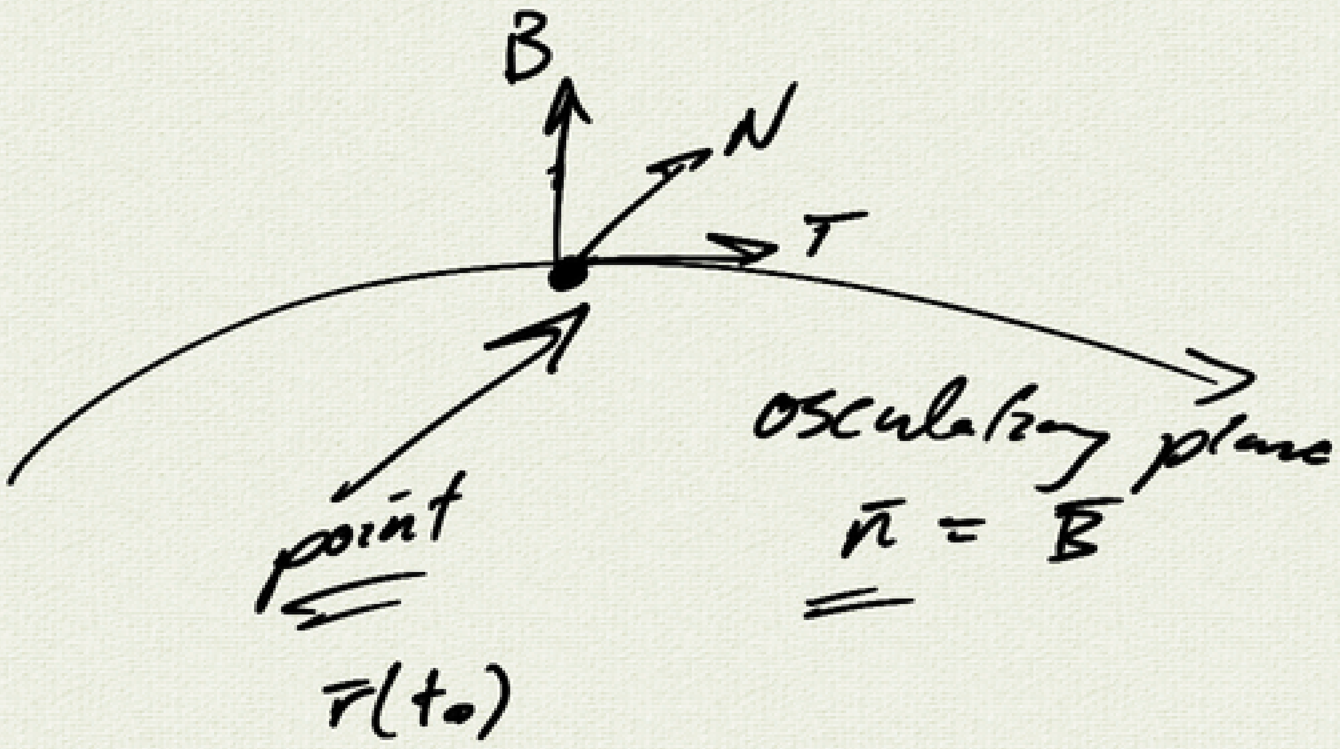


(143) $t = \pi/4$ $\quad \bar{r}(t) = \begin{pmatrix} \cos 2t \\ \sin 2t \\ t \end{pmatrix}$

$\bar{r}'(t) = \begin{pmatrix} -2\sin 2t \\ 2\cos 2t \\ 1 \end{pmatrix} \quad |\bar{r}'(t)| = \sqrt{5}$

$|\bar{r}'(t)|^2 = (-2\sin 2t)^2 + (2\cos 2t)^2 + 1^2$
$= 4\sin^2 2t + 4\cos^2 2t + 1$
$= 4(\sin^2 2t + \cos^2 2t) + 1$
$= 5$

$T(t) = \frac{1}{\sqrt{5}} \begin{pmatrix} -2\sin 2t \\ 2\cos 2t \\ 1 \end{pmatrix}$

$T'(t) = \frac{1}{\sqrt{5}} \begin{pmatrix} -4\cos 2t \\ -4\sin 2t \\ 0 \end{pmatrix}$

