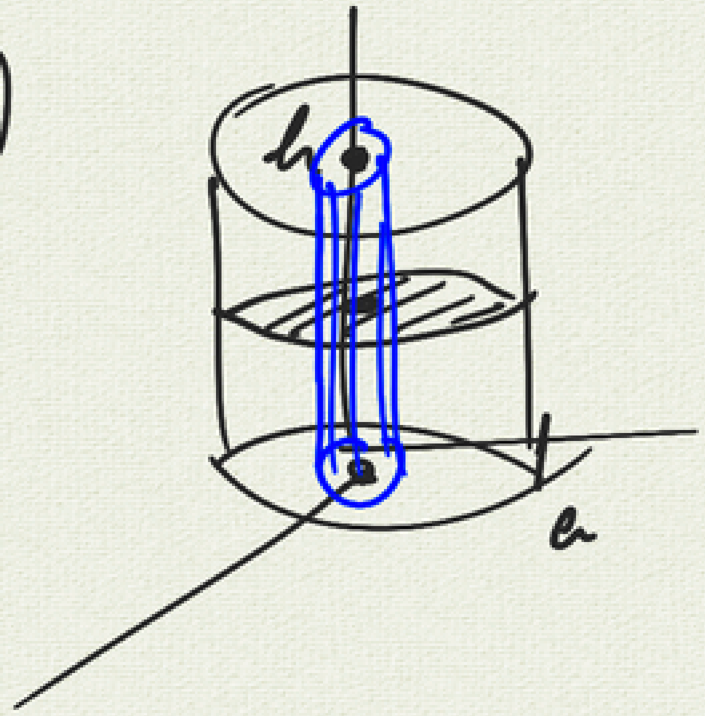
(1)



$$V = \int_0^h \int_0^a \int_0^{2\pi} r \, d\theta \, dr \, dz$$

$$= \int_0^h \int_0^a 2\pi r \, dr \, dz$$

$$= \int_0^h \pi a^2 \, dz \qquad \text{(disks)}$$

$$= \int_0^a \underbrace{\int_0^h \int_0^{2\pi} d\theta \, dz}_{} \, r \, dr$$

$$= \int_0^a \underbrace{2\pi r h}_{\text{cylindrical shell}} \, dr$$

$V = \pi r^2 h = \pi(25) \cdot 6 = 150\pi$ || $SA = 2\pi r h = 2\pi \cdot 5 \cdot 6 = 60\pi$

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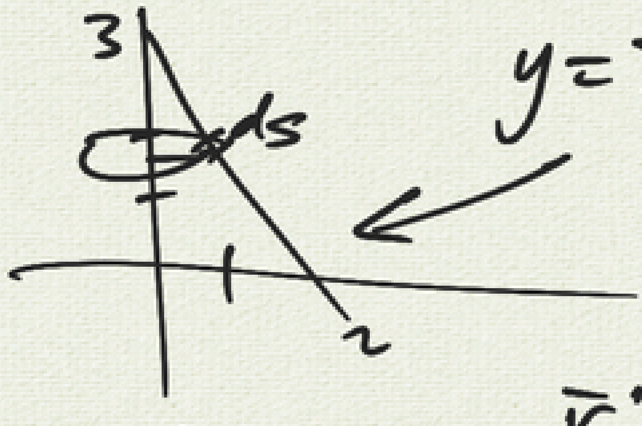
(2) $V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \cdot 4 \cdot 3 = 4\pi$

$SA = \pi r l = \pi \cdot 2 \cdot \sqrt{13}$

$l = \sqrt{2^2 + 3^2} = \sqrt{13}$

$SA = \int 2\pi x \, ds$



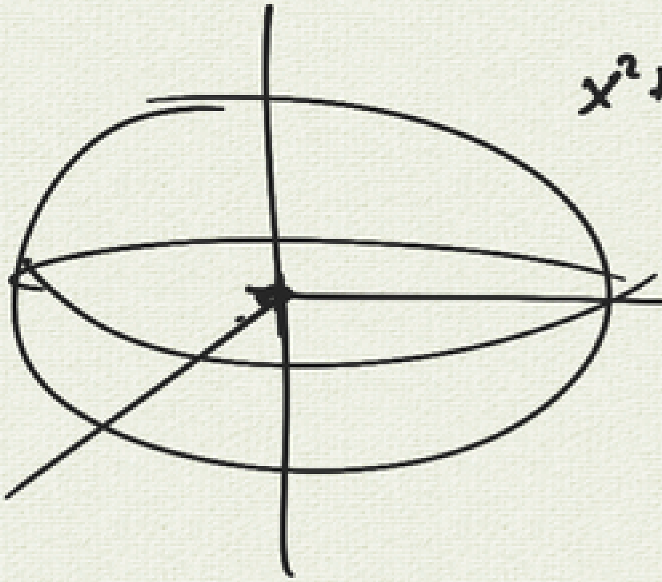
$y = 3 - \frac{3}{2} x$

$$\vec{r}(t) = \begin{pmatrix} 2t \\ 3 - 3t \end{pmatrix}$$

$$\vec{r}\,'(t) = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \quad |\vec{r}\,'(t)| = \sqrt{13}$$

