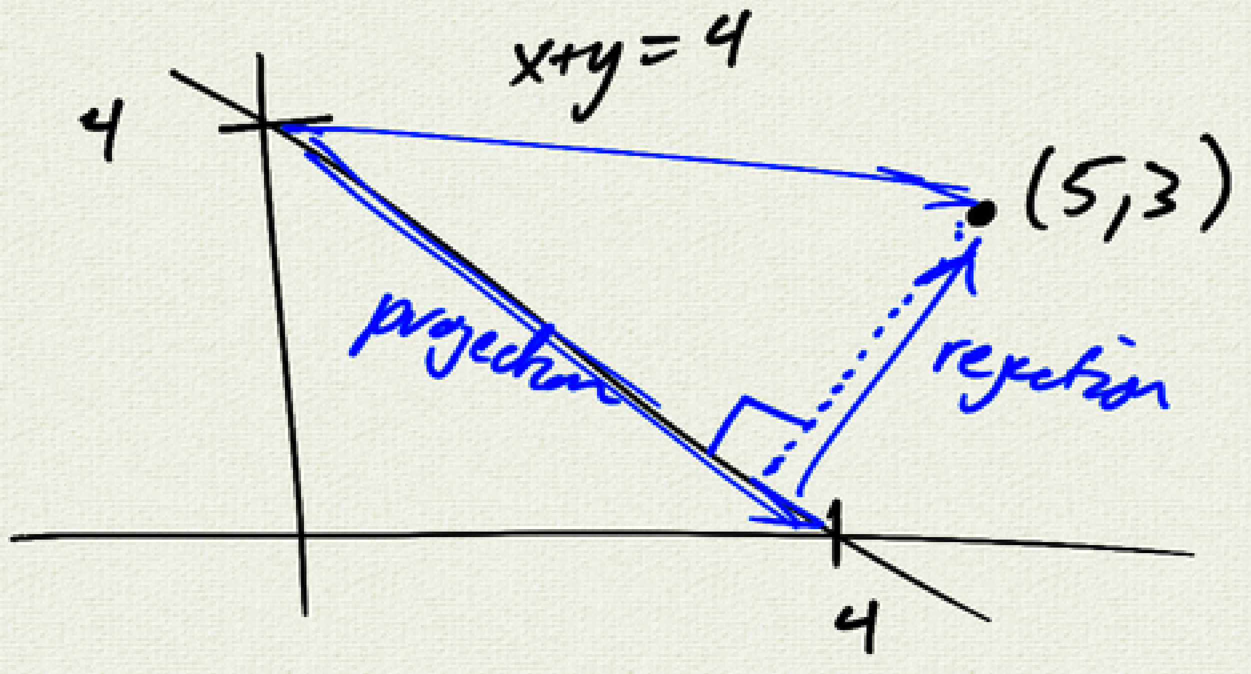
(325) $f(x,y) = 9 - x^4 y^4$ ← positive

$\longrightarrow d^2 f(0,0) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

## 4.3 Constrained optimization

example:




find distance from (5,3) to line $x+y=4$

① could find length of rejection (using vectors & dot product)

or ② minimize distance to (5,3), subject to the constraint $x+y=4$

minimize $d(x,y) = (x-5)^2 + (y-3)^2$
(minimize squared distance)
subject to $x+y=4$

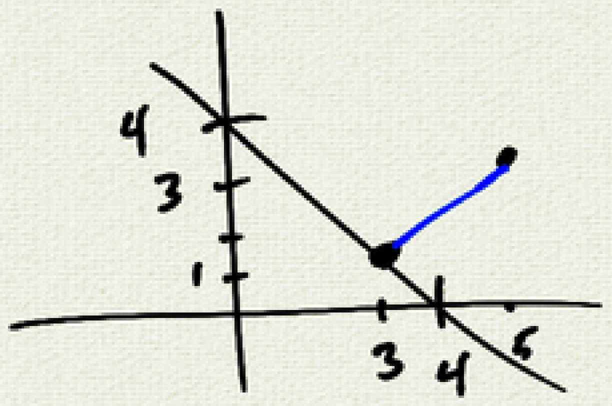
substitute: $y = 4-x$

$$d(x) = (x-5)^2 + ((4-x)-3)^2$$
$$= (x-5)^2 + (1-x)^2$$
$$= (x^2-10x+25) + (x^2-2x+1)$$

