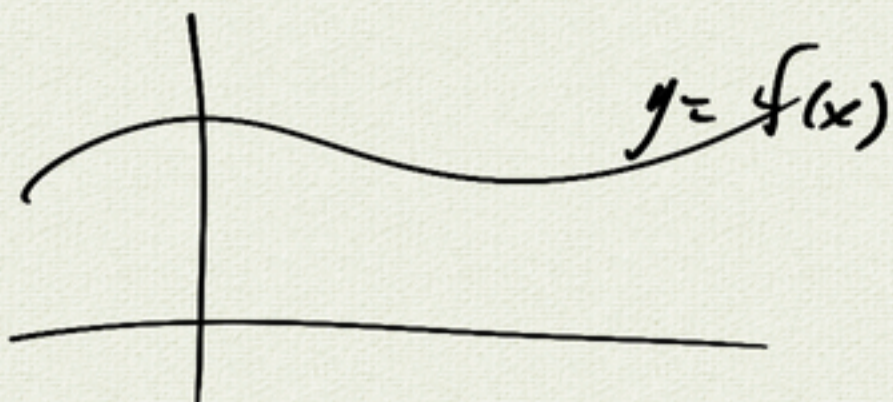
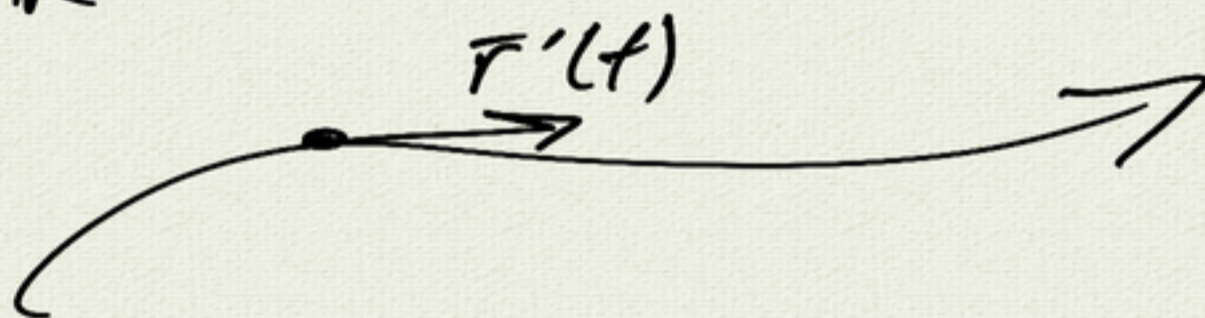


