(65) $\lim_{(x,y)\to(0,1)} \frac{y^2 \sin x}{x}$

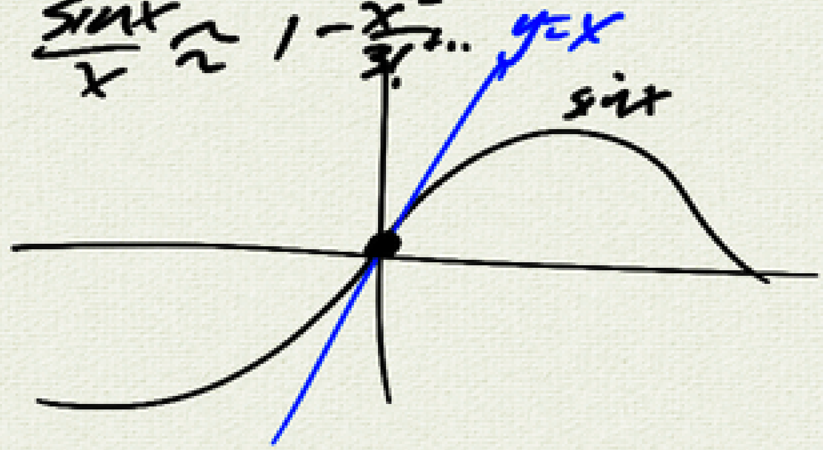
$= \left(\lim_{x\to 0} \frac{\sin x}{x}\right)\left(\lim_{y\to 1} y^2\right) = 1$