$N(t) = \begin{pmatrix} -\cos 2t \\ -\sin 2t \\ 0 \end{pmatrix}$

$B = T \times N = \frac{1}{\sqrt{5}} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2\sin 2t & 2\cos 2t & 1 \\ -\cos 2t & -\sin 2t & 0 \end{vmatrix}$

$= \frac{1}{\sqrt{5}} \begin{pmatrix} \sin 2t \\ \cos 2t \\ 2 \end{pmatrix}$

osculating plane:
$\bar{n} = \bar{B}$ at $t = \pi/4$
$= \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \leftarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$\bar{r}(\pi/4) = \begin{pmatrix} 0 \\ 1 \\ \pi/4 \end{pmatrix} = \bar{r}_0$

plane $\quad \bar{n} \cdot (\bar{r} - \bar{r}_0) = 0$

$\frac{1}{\sqrt{5}} \left(1(x-0) + 0(y-1) + 2(z - \frac{\pi}{4}) \right) = 0$

$x + 2(z - \frac{\pi}{4}) = 0$

$x + 2z = \frac{\pi}{2}$

$B = T \times N$

---

2 curves $\bar{r}(t)$, $\bar{s}(t)$

$\Rightarrow$ consider $f(t) = \bar{r}(t) \cdot \bar{s}(t)$

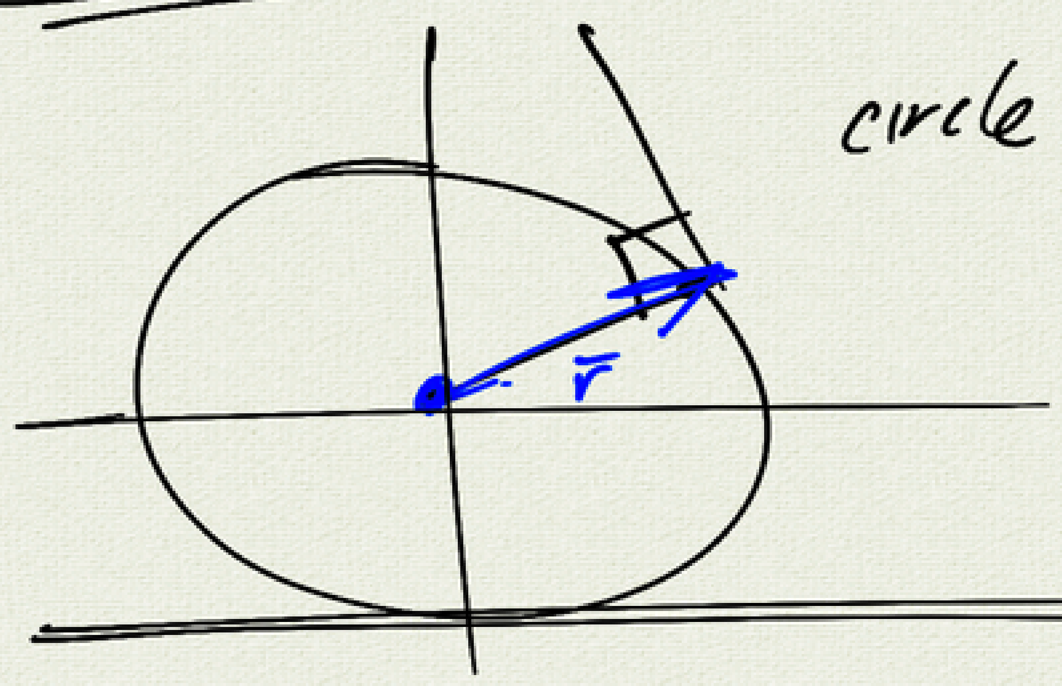
$\Rightarrow f'(t) = ?$

$= \bar{r}'(t) \cdot s(t) + \bar{r}(t) \cdot \bar{s}'(t)$

product rule

$$\bar{r}(t) = \begin{pmatrix} x_r(t) \\ y_r(t) \\ z_r(t) \end{pmatrix} \quad \bar{s}(t) = \begin{pmatrix} x_s(t) \\ y_s(t) \\ z_s(t) \end{pmatrix}$$

$$f(t) = \bar{r}(t) \cdot \bar{s}(t) = x_r(t) x_s t + y_r(t) y_s(t) + z_r(t) z_s(t)$$



circle: tangent $\perp$ radius

$\bar{r}(t)$ curve in $\mathbb{R}^2$

$|\bar{r}(t)| = 1$

$|\bar{r}(t)|^2 = \bar{r}(t) \cdot \bar{r}(t)$

$\Rightarrow \frac{d}{dt} |\bar{r}(t)|^2 = \frac{d}{dt}\left(\bar{r}(t) \cdot \bar{r}(t)\right)$

