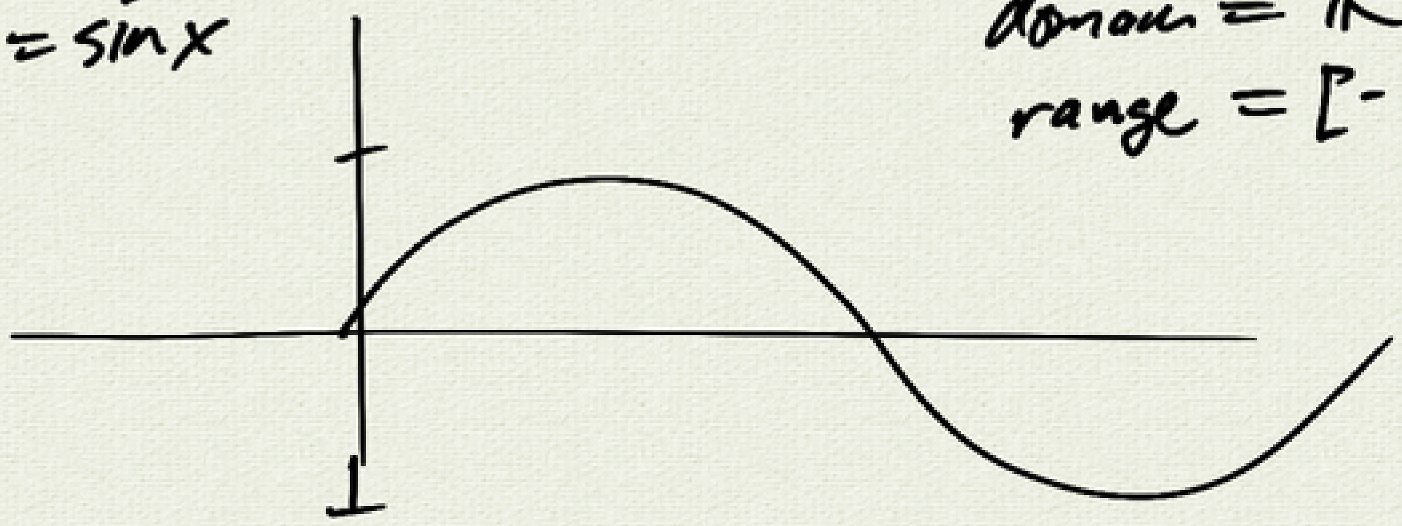
2.2 Calculus

domain/range

<u>domain</u>: where the function is defined

<u>range</u>: set of possible values for the function

$f(x) = \sin x$



domain $= \mathbb{R}$

range $= [-1, 1]$

$f: \mathbb{R} \to \mathbb{R}$

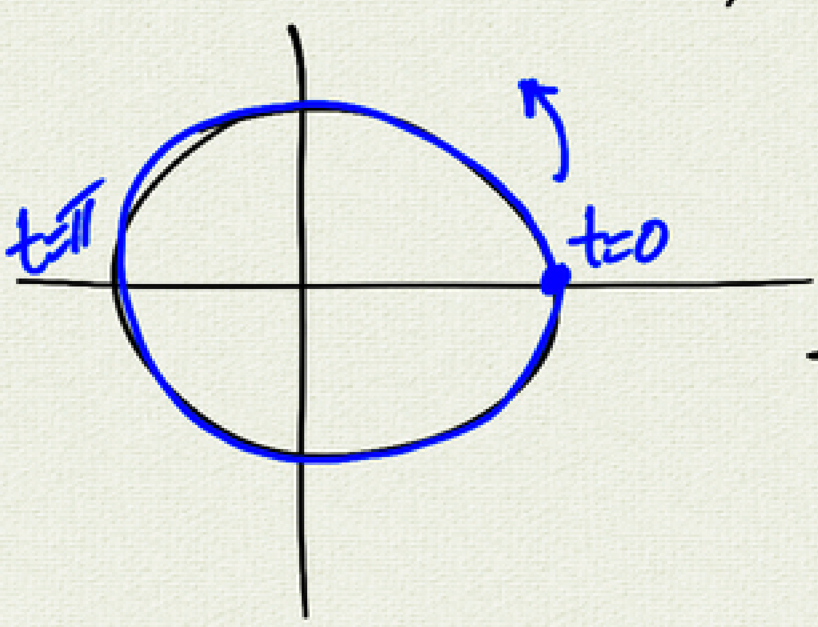
$$\bar{r}: \mathbb{R} \to \mathbb{R}^2$$
$$t \to \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