$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$

$\Longleftrightarrow \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$

$\sin x \simeq x$

$\frac{\sin x}{x} \simeq 1 - \frac{x^2}{3!} \dots$

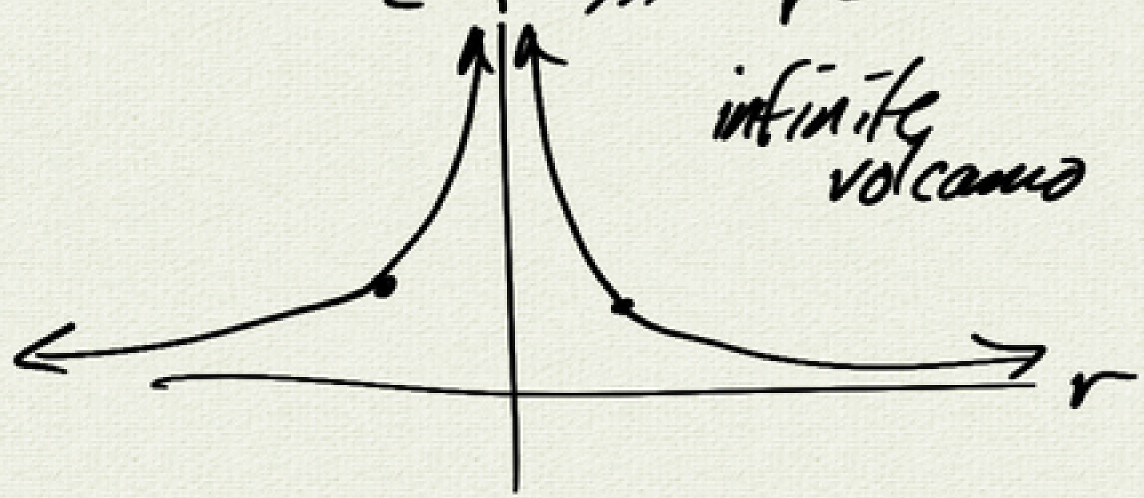


$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\cos x}{1} = 1$

(103) $f(x,y) = \frac{1}{x^2+y^2}$

$= \frac{1}{r^2}$

$z = f(x,y) = \frac{1}{r^2}$

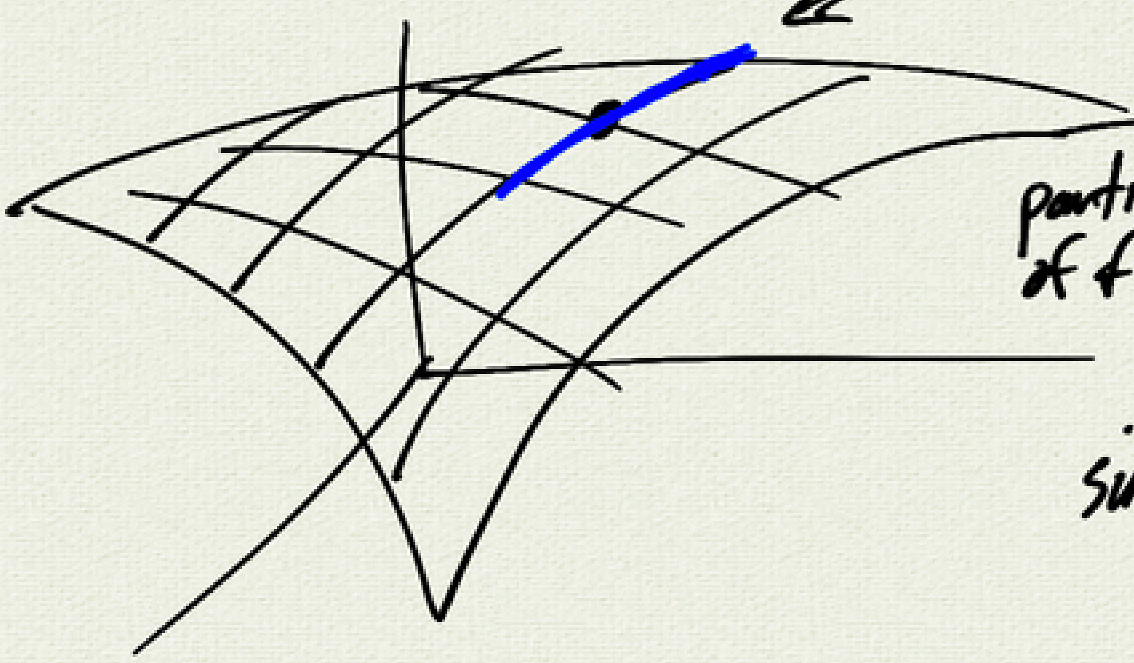


3.3 Partial derivatives

$z = f(x,y)$ surface

keep y constant

$$\boxed{\frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}}$$



partial derivative of f with respect to x

similar: $\frac{\partial f}{\partial y}$

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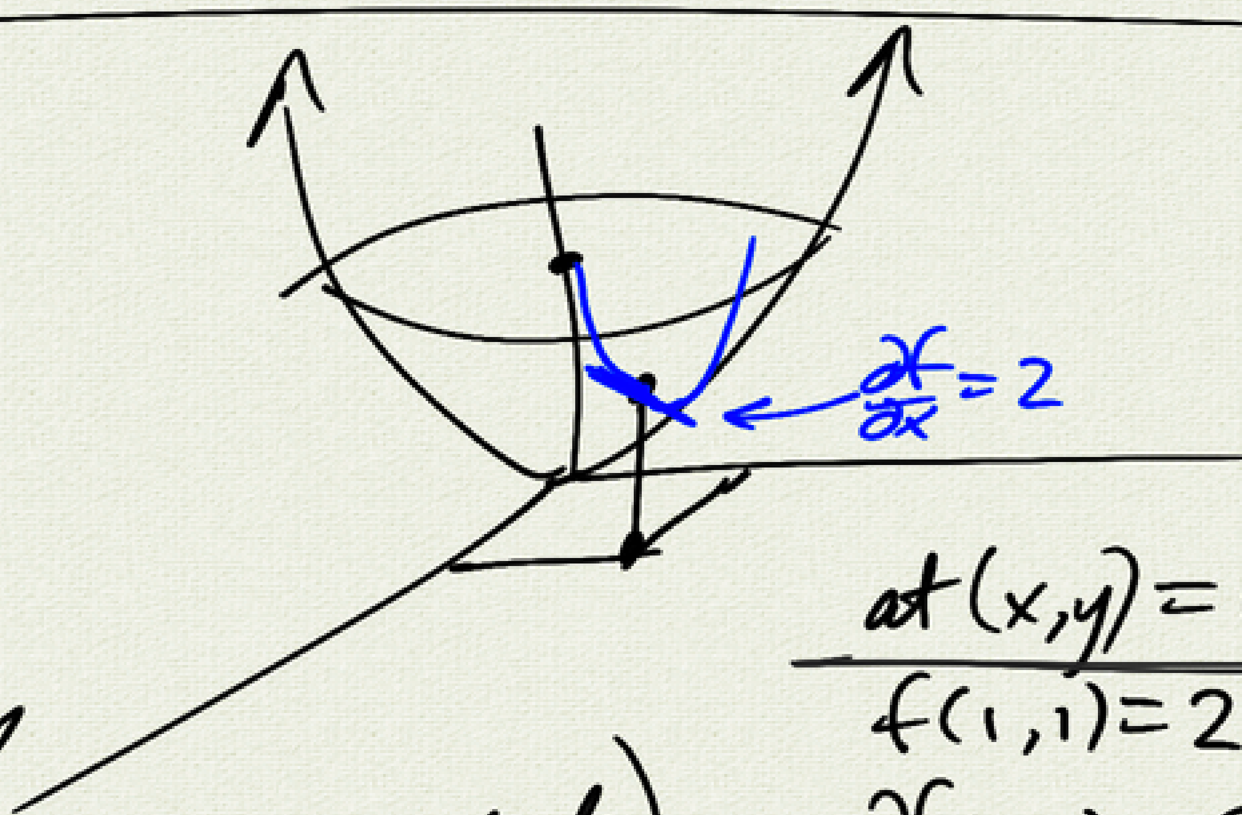
examples

