Linear Transformations Homework
MultiV 2021-22 / Dr. Kessner

First, some notation: let $R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ represent rotation in the plane.

For example, $R_{\frac{\pi}{6}}$ means rotation by $\frac{\pi}{6}$:

$$R_{\frac{\pi}{6}} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

1. Apply each of the following matrices to the specified vectors. Think about how the vectors are transformed, and make sure your answers make sense. Describe the linear transformation represented by the matrix.

a. $S = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ applied to $\mathbf{i}, \mathbf{j}$, and $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

b. S^2 applied to $\mathbf{i}, \mathbf{j}$, and $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

c. $R_{\frac{\pi}{6}}$ applied to $\mathbf{i}, \mathbf{j}$, and $\begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$

d. $R^2_{\frac{\pi}{6}}$ applied to $\mathbf{i}, \mathbf{j}$, and $\begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$

e. $R^3_{\frac{\pi}{6}}$ applied to $\mathbf{i}, \mathbf{j}$, and $\begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$

f. $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ applied to $\mathbf{i}, \mathbf{j}$, and $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

g. A^2 applied to $\mathbf{i}, \mathbf{j}$, and $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

h. $P = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$ applied to $\mathbf{i}, \mathbf{j}, \mathbf{k}$, and $\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$

i. $C = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ applied to $(0), (1), (2)$. Yes, these are 1×1 matrices, otherwise known as real numbers.

j. PC applied to $(0), (1), (2)$

2. Verify the following:

$$R^{-1}_{\frac{\pi}{6}} = R_{-\frac{\pi}{6}}$$

$$R^2_{\frac{\pi}{4}} = R_{\frac{\pi}{2}}$$

$$R^2_{\frac{\pi}{2}} = R_\pi$$

$$R^2_\pi = I$$

$$R_\theta R_\phi = R_{\theta+\phi}$$