3.1 Functions of Several Variables

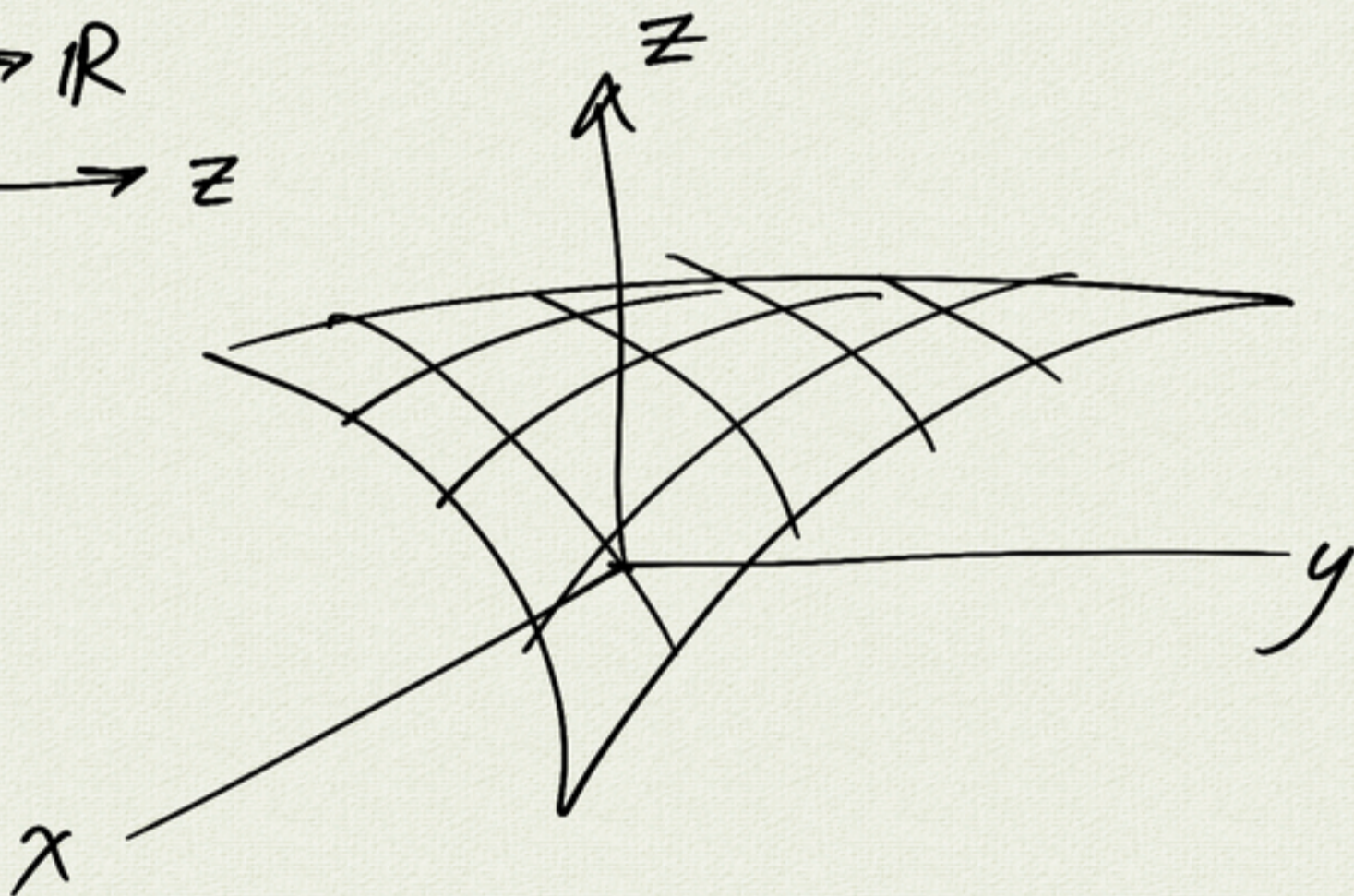
so far: $f: \mathbb{R} \rightarrow \mathbb{R}$



$F: \mathbb{R} \rightarrow \mathbb{R}^3$



$f: \mathbb{R}^2 \rightarrow \mathbb{R}$
 $(x, y) \mapsto z$

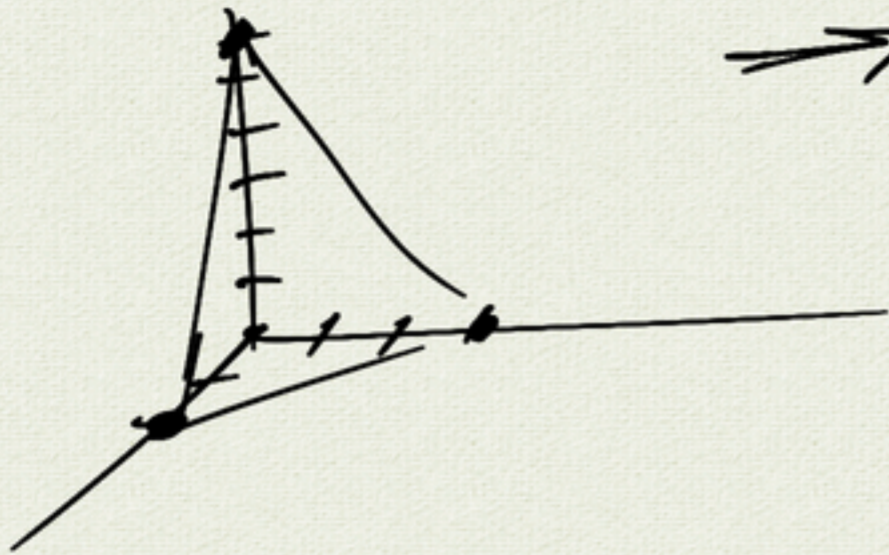


$f: \mathbb{R}^3 \rightarrow \mathbb{R}$
 $(x, y, z) \rightarrow \text{value}$
 w

