229 $u = e^x \sin y$ $\quad x = -\ln 2t$ $\quad y = \pi t$ $\quad$ at $(\ln 2, \pi/4)$

$t = \frac{1}{4}$

$\ln(xy) = \ln x + \ln y$

$x(\frac{1}{4}) = -\ln(\frac{2}{4})$
$= -\ln(\frac{1}{2})$
$= \ln 2$

$\log_{10}(100) = 2$

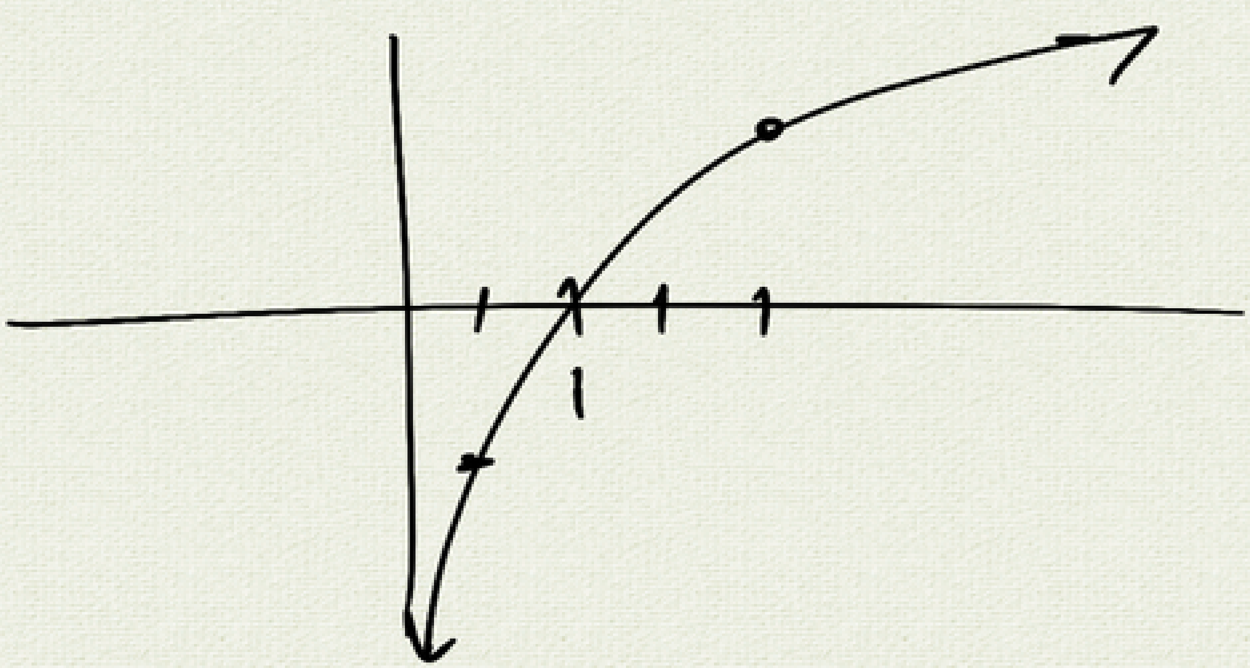
$\log_{10}(1000) = 3$

$\log_{10}(100{,}000) = 5$
$= 10^5$

$\log_{10}(\underset{2}{100} \cdot \underset{3}{1000}) = 2 + 3$

$\ln(x^n) = n \ln x$

$\ln(\frac{1}{2}) = \ln(2^{-1})$
$= -\ln 2$



(229) $\boxed{u = e^x \sin y}$ $x = -\ln 2t$ at $x = \ln 2$

$y = \pi t$ $y = \pi/4$

$\boxed{t = 1/4}$

find $\frac{du}{dt}\left(\frac{1}{4}\right)$

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

$$= \underbrace{e^x}_{2}\underbrace{\sin y}_{\sqrt{2}/2}\underbrace{\left(-\frac{1}{t}\right)}_{-4} + \underbrace{e^x}_{2}\underbrace{\cos y}_{\sqrt{2}/2}\,\pi$$

