HW 4.3

349

$x^2 - yz = 5$

minimize $d(x,y,z) = x^2 + y^2 + z^2$

$x^2 = yz + 5$

$d(y,z) = (yz+5) + y^2 + z^2$

$d_y = z + 2y$

$d_z = y + 2z$

critical pt: $d_y = 0 = d_z$

$z + 2y = 0$

$y + 2z = 0$

$2y + z = 0$

$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\det A = -3 \neq 0$

$\Rightarrow A^{-1}$ exists

$A \begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

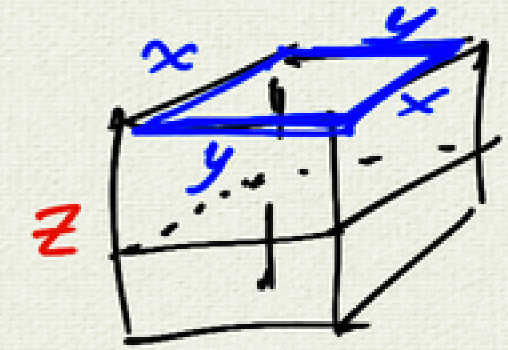
$\begin{pmatrix} y \\ z \end{pmatrix} = A^{-1} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$y = 0 = z = 0$

$x^2 = yz + 5 = 5 \Rightarrow x = \pm\sqrt{5}$

(351) $2x+2y+z=108$

maximize $V(x,y,z)=xyz$



$P+Z=108$

perimeter + length

$(2x+2y)+z$

$z=108-2x-2y$

$V=xy(108-2x-2y)$

$V_x=108y-4xy-2y^2$

$V_y=108x-2x^2-4xy$

critical pt: $V_x=0=V_y$

$108y-4xy-2y^2=\boxed{108x-2x^2-4xy}$

$\to 2y(54-y)=2x(54-x)$

?, $\Rightarrow x=y$

$4x^2=x(108-2x)$

$=108x-2x^2$

$6x^2-108x=0$

$6x(x-18)=0$

$\Rightarrow x=y=18$

$108 \Rightarrow z=36$

357

$$h + C = 120$$

$$h + 2\pi r = 120$$

max $V = \pi r^2 h$

$$V = \pi r^2 (120 - 2\pi r)$$

$$= 120\pi r^2 - 2\pi^2 r^3$$