$= \bar{r}'(t) \cdot \bar{r}(t) + \bar{r}(t) \cdot \bar{r}'(t)$

$= 2\, \bar{r} \cdot \bar{r}'$

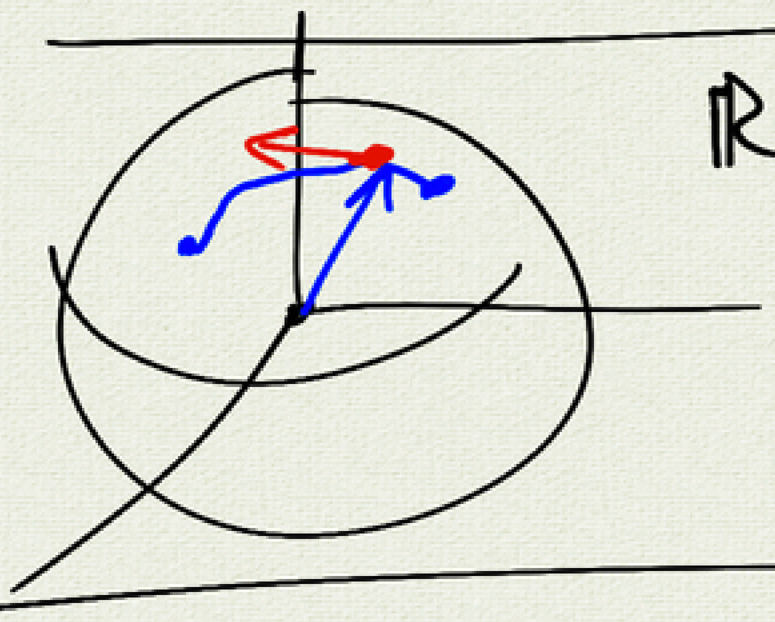
Suppose

$|\bar{r}| = 1$ (const)

$\Rightarrow \bar{r} \cdot \bar{r}' = 0$

curve $\perp$ tangent

not rely 2 dimension



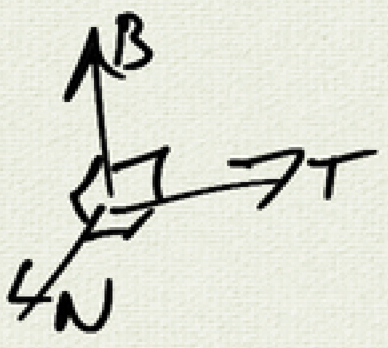
$\mathbb{R}^3$

$|\bar{r}| = 1$

$T(t) = \dfrac{\bar{r}'(t)}{|\bar{r}'(t)|}$ unit vector | T is on the unit sphere