$$= \sqrt{2}(\pi - 4)$$

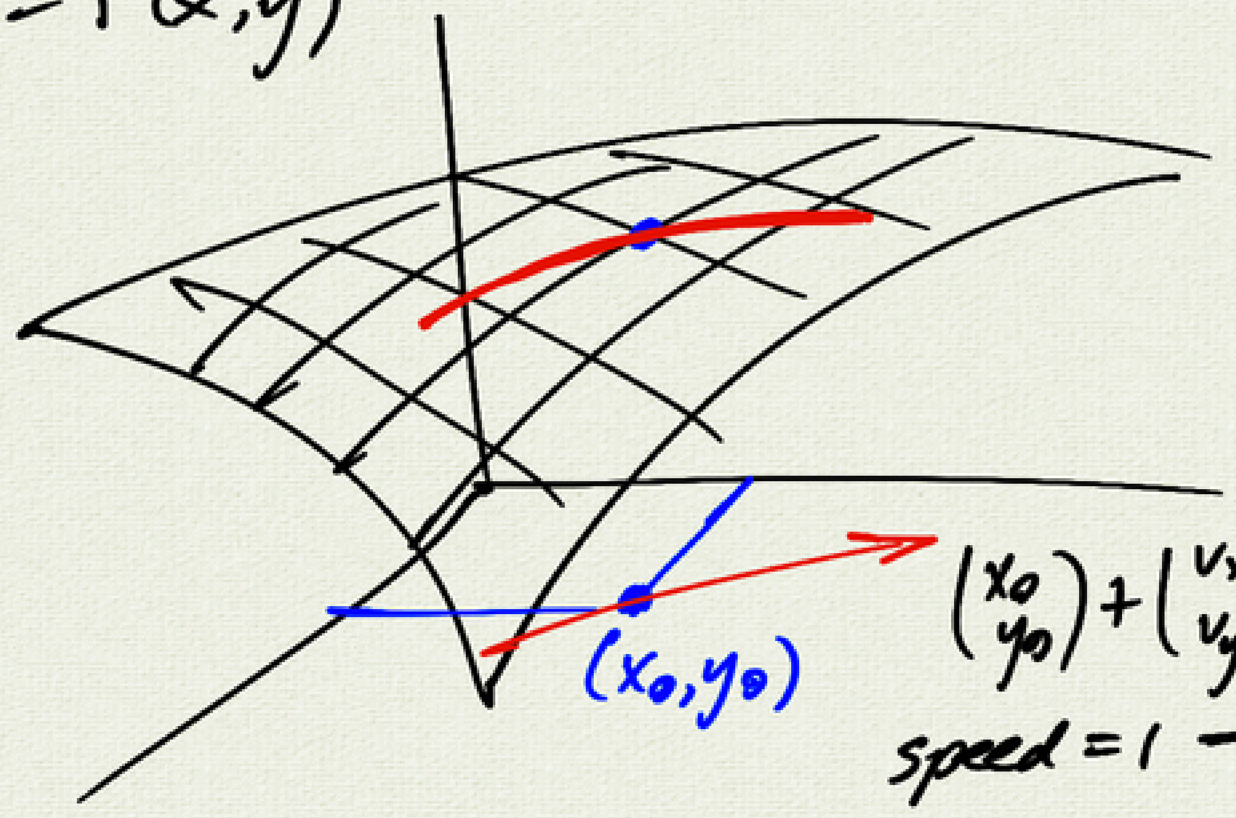
$\frac{\partial u}{\partial x} = e^x \sin y$ $\quad \frac{dx}{dt} = -\frac{1}{2t}\cdot 2$

$\frac{\partial u}{\partial y} = e^x \cos y$ $\quad = -1/t$

$\frac{dy}{dt} = \pi$

3.7 Directional derivatives

$z = f(x,y)$



$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} v_x \\ v_y \end{pmatrix} t = \bar{r}(t)$$

speed $= 1 \longrightarrow |\bar{v}| = 1$

$v_x^2 + v_y^2 = 1$

$$f(t) = f(x(t), y(t))$$

$$f'(t) = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

$$= \underbrace{\begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{pmatrix}}_{df} \begin{pmatrix} x' \\ y' \end{pmatrix} \longleftarrow \bar{r}'(t)$$

$$= \begin{pmatrix} f_x \\ f_y \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \end{pmatrix} \qquad |\bar{r}'(t)| = 1 \text{ speed } 1$$

∇f

"del" gradient

$$= |\nabla f| \underbrace{|r'(t)|}_{1} \cos\theta$$

$$f'(t) = |\nabla f| \underbrace{\cos\theta}_{\text{in } [-1,1]}$$

$$\Longrightarrow |f'(t)| \le |\nabla f|$$

(magnitude of) gradient is the largest the derivative can be (when $\bar{r}(t)$ is in direction of ∇f)