$$V'(r) = 240\pi r - 6\pi^2 r^2$$

$$= 6\pi r(40 - \pi r)$$

critical pt's $r = 0$

$r = 40/\pi$

$$\Longrightarrow h = 120 - 2\pi r$$

$$= 120 - 2\pi\left(\frac{40}{\pi}\right)$$

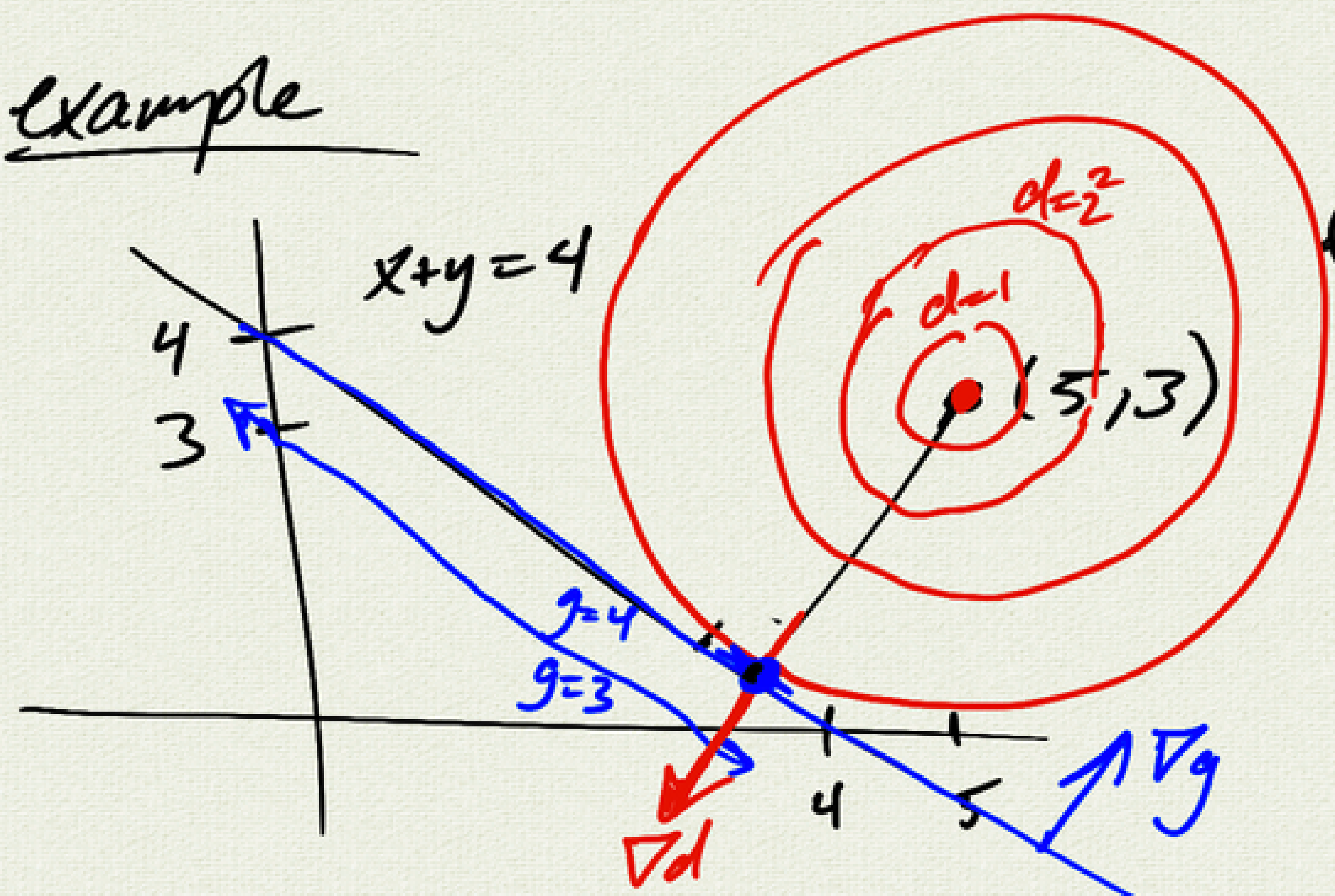
$$= 40$$

$$V = \pi r^2 h = \pi\left(\frac{40}{\pi}\right)^2 (40)$$

$$= \frac{64000}{\pi}$$

4.4 Lagrange multipliers

example



find distance from (5,3) to $x+y=4$

minimize $d(x,y) = (x-5)^2 + (y-3)^2$

constrained to $x+y=4$

$g(x,y) = 4$ level set of $g(x,y)$

---

look for places where

$$\nabla g = \lambda \nabla d$$

"lambda" Lagrange multiplier

$$\nabla g = \lambda \nabla d$$

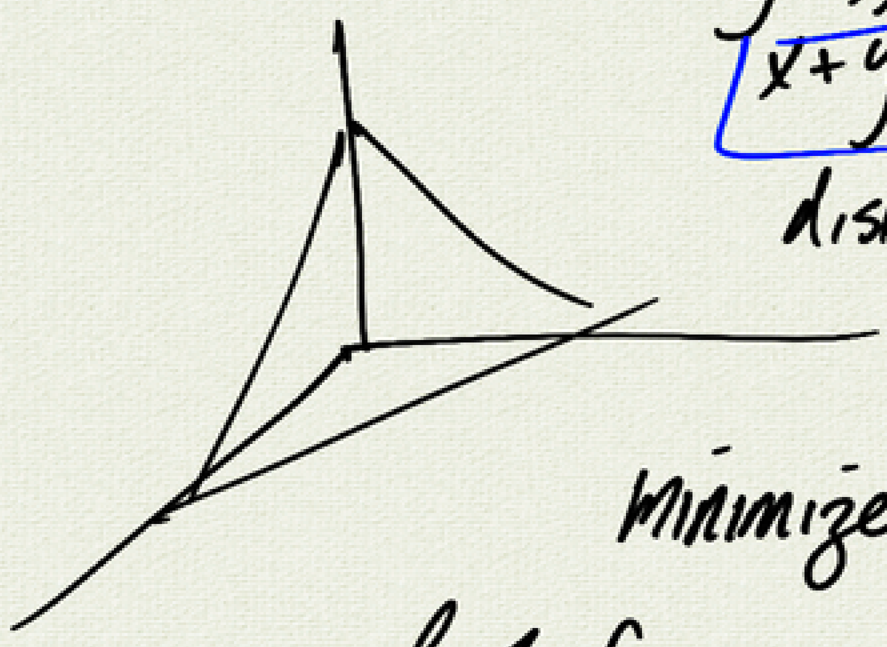
$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \lambda \begin{pmatrix} 2(x-5) \\ 2(y-3) \end{pmatrix}$$

$$\left.\begin{array}{l} 1 = \lambda \cdot 2(x-5) \\ 1 = \lambda \cdot 2(y-3) \\ x+y=4 \end{array}\right]$$

$$\left.\begin{array}{l} \frac{1}{2\lambda} = x-5 \\ \frac{1}{2\lambda} = y-3 \end{array}\right] \begin{array}{l} x-5=y-3 \\ x-y=2 \end{array}$$

$$\begin{array}{l} x-y=2 \\ \underline{x+y=4} \\ 2x=6 \quad y=1 \\ x=3 \end{array}$$