example: temperature
in a room

example: plane

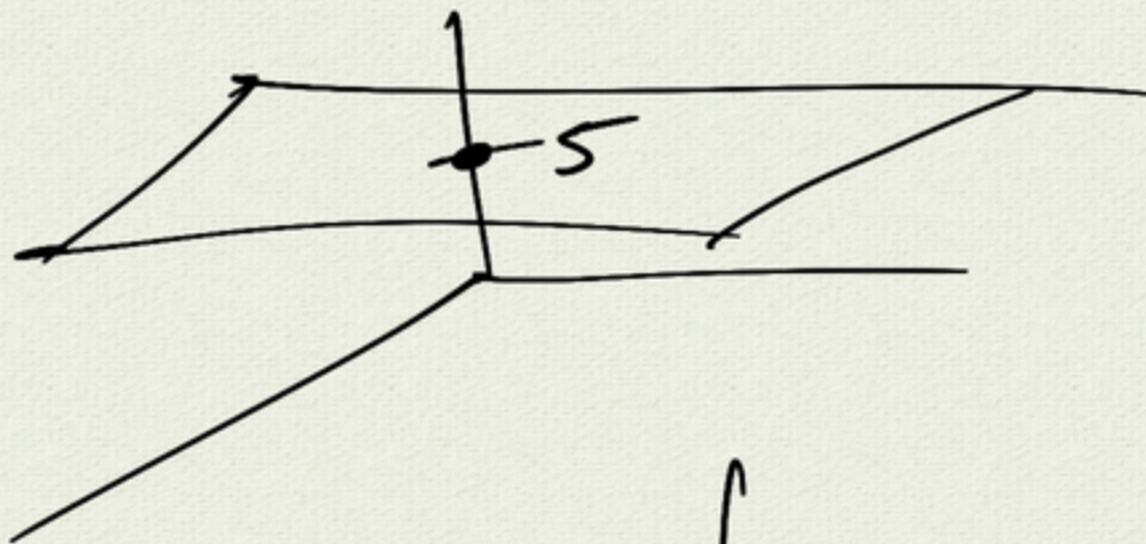
$$3x + 2y + z = 6$$



$$\begin{aligned} \Rightarrow z &= 6 - 3x - 2y \\ &= f(x, y) \end{aligned}$$

examples:

plane $f(x, y) = 5$



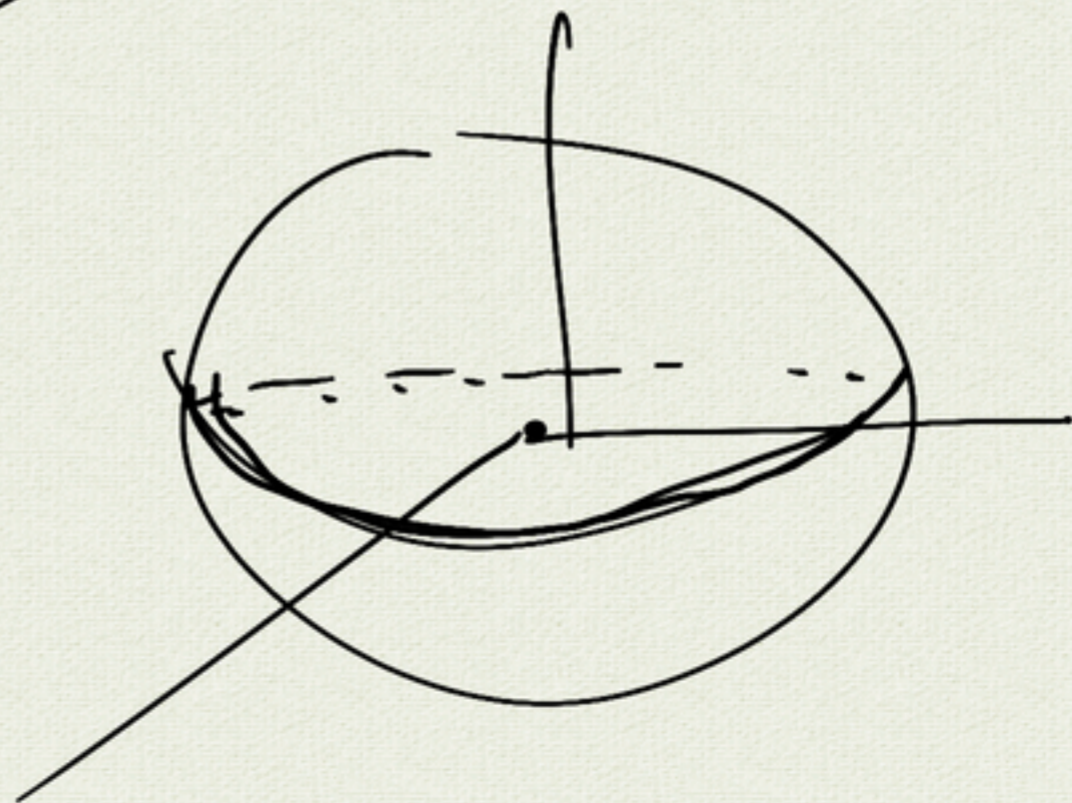
sphere

$$x^2 + y^2 + z^2 = 9$$

$$z^2 = 9 - x^2 - y^2$$

$$z = \pm \sqrt{9 - x^2 - y^2}$$

↑ top / bottom



$$z = \sqrt{x^2 + y^2}$$

