381 $x^2+y^2+z^2=9$

$\underbrace{x^2+y^2}$ $\longrightarrow r^2=x^2+y^2$

$r^2+z^2=9$

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367 $\overset{x}{1}, \overset{y}{\sqrt{3}}, \overset{z}{2}$: $(1,\sqrt{3},2) \longrightarrow (r,\theta,z)$

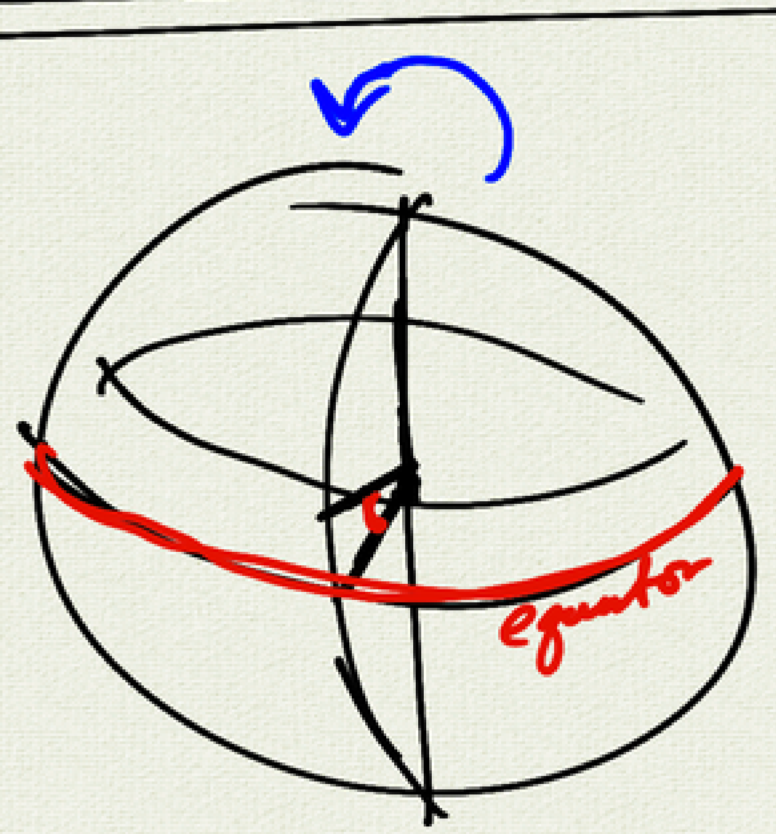
$r^2=x^2+y^2=1+3=4$

$r=2$

$\tan\theta = \frac{y}{x} = \sqrt{3}$

$\Longrightarrow \theta = \frac{\pi}{3}$

cylindrical
$(r,\theta,z)=(2,\frac{\pi}{3},2)$



latitude

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plane through 3 points

distance to plane from point

equation of line

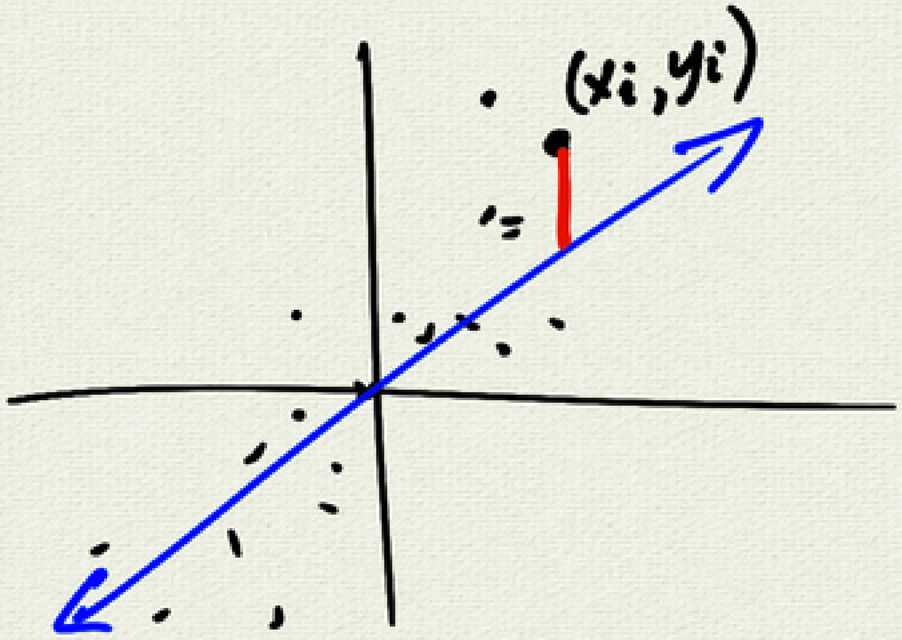
distance to line from point

cylindrical + spherical coords.