Example:



$g(x,y,z) = \boxed{x+y+z} = 1$

distance to origin

$f(x,y,z) = x^2 + y^2 + z^2$

minimize f, constrained to $g(x,y,z) = 1$

look for:

$$\nabla g = \lambda \nabla f$$

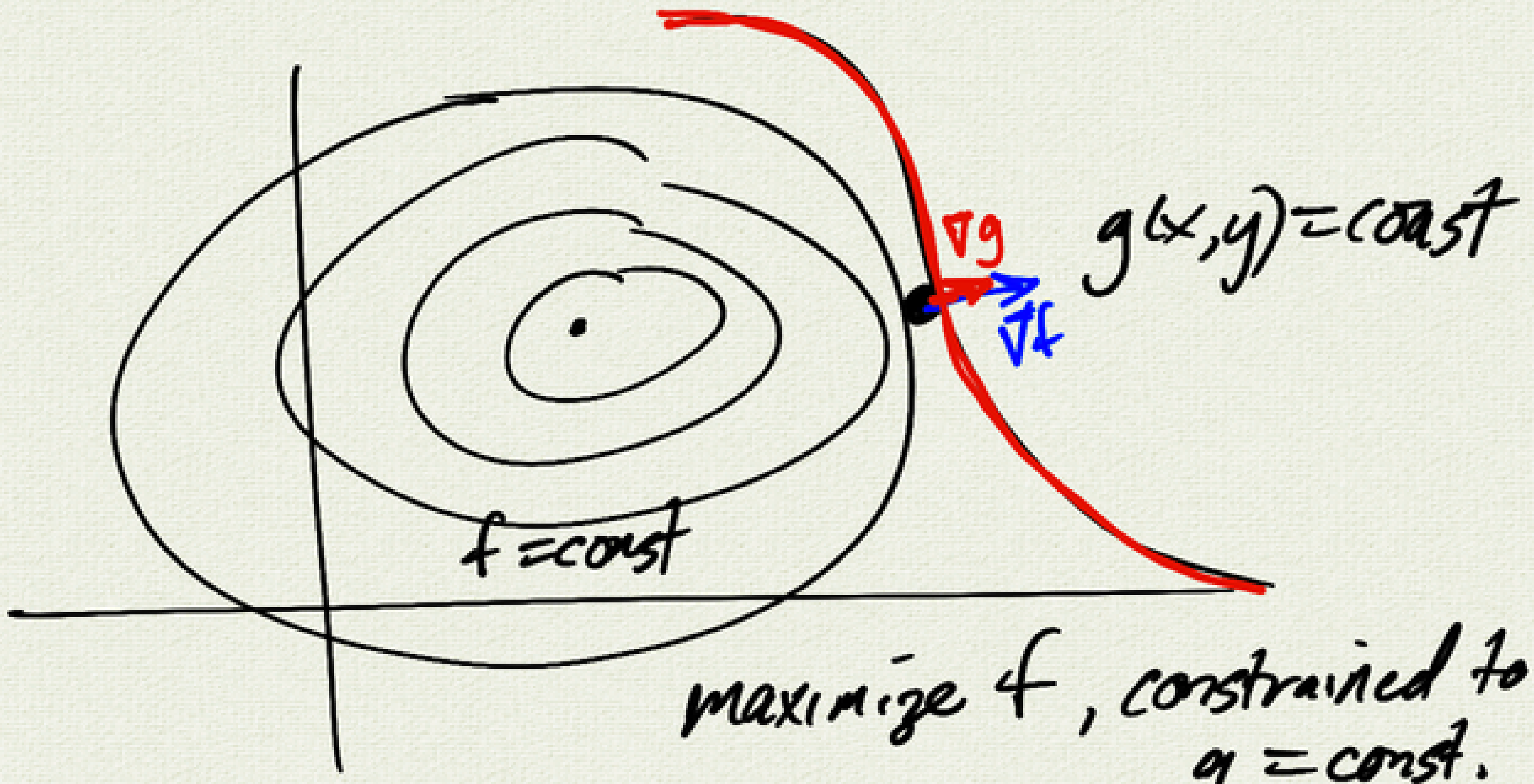
$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \lambda \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$$

$$\left.\begin{aligned} 1 &= 2x\lambda \\ 1 &= 2y\lambda \\ 1 &= 2z\lambda \end{aligned}\right\} \quad x = y = z \left(= \frac{1}{2\lambda}\right)$$

$x + y + z = 1$

$\longrightarrow (x,y,z) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

$\Longrightarrow f(x,y,z) = \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 = \frac{1}{3}$



maximize f, constrained to $g = \text{const}$.

$\Longrightarrow$ look for points where

$$\nabla f = \lambda \nabla g$$