$\rightarrow |T| = 1 \Rightarrow T' \cdot T = 0$

$\Rightarrow N \cdot T = 0$



2.5 Motion

$\bar{r} : \mathbb{R} \xrightarrow[\text{curve}]{} \mathbb{R}^3$

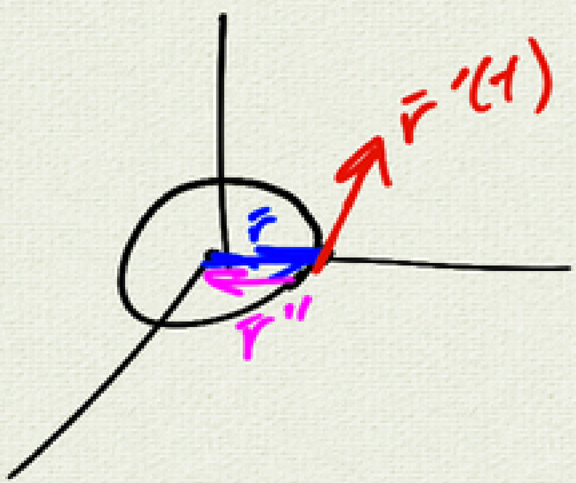
$\bar{r}(t) =$ position

$\bar{r}'(t) =$ velocity

$|\bar{r}'(t)| =$ speed

$\bar{r}''(t) =$ acceleration

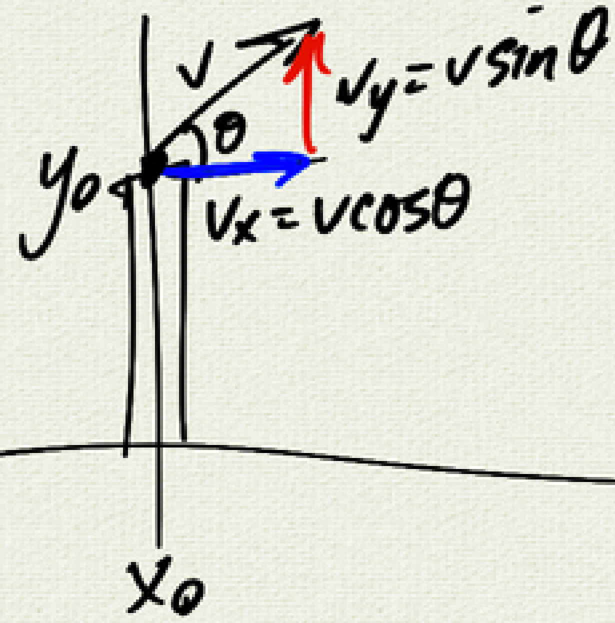
<u>example</u>: $\bar{r}(t) = \begin{pmatrix} \cos t \\ \sin t \\ 0 \end{pmatrix}$



$\bar{r}'(t) = \begin{pmatrix} -\sin t \\ \cos t \\ 0 \end{pmatrix}$

$\bar{r}''(t) = \begin{pmatrix} -\cos t \\ -\sin t \\ 0 \end{pmatrix} = -\bar{r}(t)$

projectile motion



equations of motion $\bar{r}(t)$

assumption:

gravity $\quad \bar{r}''(t) = \begin{pmatrix} 0 \\ g \end{pmatrix}$ const

$$\bar{r}''(t) = \begin{pmatrix} 0 \\ g \end{pmatrix}$$

initial velocity

$$\bar{r}'(t) = \begin{pmatrix} c_1 \\ gt + c_2 \end{pmatrix} \qquad c_1 \to v_x, \quad c_2 \to v_y$$

$$= \begin{pmatrix} v_x \\ gt + v_y \end{pmatrix}$$

initial position

$$\bar{r}(t) = \begin{pmatrix} v_x t + c_3 \\ \frac{1}{2} g t^2 + v_y t + c_4 \end{pmatrix}$$

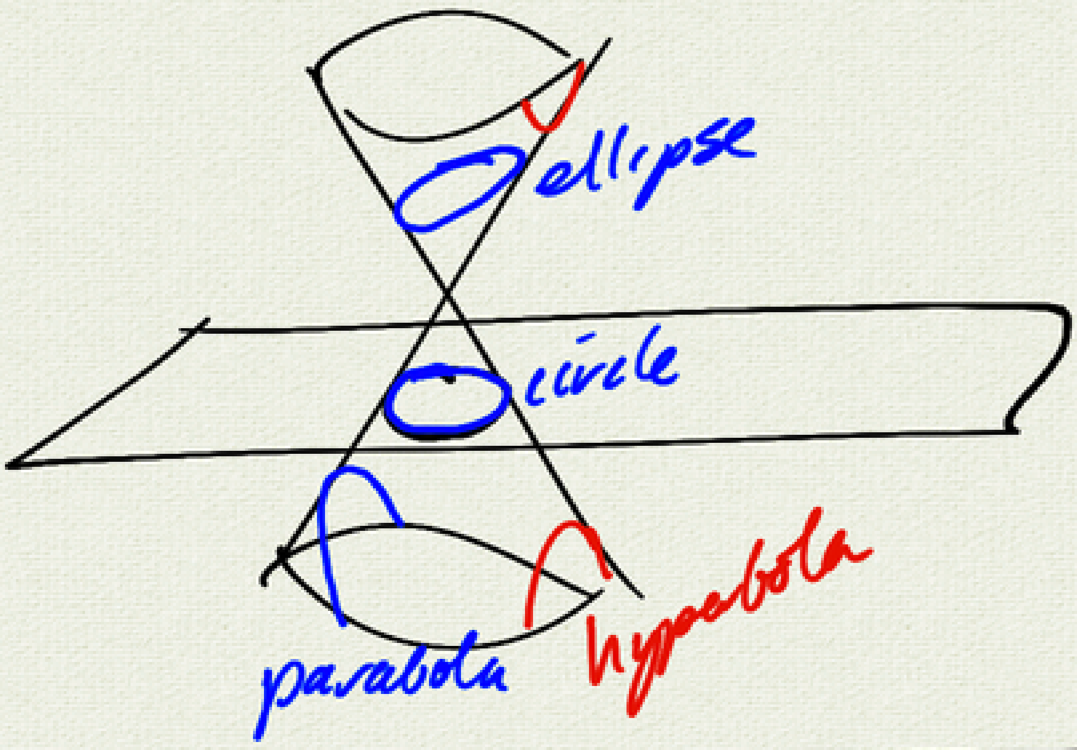
$$\bar{r}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} v_x t + x_0 \\ \frac{1}{2} g t^2 + v_y t + y_0 \end{pmatrix}$$