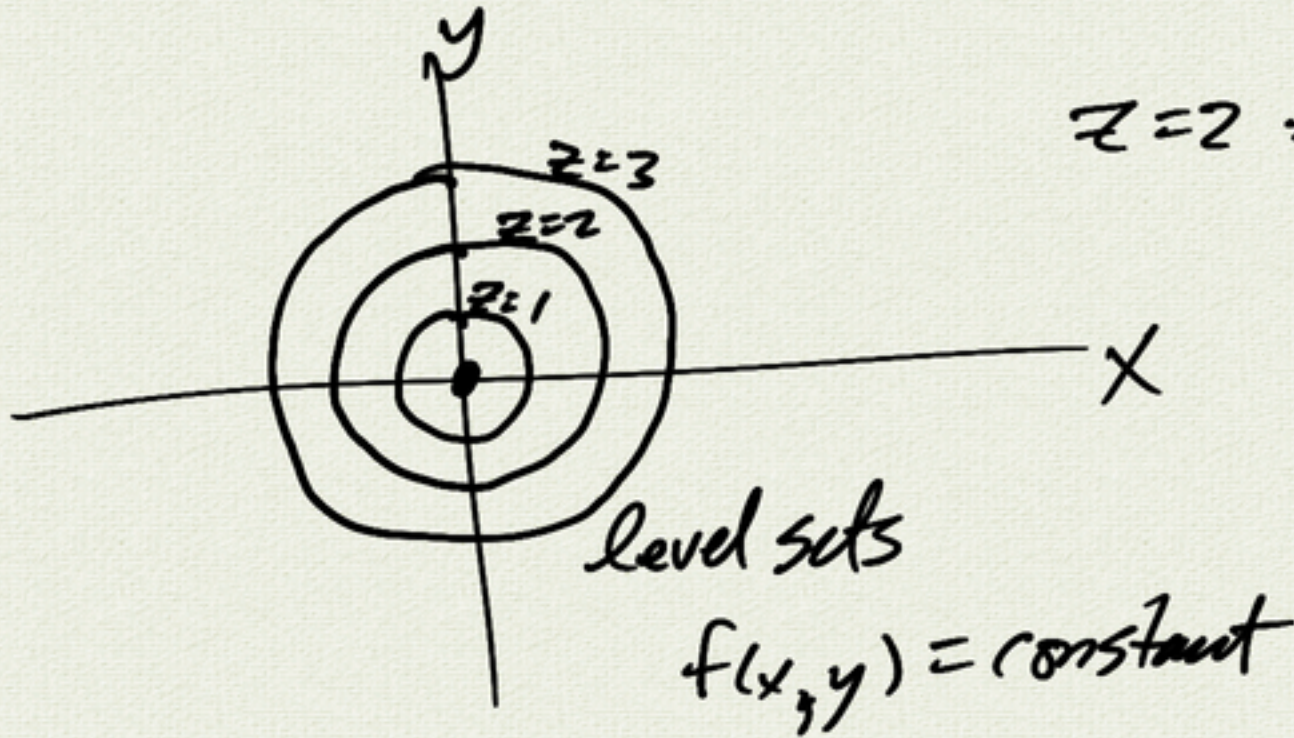
$$z = 0$$

$$z = 1 \Rightarrow \sqrt{x^2 + y^2} = 1$$

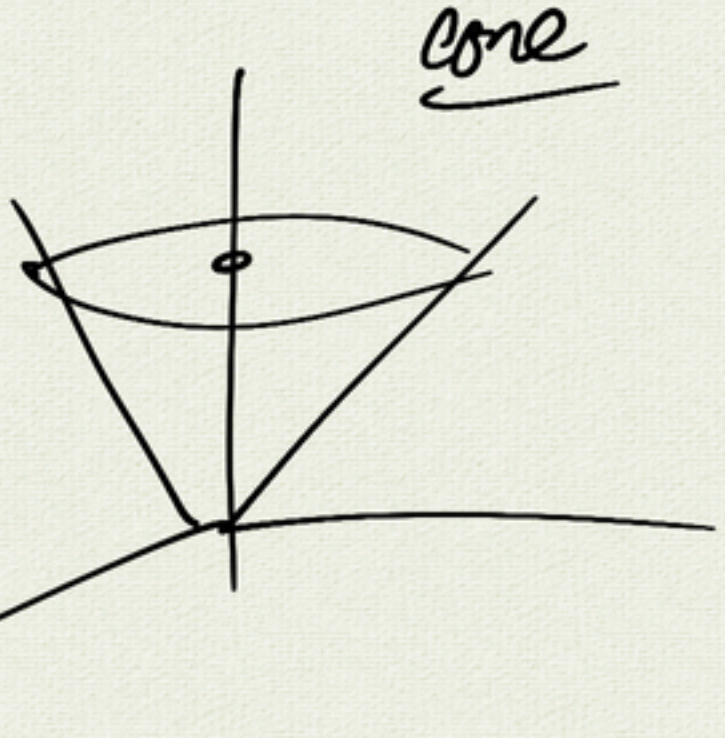
$$x^2 + y^2 = 1$$

$$z = 2 = \sqrt{x^2 + y^2} = 2$$

$$x^2 + y^2 = 4$$



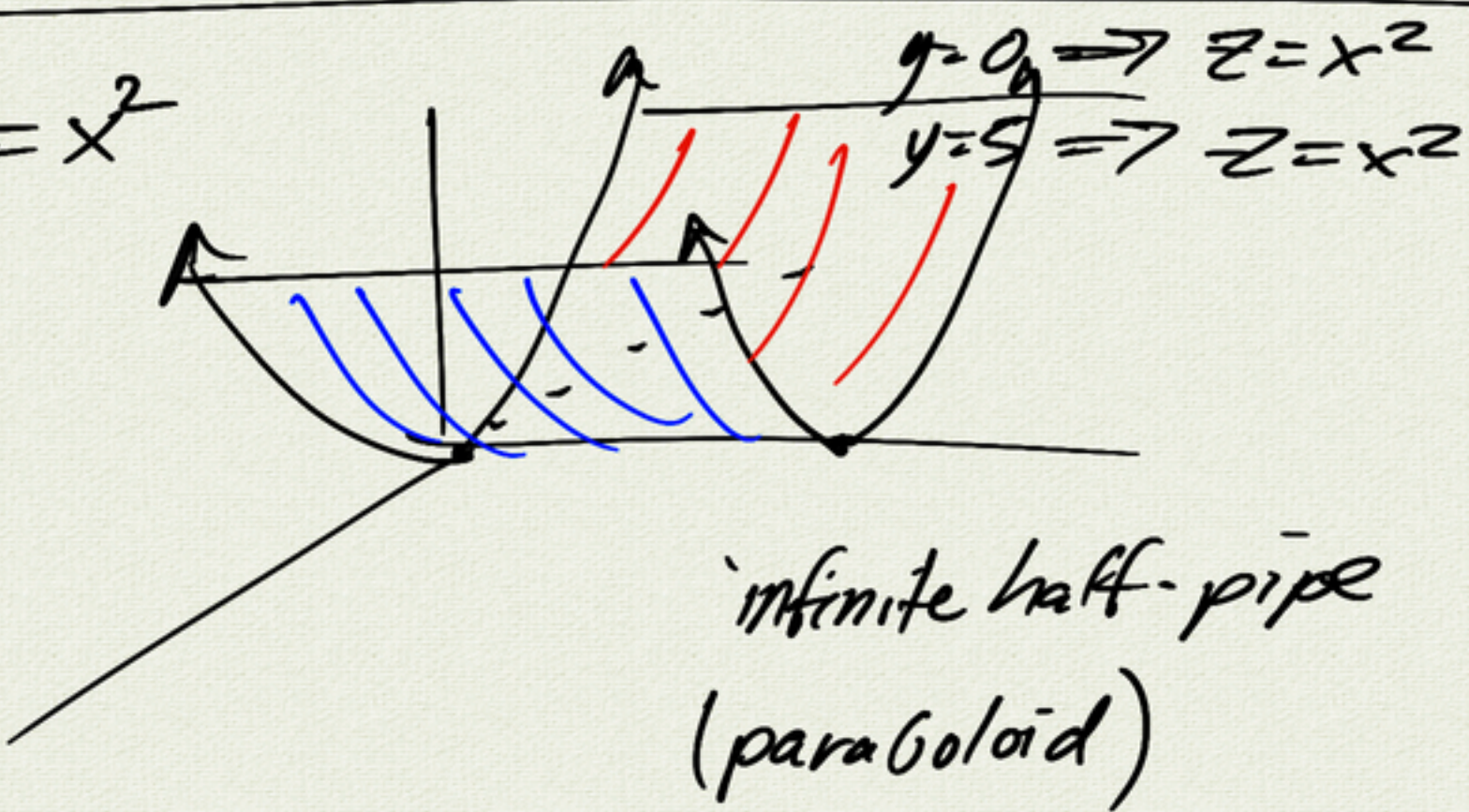