$\nabla f = \begin{pmatrix} f_x \\ f_y \end{pmatrix}$ gradient

1. direction of greatest change (and magnitude)
2. define directional derivative

$$D_{\bar{u}}(f) = \nabla f \cdot \bar{u}$$

$\bar{u}$ unit vector, $|\bar{u}| = 1$

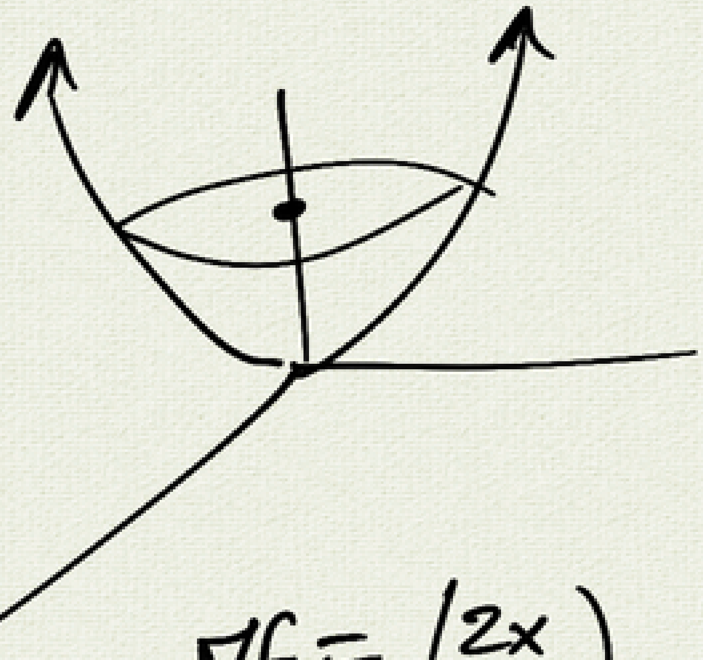
example: $\bar{u} = \vec{i}$

$$D_{\vec{i}}(f) = \nabla f \cdot \vec{i} = f_x$$

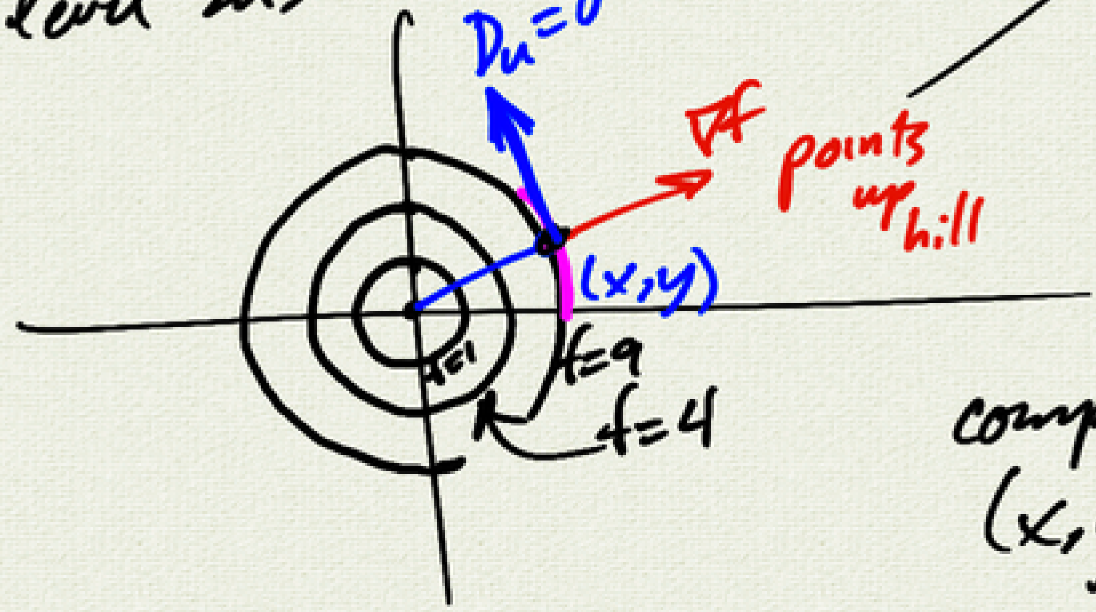
example:

$$f(x,y) = x^2 + y^2$$

paraboloid



level sets



$$\nabla f = \begin{pmatrix} 2x \\ 2y \end{pmatrix}$$

compute:

$$(x,y) = 3\left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$$

$$= \left(\frac{3\sqrt{3}}{2}, \frac{3}{2} \right)$$

$$\vec{u} = \left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$$

compute:

$D_{\vec{u}}(f)$ at $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2} \right)$

$$\nabla f\left(\frac{3\sqrt{3}}{2}, \frac{3}{2} \right) = \begin{pmatrix} 2\left(\frac{3\sqrt{3}}{2}\right) \\ 2\left(\frac{3}{2}\right) \end{pmatrix}$$