---

gravity $\quad -32 \text{ ft/s}^2$

$\quad -9.8 \text{ m/s}^2$

Conic sections



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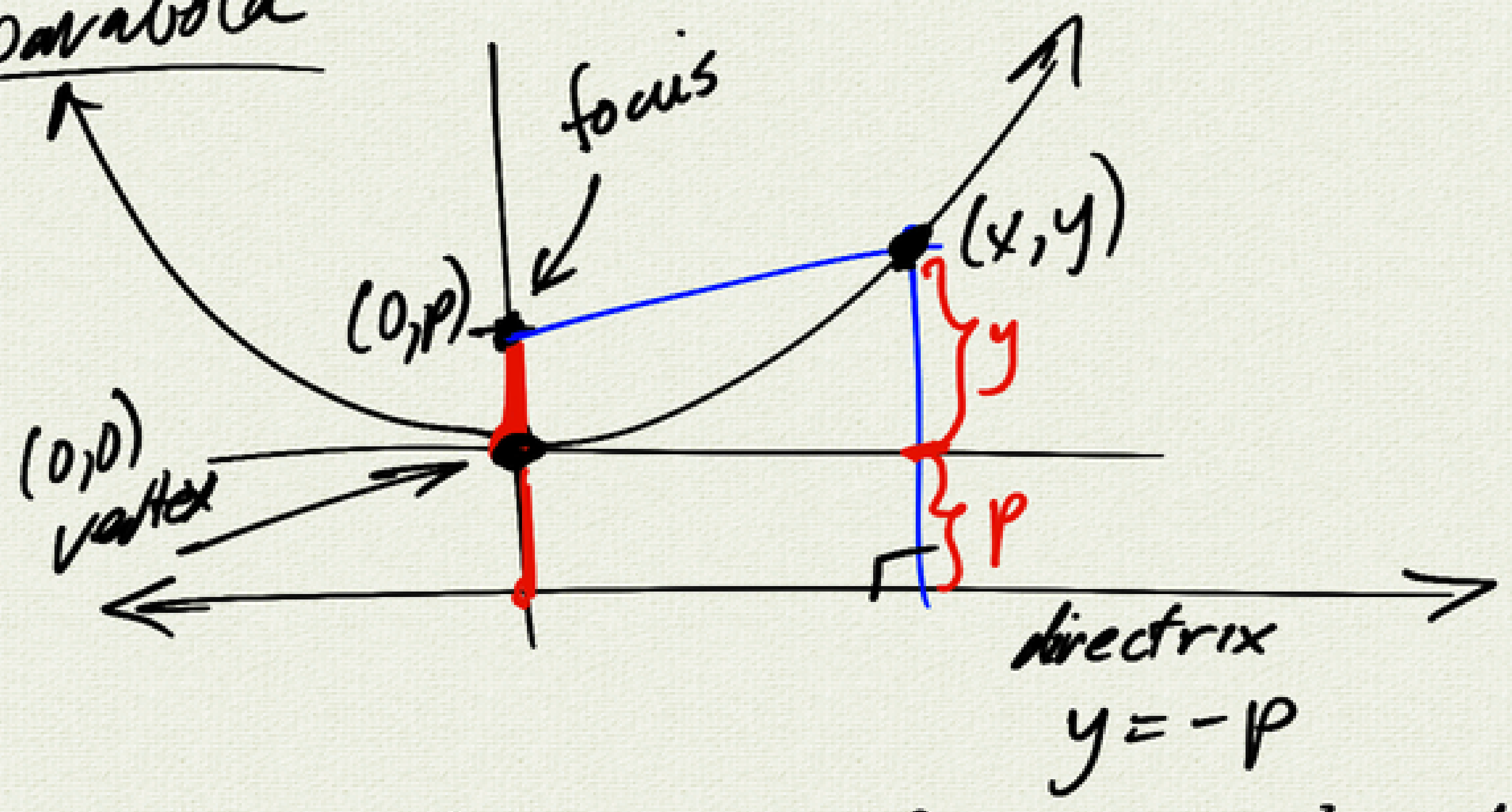
parabola