$$z = \sqrt{x^2 + y^2} = r$$



$$f(x,y) = x^2$$

$$y = 0 \Rightarrow z = x^2$$

$$y = 5 \Rightarrow z = x^2$$

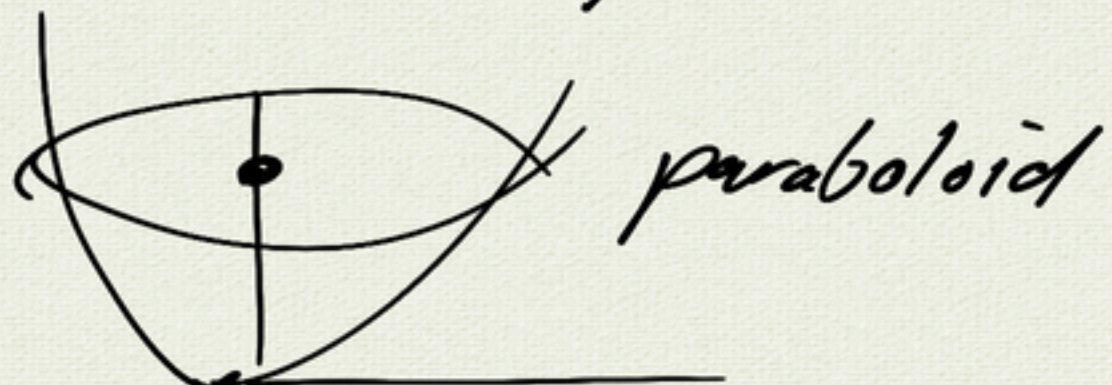
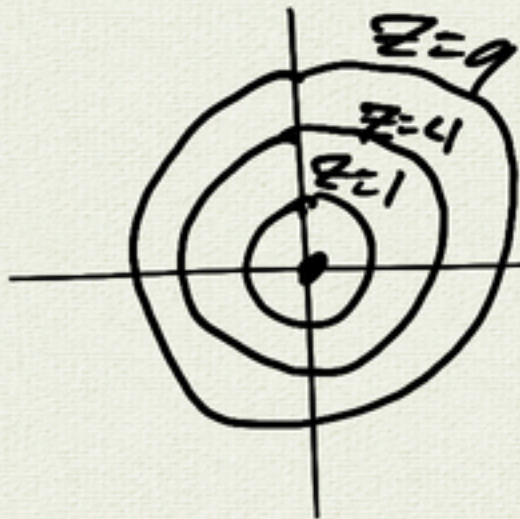