dot product
cross product

projection

Linear regression



best fit line
(through origin;
choose slope)

date: $(x_1, y_1) \ldots (x_n, y_n)$

---

excursion into higher dimensions

2D: $\langle x_1, y_1 \rangle \cdot \langle x_2, y_2 \rangle = x_1 x_2 + y_1 y_2$

3D: $\langle x_1, y_1, z_1 \rangle \cdot \langle x_2, y_2, z_2 \rangle = x_1 x_2 + y_1 y_2 + z_1 z_2$

---

change notation

$$\left.\begin{array}{l} \vec{u} = \langle u_1, u_2, u_3 \rangle \\ \vec{v} = \langle v_1, v_2, v_3 \rangle \end{array}\right\} \quad \vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

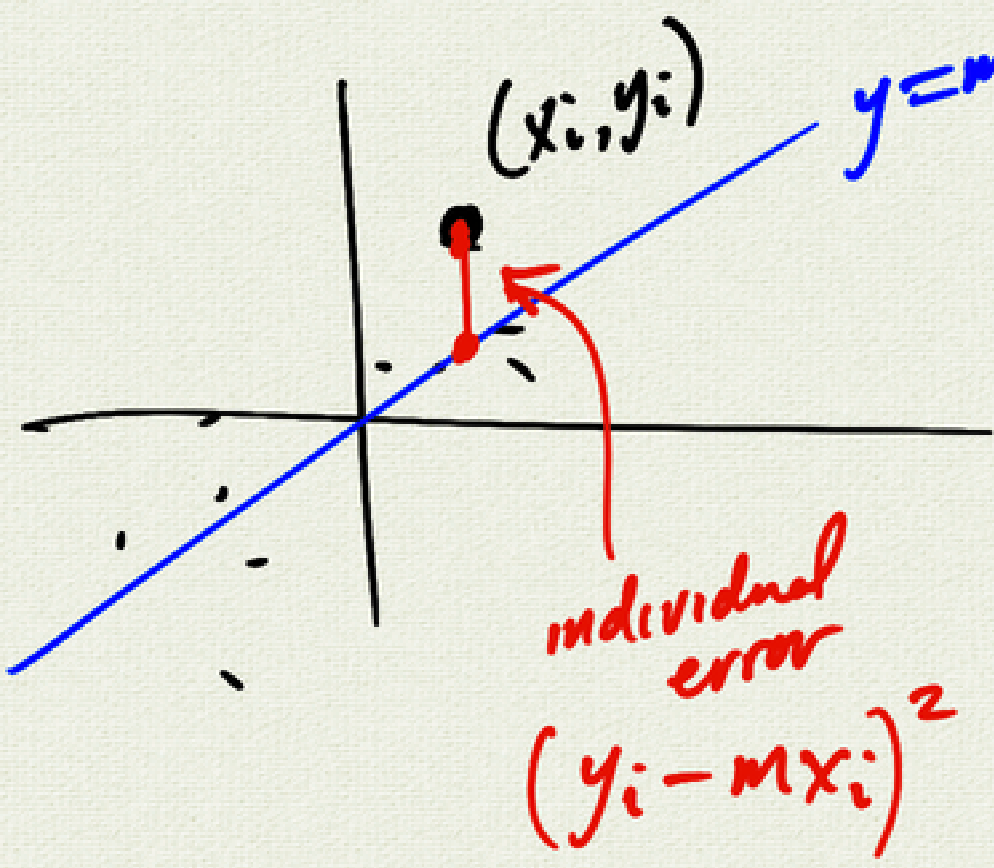
$$= \sum_{i=1}^{3} u_i v_i$$

4D: $\vec{u} \cdot \vec{v} = \sum_{i=1}^{4} u_i v_i$

$$|\vec{u}|^2 = \vec{u} \cdot \vec{u} = \sum_{i=1}^{4} u_i^2$$

$\vec{u} = \langle u_1, u_2, u_3, u_4 \rangle$

$\vec{v} = \langle v_1, v_2, v_3, v_4 \rangle$

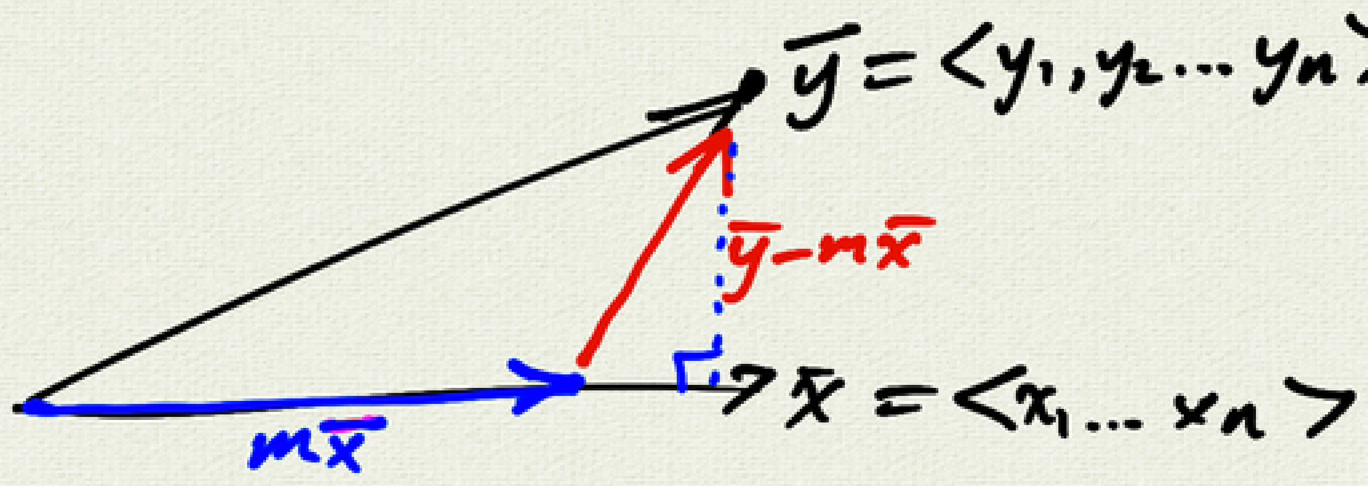


find best m

how?

minimize "total error"

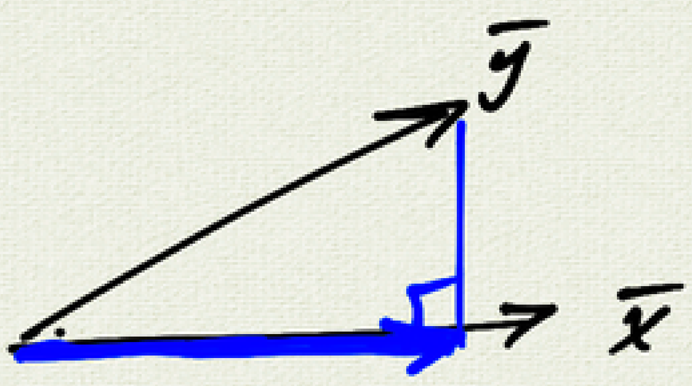
$$\boxed{E(m) = \sum_{i=1}^{n} (y_i - mx_i)^2}$$