geometric def: distance to focus = distance to directrix

$$\sqrt{(x-0)^2+(y-p)^2} = y+p$$

$$x^2 + (y-p)^2 = (y+p)^2$$

$$x^2 + (y^2 - 2py + p^2) = (y^2 + 2py + p^2)$$

$$x^2 = 4py$$

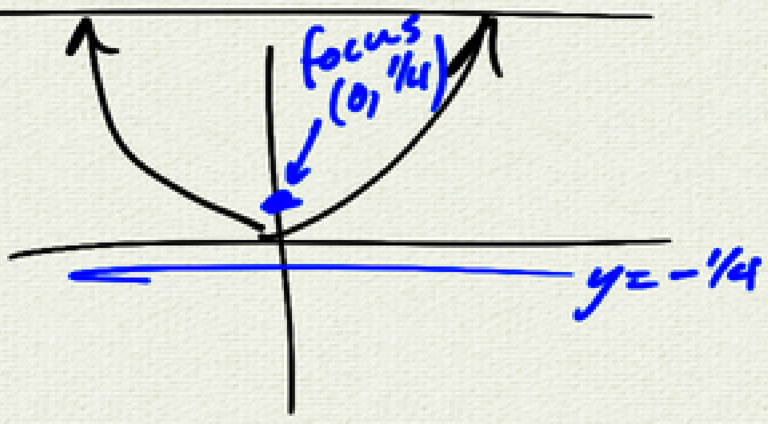
$$y = \frac{1}{4p}x^2$$

---

example:

$$y = x^2$$

$p = \frac{1}{4}$



$$y = \boxed{\frac{1}{4p}}x^2$$

$$\frac{1}{4p} = 1$$

$$p = \frac{1}{4}$$

$$\boxed{y - k = \frac{1}{4p}(x-h)^2}$$

vertex at (h, k)

circle

$$(x-h)^2+(y-k)^2=r^2$$

(h,k) center

r radius

ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \left(\begin{matrix} a=b \Rightarrow \\ \text{circle}\end{matrix}\right)$$

