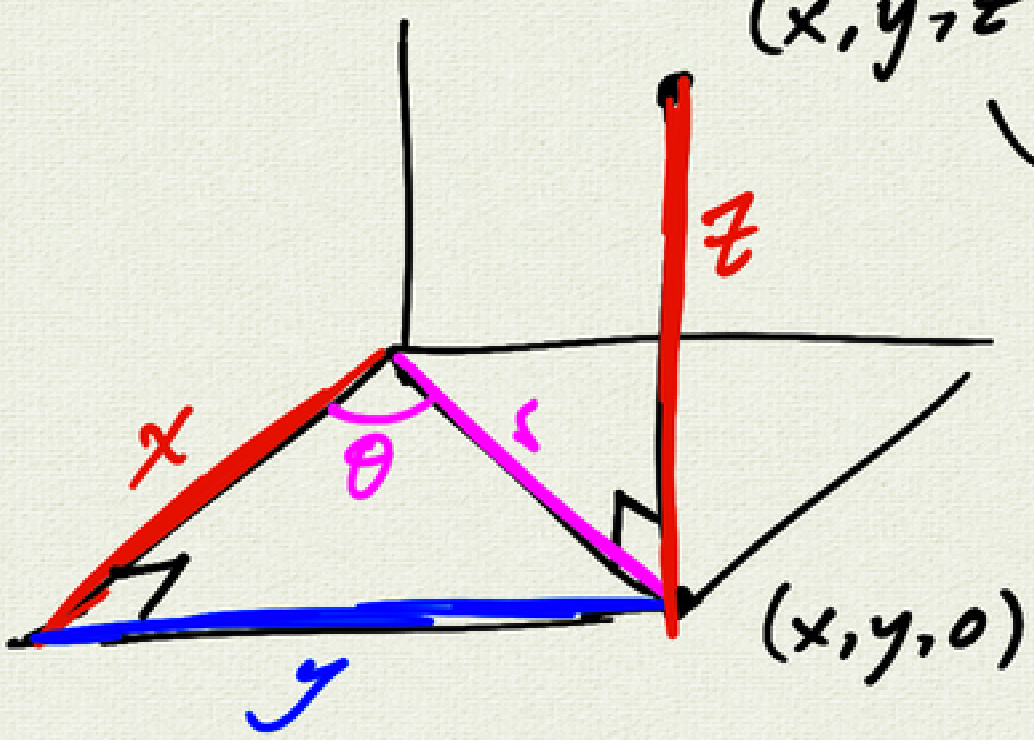
1.7 Cylindrical & Spherical Coordinates



$\tan\theta = \frac{y}{x}$

$r^2 = x^2 + y^2$

polar



$\to (r, \theta, z)$

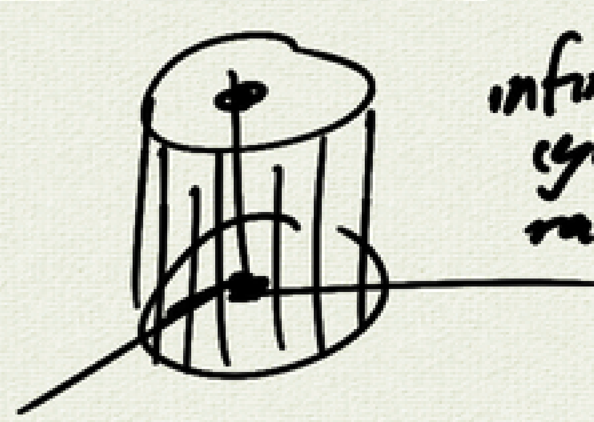
cylindrical

$(1, 1, 1)$ rectangular $\longrightarrow$ cylindrical

(r, θ, z)