$$f(x,y) = x^2 + y^2$$

$$z = 1 \Rightarrow x^2 + y^2 = 1$$

$$z = 4 \Rightarrow x^2 + y^2 = 4$$

$$z = 9 \Rightarrow x^2 + y^2 = 9$$

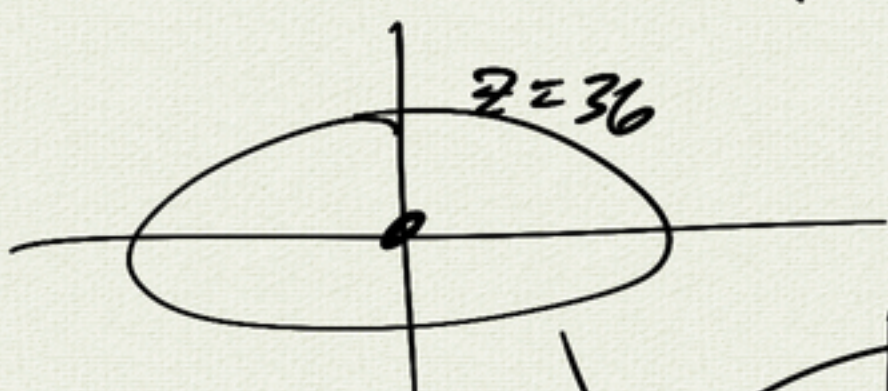


$$f(x,y) = 4x^2 + 9y^2$$

level sets: $z = 1$ $4x^2 + 9y^2 = 1$

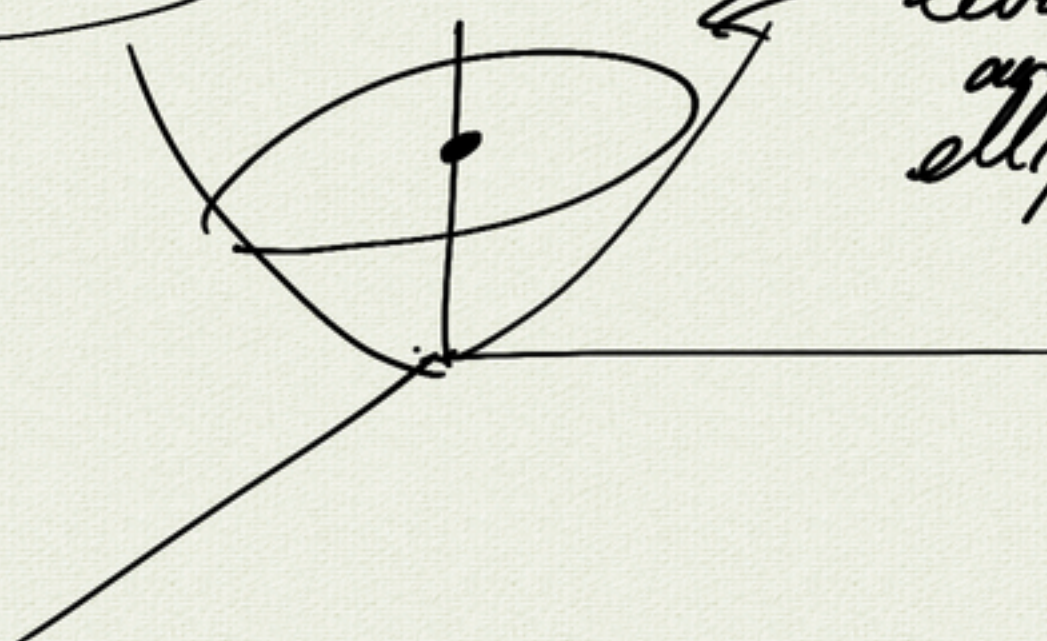
$z = 36$ $4x^2 + 9y^2 = 36$

$$\frac{x^2}{9} + \frac{y^2}{4} = 1 \text{ ellipse}$$



elliptic paraboloid

level sets are ellipses



$$f(x, y, z) = x^2 + y^2 + z^2$$