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

$y=0 \Rightarrow$

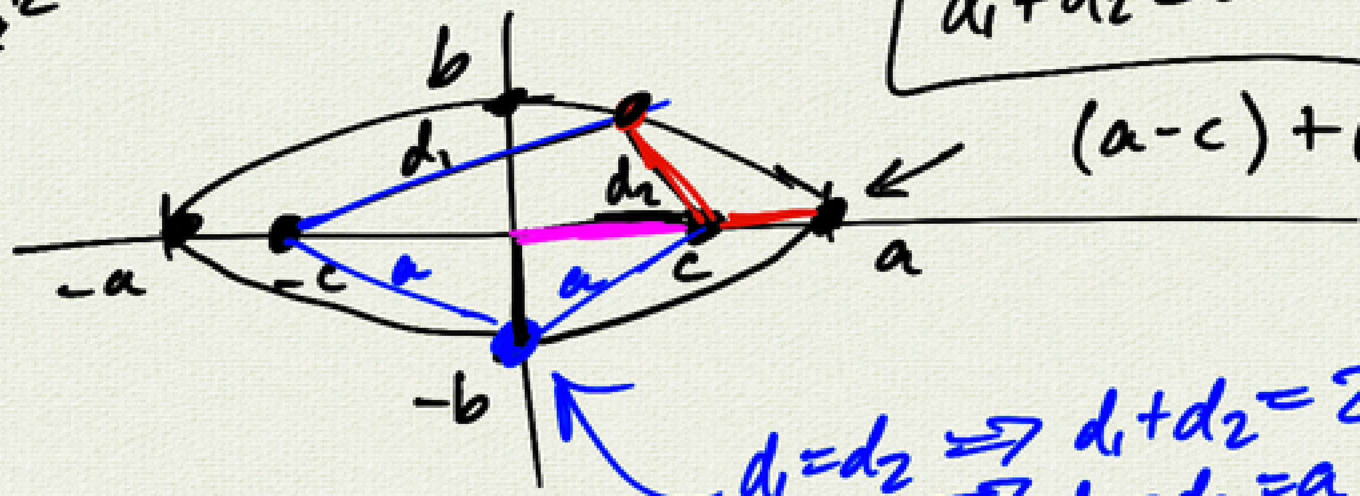
$\frac{x^2}{a^2}=1$

$x^2=a^2$

$x=\pm a$

$$\boxed{d_1+d_2=\text{const}}$$ geometric definition

$(a-c)+(a+c)=2a=\text{const}$



$d_1=d_2 \Rightarrow d_1+d_2=2a$

$\Rightarrow d_1=d_2=a$

$$a^2=b^2+c^2$$

$$c^2=a^2-b^2 \qquad \left(\text{find } c \text{ from } a, b\right)$$

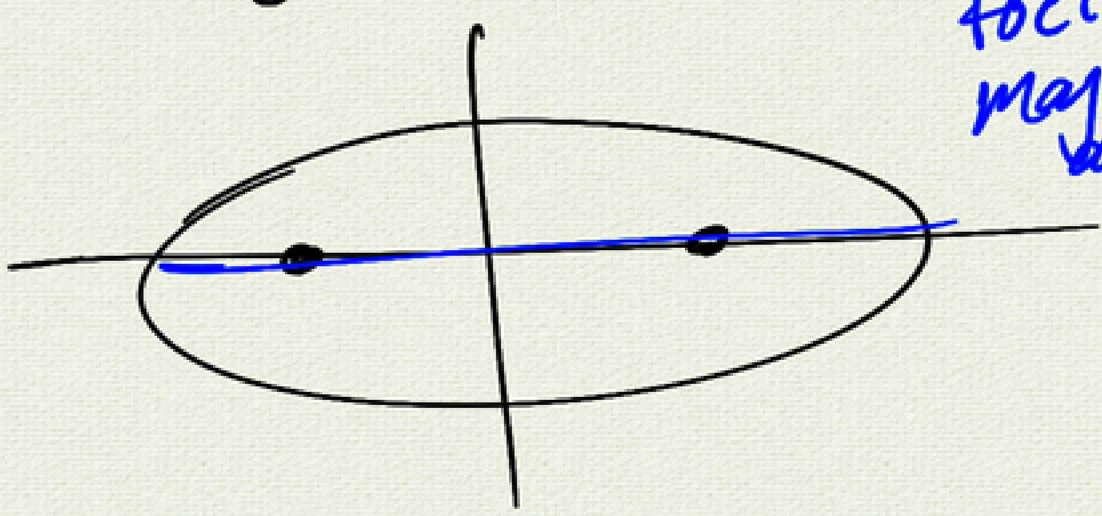
$$\text{eccentricity}=\frac{c}{a}<1 \quad (\text{for ellipse})$$