<u>example</u>:

$$\bar{r}(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$$

domain $= \mathbb{R}$

range $=$ all vectors $\bar{v}$ such that $|\bar{v}| = 1$

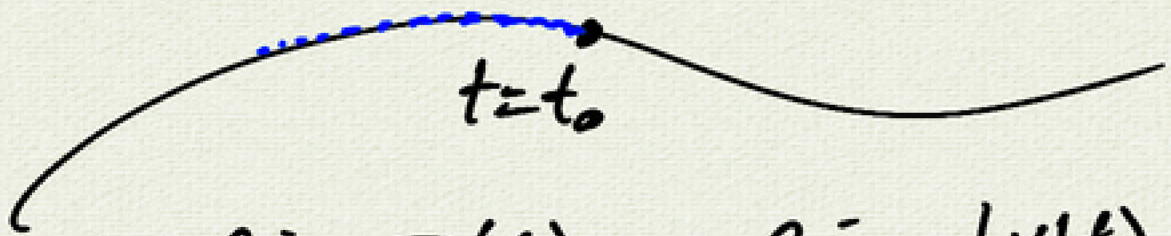


$$\bar{r}: \mathbb{R} \to \mathbb{R}^3$$

$$\bar{r}(t) = \begin{pmatrix} \cos t \\ \sin t \\ 0 \end{pmatrix}$$

domain $= \mathbb{R}$

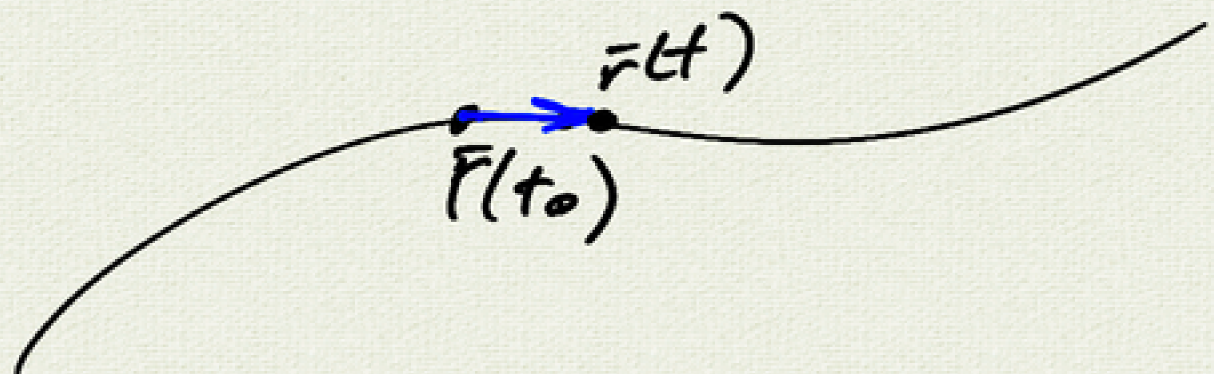
range $=$ all vectors $\bar{v}$ such that $|\bar{v}| = 1$ and $v_z = 0$



$$\lim_{t\to t_0} \bar{r}(t) = \lim_{t\to t_0} \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}$$

$\bar{r}: \mathbb{R} \to \mathbb{R}^3$

curve

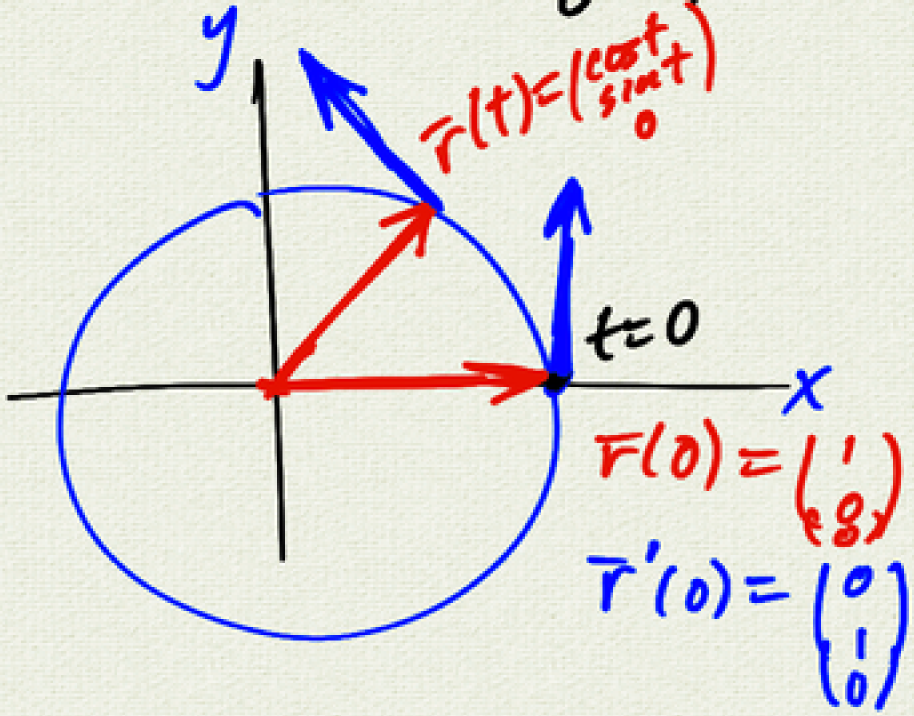


define $\displaystyle \bar{r}'(t_0) = \lim_{t\to t_0} \frac{\bar{r}(t)-\bar{r}(t_0)}{t-t_0}$