$f(x,y) = x^2 + y^2$

$z = f(x,y)$

paraboloid



partial derivatives

$\frac{\partial f}{\partial x} = 2x$ (treat y as constant)

$\frac{\partial f}{\partial y} = 2y$

at $(x,y) = (1,1)$

$f(1,1) = 2$

$\frac{\partial f}{\partial x}(1,1) = 2$

$\frac{\partial f}{\partial y}(1,1) = 2$

at $(x,y) = (0,0)$:

$f(0,0) = 0$

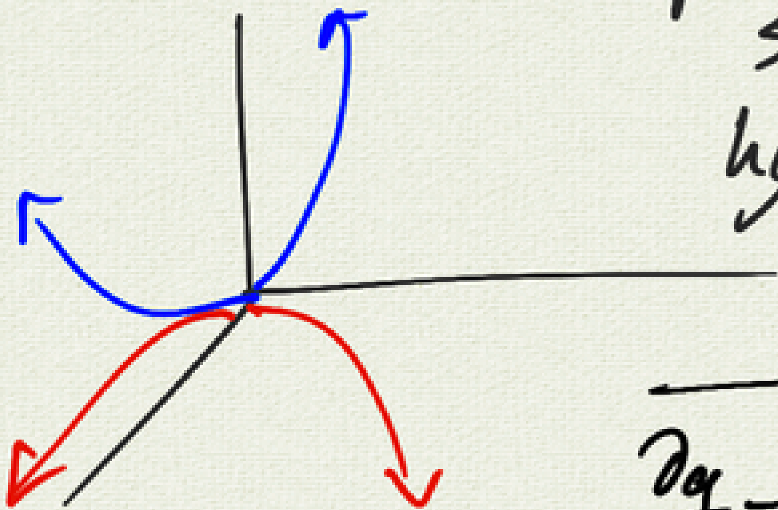
$\frac{\partial f}{\partial x} = 0$

$\frac{\partial f}{\partial y} = 0$

(local minimum)

$$g(x,y) = x^2 - y^2$$

pringle / saddle / hyperbolic paraboloid



$$\frac{\partial g}{\partial x} = 2x$$

$$\frac{\partial g}{\partial y} = -2y$$

<u>at $(0,0)$</u>:

$$\frac{\partial g}{\partial x} = 0 = \frac{\partial g}{\partial y}$$

<u>not</u> a local min

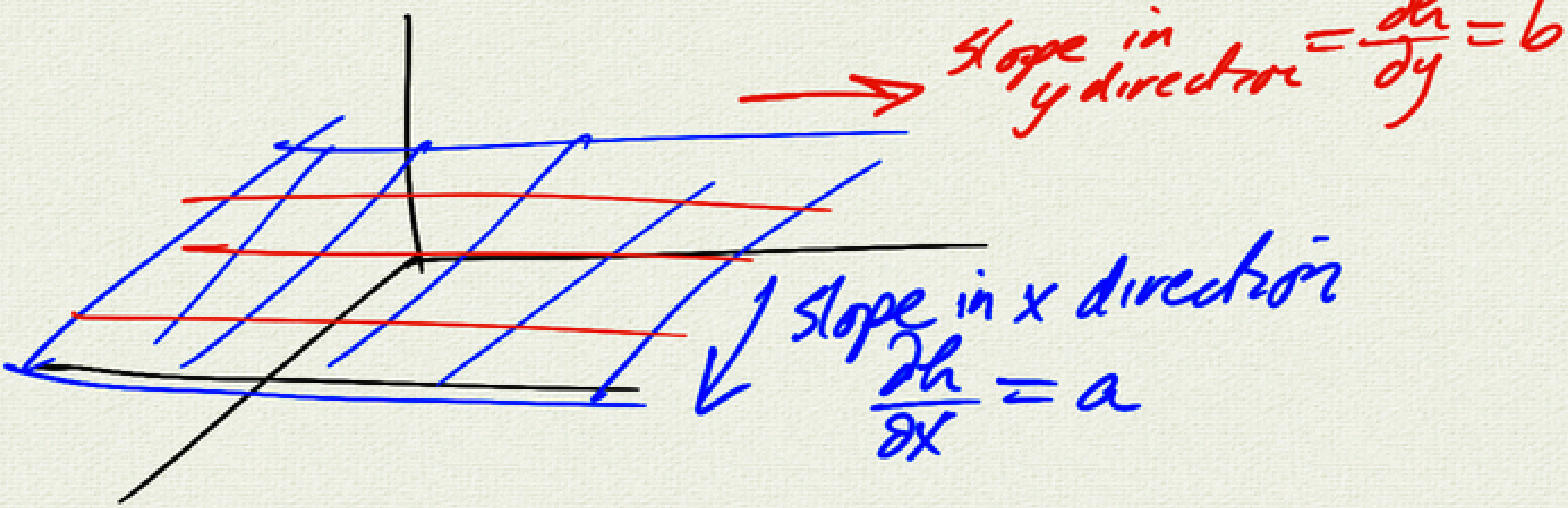
saddle point

$$h(x,y) = ax + by$$

$$z = ax + by \quad \text{plane} \quad (ax + by - z = 0)$$

$$\frac{\partial h}{\partial x} = \frac{\partial z}{\partial x} = a \quad (= h_x) \text{ notation}$$

$$\frac{\partial z}{\partial y} = b \quad (= h_y)$$



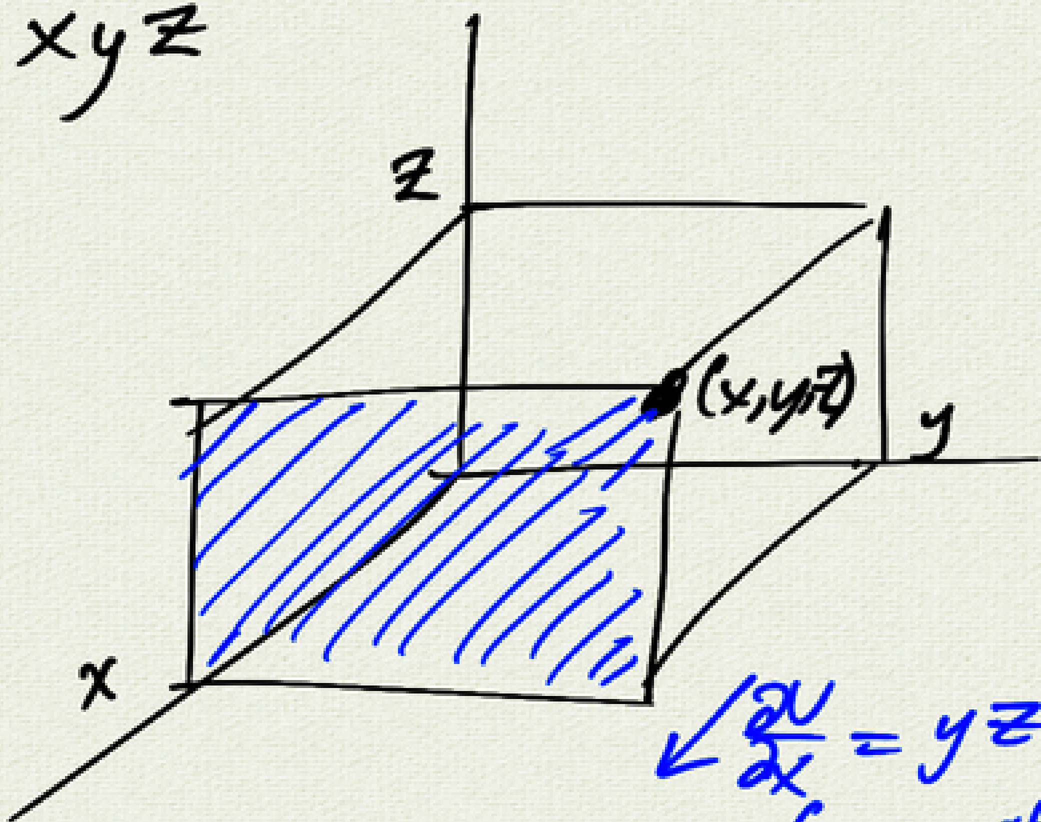
volume of rectangular prism

$$V(x,y,z) = xyz$$

$$\boxed{\frac{\partial V}{\partial x} = yz}$$

$$\frac{\partial V}{\partial y} = xz$$

$$\frac{\partial V}{\partial z} = xy$$



$$\frac{\partial V}{\partial x} = yz$$

(area of face)