level sets: $f=0$ $\{(0,0,0)\}$

$f(x, y, z)=1$ $x^2 + y^2 + z^2 = 1$
Sphere radius 1

$f=4$ $x^2 + y^2 + z^2 = 4$
Sphere radius 2

z $f(x, y) = x^2 - y^2$

$z=0$ $x^2 - y^2 = 0$

$$x^2 = y^2$$

$$x = \pm y$$

$z=1$ $x^2 - y^2 = 1$

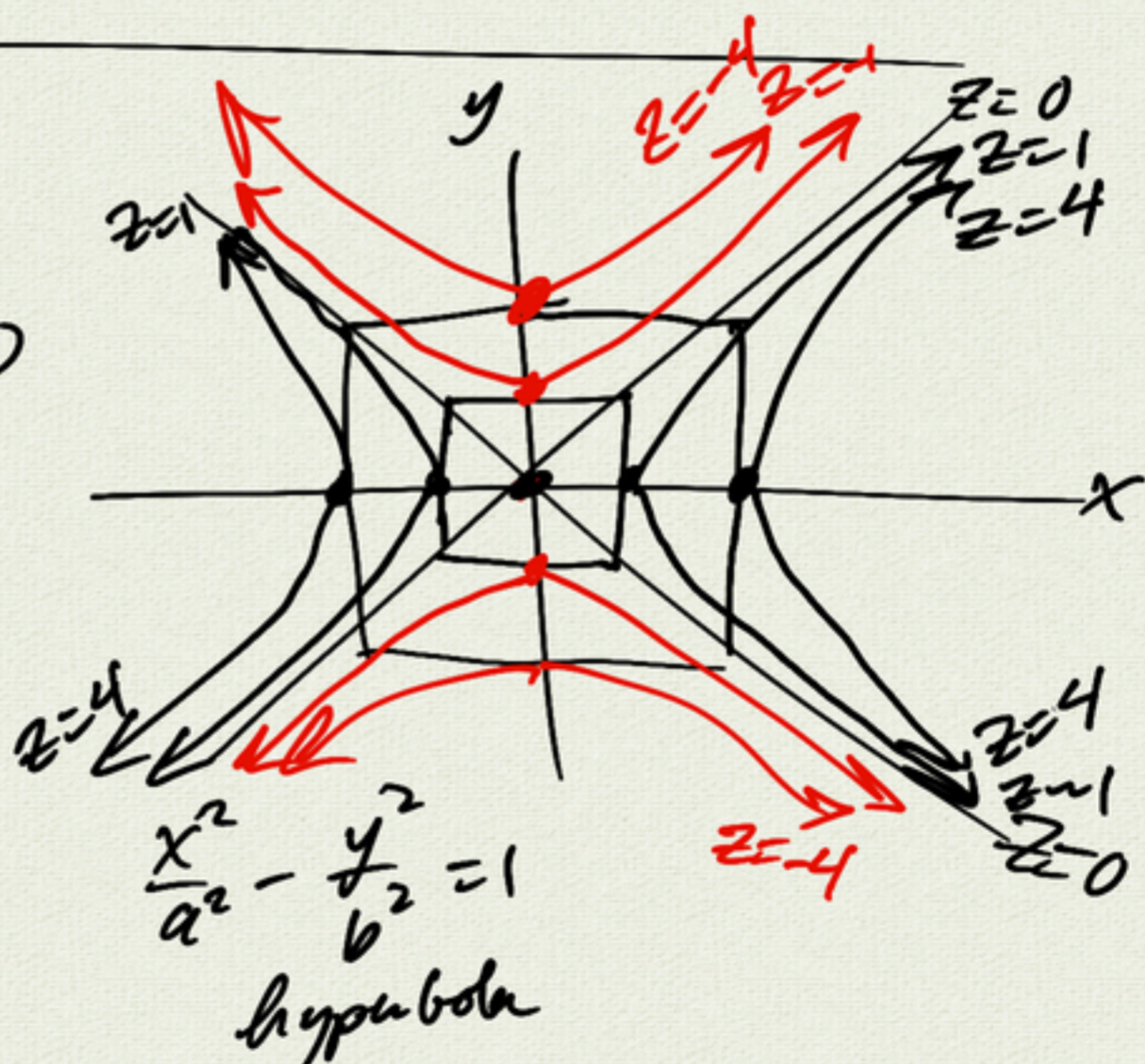
$z=4$ $x^2 - y^2 = 4$

$$\frac{x^2}{4} - \frac{y^2}{4} = 1$$

$z=-1$ $x^2 - y^2 = -1$

$$y^2 - x^2 = 1$$

$z=-4$ $y^2 - x^2 = 4$



hyperbolic
paraboloid