$$= \lim_{t\to t_0} \left\langle \frac{x(t)-x(t_0)}{t-t_0}, \frac{y(t)-y(t_0)}{t-t_0}, \frac{z(t)-z(t_0)}{t-t_0} \right\rangle$$

$$= \langle x'(t), y'(t), z'(t) \rangle$$

$$\bar{r}(t) = \begin{pmatrix} \cos t \\ \sin t \\ 0 \end{pmatrix} \qquad \bar{r} : \mathbb{R} \longrightarrow \mathbb{R}^3$$



$$\bar{r}'(t) = \begin{pmatrix} -\sin t \\ \cos t \\ 0 \end{pmatrix}$$

$\bar{r}'(t) \perp \bar{r}(t)$:

$$\bar{r}(t) \cdot \bar{r}'(t) = \begin{pmatrix} \cos t \\ \sin t \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -\sin t \\ \cos t \\ 0 \end{pmatrix} = 0$$

orthogonal

$\boxed{\bar{r}'(t) \text{ is tangent to the curve } \bar{r}(t)}$

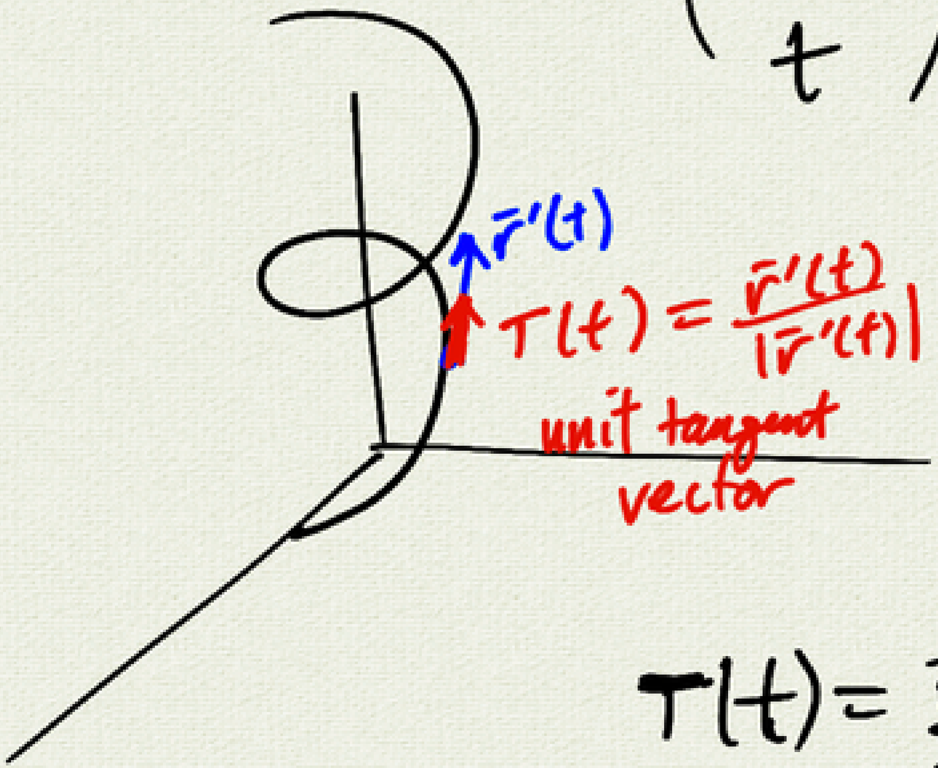
<u>helix</u>

$$\bar{r}(t) = \begin{pmatrix} \cos t \\ \sin t \\ t \end{pmatrix} \qquad \bar{r}'(t) = \begin{pmatrix} -\sin t \\ \cos t \\ 1 \end{pmatrix}$$

$\Uparrow$

velocity

tangent vector



$$\text{speed} = |\bar{r}'(t)| = \sqrt{2}$$

$$T(t) = \frac{\bar{r}'(t)}{|\bar{r}'(t)|} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\sin t \\ \cos t \\ 1 \end{pmatrix}$$