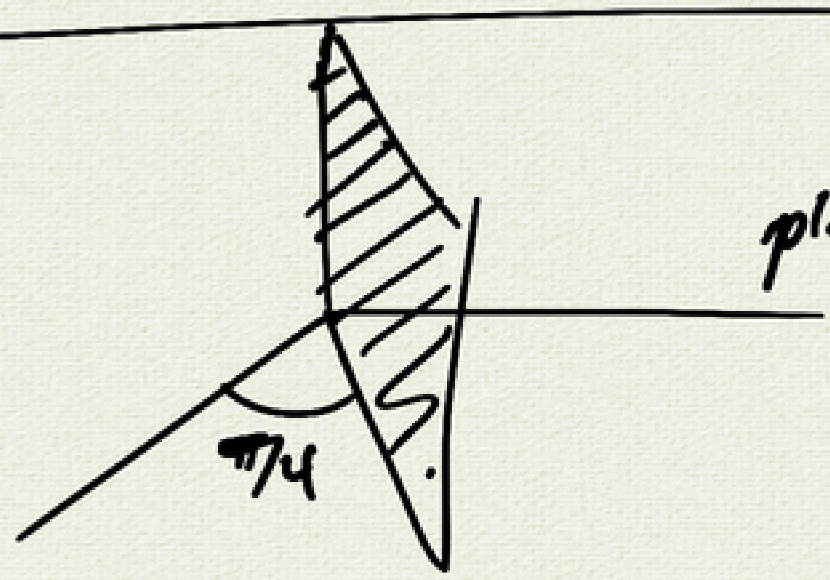
$(\sqrt{2}, \frac{\pi}{4}, 1)$

equations $r = 5$ cylindrical



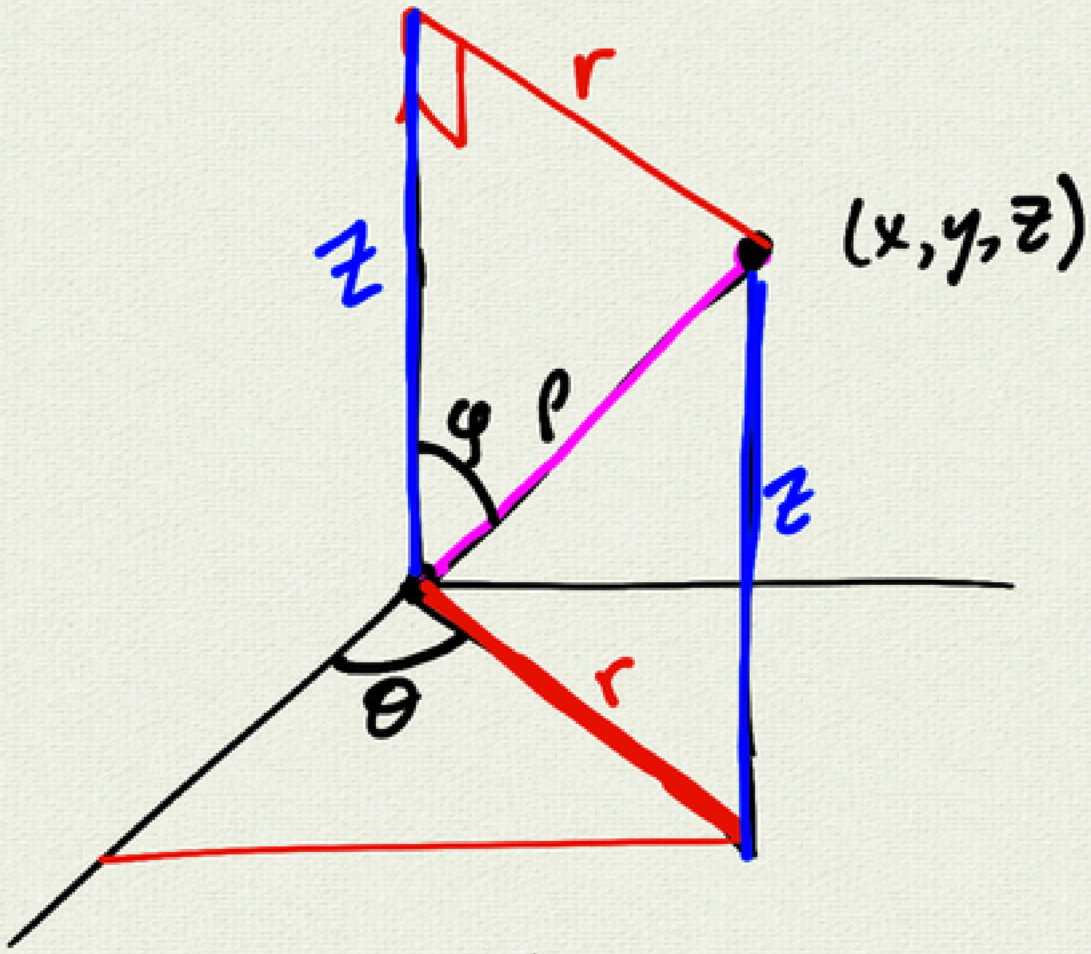
infinite cylinder radius 5

$z = 5$ plane



$\theta = \pi/4$

plane



ρ rho

φ phi Φ ϕ

θ theta

spherical: (ρ, θ, φ)

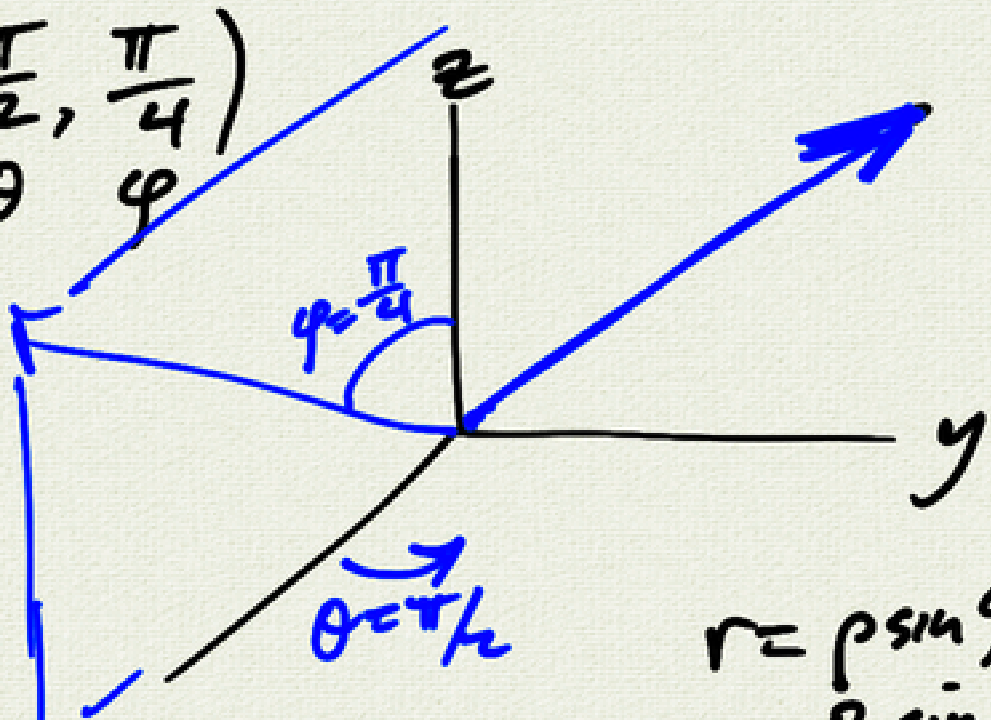
$$\boxed{r = \rho \sin\varphi}$$

$$z = \rho\cos\varphi$$

$$\Longrightarrow \boxed{\begin{aligned} x &= r\cos\theta = \rho\sin\varphi\cos\theta \\ y &= r\sin\theta = \rho\sin\varphi\sin\theta \\ z &= \rho\cos\varphi \end{aligned}}$$

$\left(8, \frac{\pi}{2}, \frac{\pi}{4}\right)$

$\rho \quad \theta \quad \varphi$



$$\begin{aligned} r &= \rho\sin\varphi \\ &= 8\sin\frac{\pi}{4} \\ &= 8\frac{\sqrt{2}}{2} \\ &= 4\sqrt{2} = z \end{aligned}$$

$x = r\cos\theta = 0$

$y = r\sin\theta = 4\sqrt{2}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 4\sqrt{2} \\ 4\sqrt{2} \end{pmatrix}$$

<u>equations</u> $\rho = 5$ sphere radius 5

$\varphi = \frac{\pi}{4}$ cone