$$|\bar{y} - m\bar{x}|^2 = \sum_{i=1}^{n} (y_i - mx_i)^2$$

$\Uparrow$

minimizing distance between $\bar{y}$ and line determined by $\bar{x}$



$\Longrightarrow$ projection of $\bar{y}$ onto $\bar{x}$

$$|\mathrm{proj}_{\bar{x}}(\bar{y})| = \mathrm{comp}_{\bar{x}}(y)$$

$$= \bar{y} \cdot \frac{\bar{x}}{|\bar{x}|}$$

$$\mathrm{proj}_{\bar{x}}(\bar{y}) = \left(\bar{y} \cdot \frac{\bar{x}}{|\bar{x}|}\right) \frac{\bar{x}}{|x|} = m\bar{x}$$

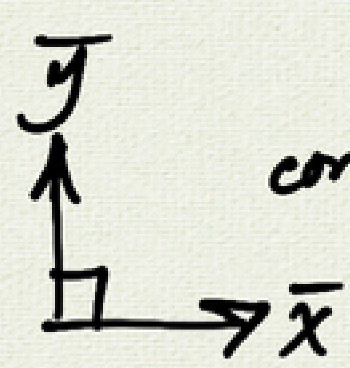
$$m = \frac{\bar{x} \cdot \bar{y}}{|x|^2}$$

$$= \frac{\sum x_i y_i}{\sum x_i^2}$$

covariance $\sum (x_i - \bar{x})(y_i - \bar{y})$

$\sum (x_i - \bar{x})^2$ variance

linear algebra	stats
vector $\bar{x} = \langle x_1 \dots x_n \rangle$	data
$\bar{x} \cdot \bar{y}$ dot product	covariance
$\lvert\bar{x}\rvert^2 = \bar{x} \cdot \bar{x}$	variance
$\lvert\bar{x}\rvert$ magnitude	standard deviation
$\cos\theta = \frac{\bar{x} \cdot \bar{y}}{\lvert x\rvert \lvert y\rvert}$ angle between vectors	correlation



correlation 0

correlation ≈ 1