minimize $d(x) = 2x^2 - 12x + 26$

$d'(x) = 4x - 12$

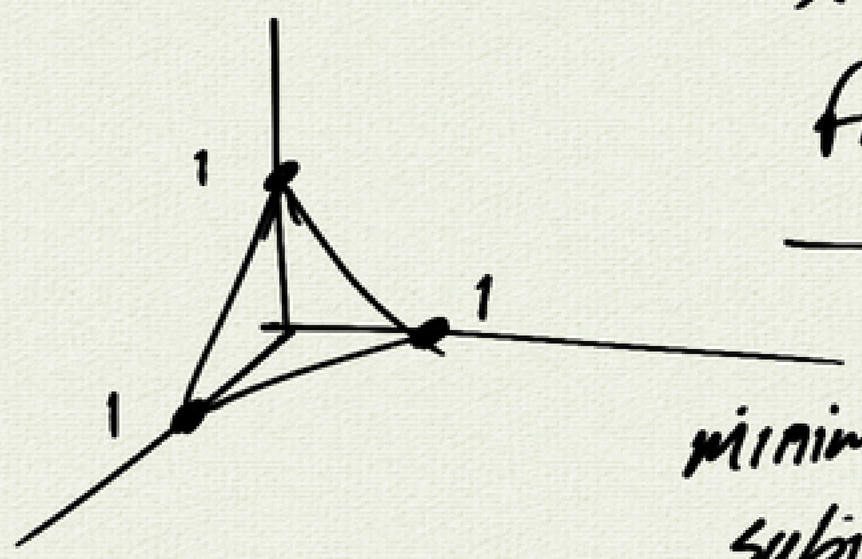
critical pts: $d'(x) = 0 \Longrightarrow \left.\begin{matrix} x=3 \\ y=1 \end{matrix}\right]$ closest point




distance

$$d^2 = (3-5)^2 + (1-3)^2$$
$$= 8$$
$$\Rightarrow d = 2\sqrt{2}$$

example 2




$x+y+z=1$ plane

find distance to origin

minimize distance to origin, subject to constraint $x+y+z=1$

$\Rightarrow$ minimize $d(x,y,z)=x^2+y^2+z^2$

subject to $x+y+z=1$

substitute $z=1-x-y$

$\Rightarrow d(x,y)=x^2+y^2+(1-x-y)^2$

critical pts:

$$d_x = 2x+2(1-x-y)(-1)$$
$$= 2x-2+2x+2y$$
$$= 4x+2y-2$$
$$d_y = 4y+2x-2$$

critical pts: $d_x = d_y = 0$

$$4x+2y-2=0$$
$$2x+4y-2=0$$

$$\begin{pmatrix}4&2\\2&4\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}2\\2\end{pmatrix}$$

$$\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{12}\begin{pmatrix}4&-2\\-2&4\end{pmatrix}\begin{pmatrix}2\\2\end{pmatrix}$$

$$=\frac{1}{12}\begin{pmatrix}4\\4\end{pmatrix}$$

$$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}1/3\\1/3\end{pmatrix}$$

$$\Rightarrow z=1/3$$
$$(z=1-x-y)$$

$$\begin{pmatrix}a&c\\b&d\end{pmatrix}^{-1}=\frac{1}{ad-bc}\begin{pmatrix}d&-c\\-b&a\end{pmatrix}$$

$$A\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}a\\b\end{pmatrix}$$

$$\Rightarrow A^{-1}A\begin{pmatrix}x\\y\end{pmatrix}=A^{-1}\begin{pmatrix}a\\b\end{pmatrix}$$

$$\begin{pmatrix}x\\y\end{pmatrix}=A^{-1}\begin{pmatrix}a\\b\end{pmatrix}$$

$2^{nd}$ deriv test:

$$d^2f=\begin{pmatrix}4&2\\2&4\end{pmatrix}$$

$$\left.\begin{array}{l}\det d^2f = 12>0\\ f_{xx}>0\end{array}\right]\text{ local min.}$$

$$d^2=\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^2$$
$$=\frac{1}{3}$$
$$\Rightarrow d=\frac{1}{\sqrt{3}}$$