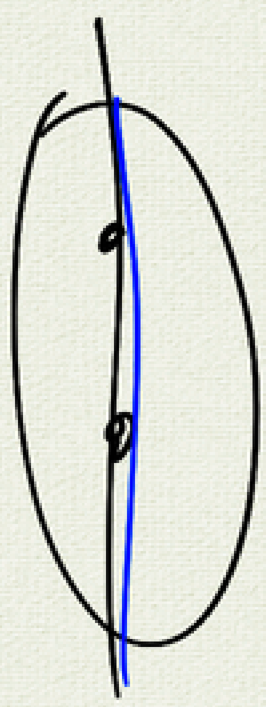
$e=0$

circle

e almost 1

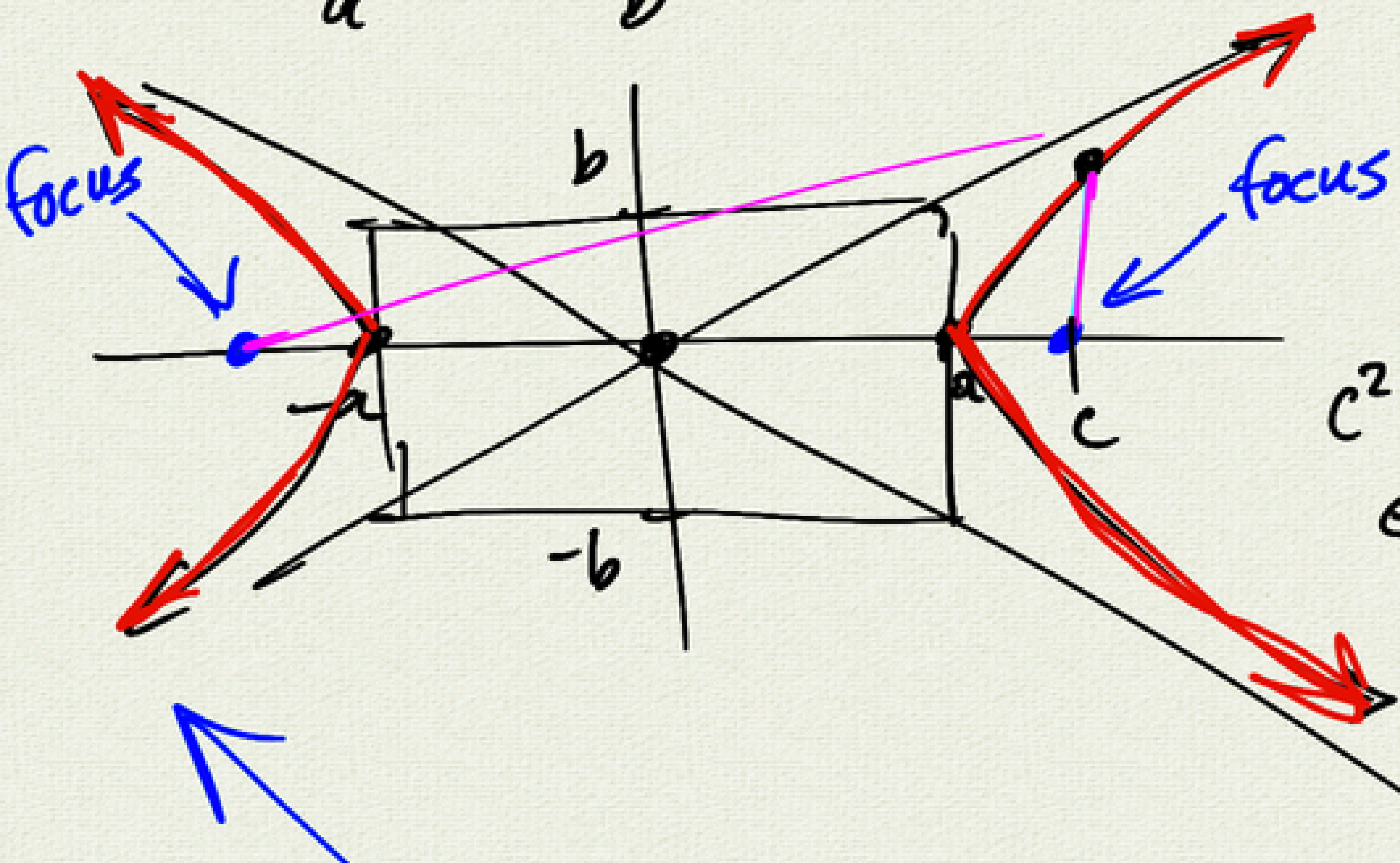
foci on major axis





Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



$c^2 = a^2 + b^2$

$e = \frac{c}{a} > 1$