$$= \begin{pmatrix} 3\sqrt{3} \\ 3 \end{pmatrix}$$

$$D_{\vec{u}}(f) = \nabla f \cdot \vec{u}$$

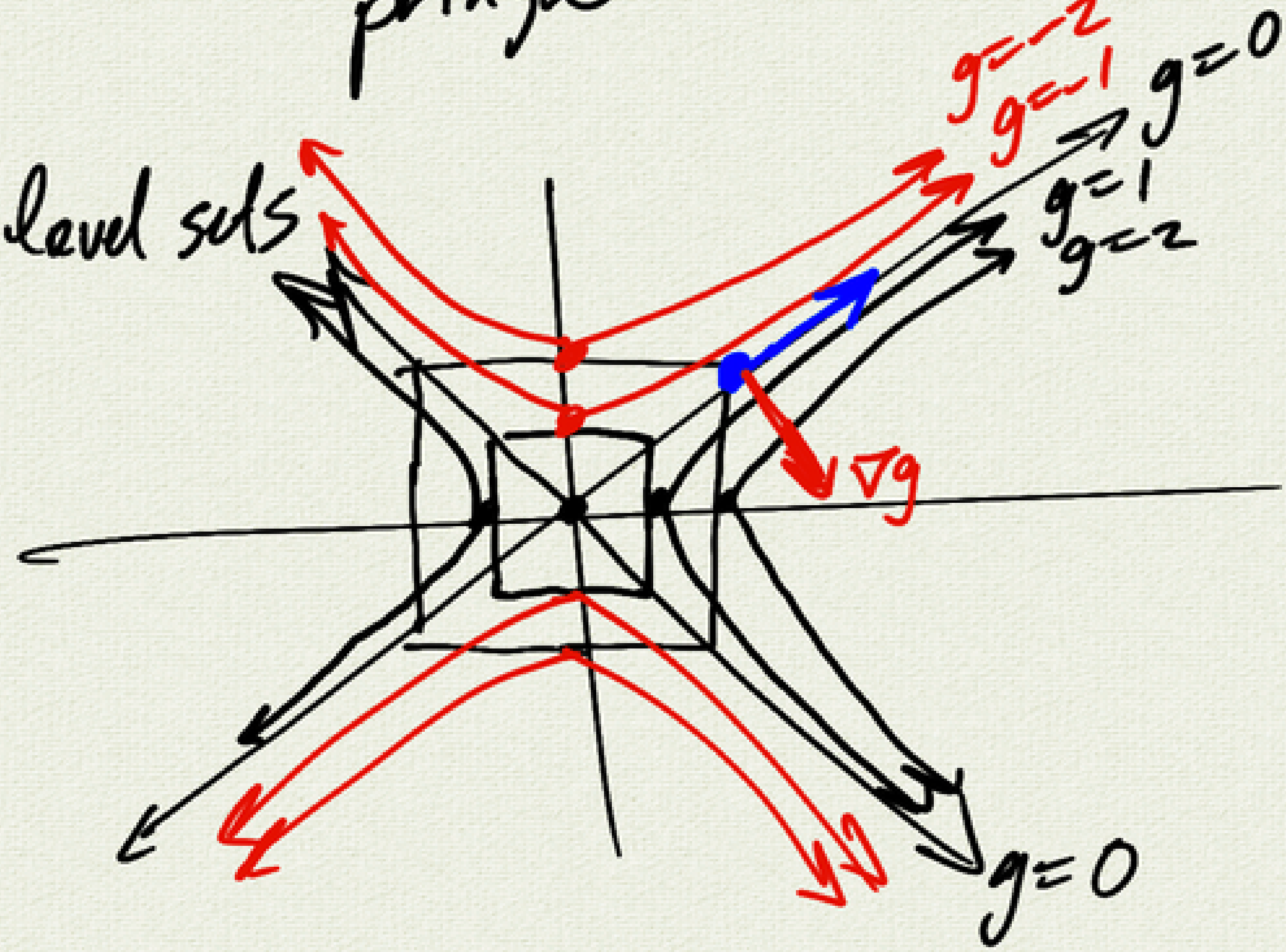
$$= \begin{pmatrix} 3\sqrt{3} \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -1/2 \\ \sqrt{3}/2 \end{pmatrix}$$

$$= 0$$

example:

$$g(x,y) = x^2 - y^2$$

pringle



$g=1$

$x^2-y^2=1$

$g=4$

$x^2-y^2=4$

$\frac{x^2}{4}-\frac{y^2}{4}=1$

$g=-1$

$x^2-y^2=-1$

$y^2-x^2=1$

calculate $D_{\bar{u}}(g)$ $\bar{u} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

at $(2,2)$

$$\nabla g = \begin{pmatrix}2x\\-2y\end{pmatrix} \qquad \nabla g(2,2) = \begin{pmatrix}4\\-4\end{pmatrix}$$

$$D_{\bar{u}}(g)(2,2) = \nabla g \cdot \bar{u}$$

$$= \begin{pmatrix}4\\-4\end{pmatrix} \cdot \begin{pmatrix}1\\1\end{pmatrix} \frac{1}{\sqrt{2}}$$

$$= 0$$