$$= \int_0^1 2\pi \, 2t \, \sqrt{13} \, dt$$

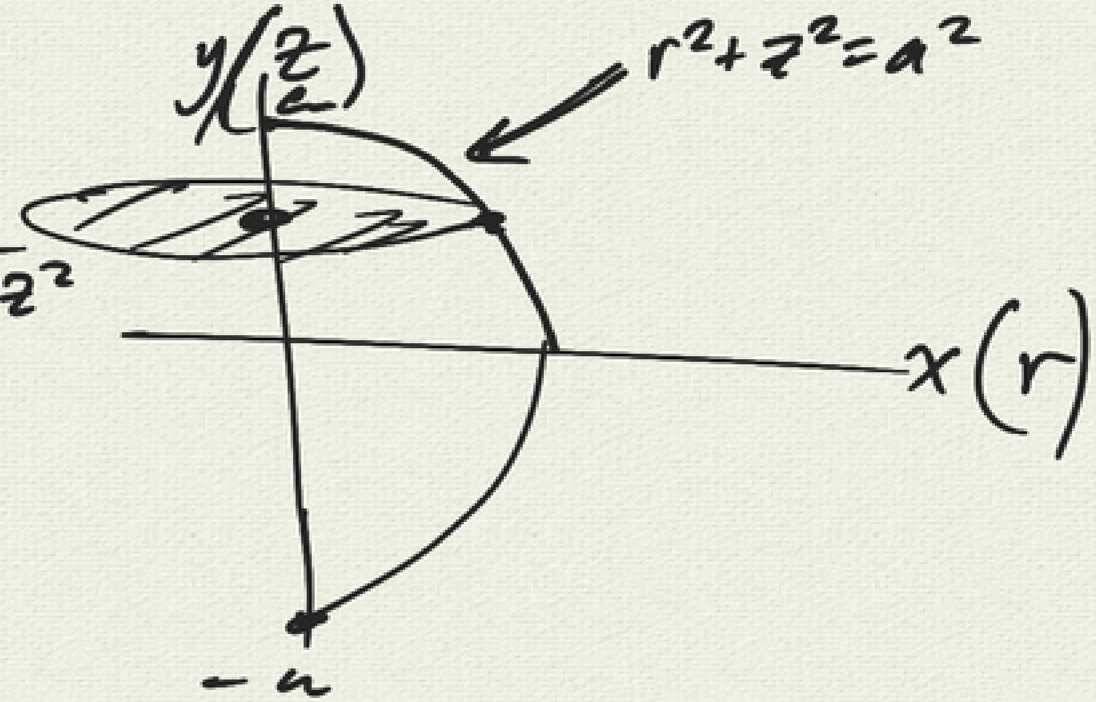
$$= 2\pi\sqrt{13}$$



$$x^2+y^2+z^2=a^2$$

$$r^2+z^2=a^2$$

$$r=\pm\sqrt{a^2-z^2}$$



$$V=2\pi\int_{-a}^{a}\int_{0}^{\sqrt{a^2-z^2}} r\,dr\,dz$$

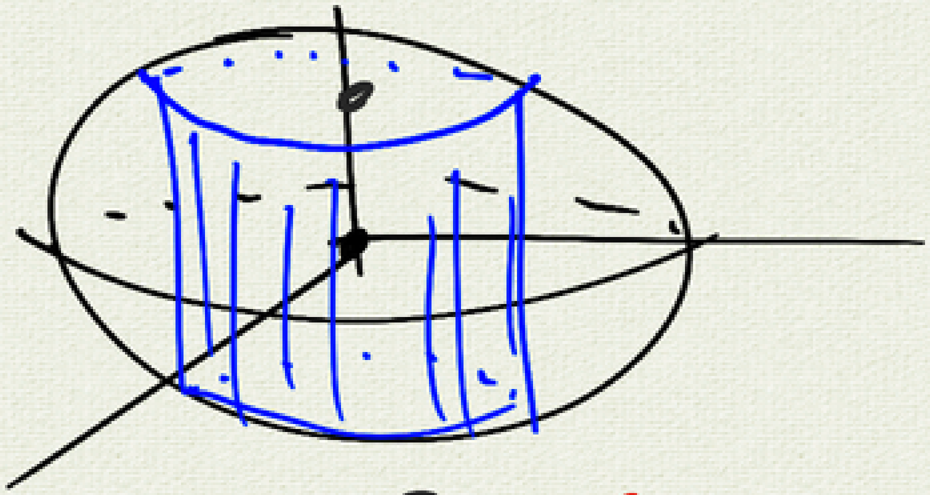
$$=2\pi\int_{-a}^{a}\frac{a^2-z^2}{2}\,dz$$

$$=\int_{-a}^{a}\underbrace{\pi\,(a^2-z^2)}_{\text{disk}}\,dz$$

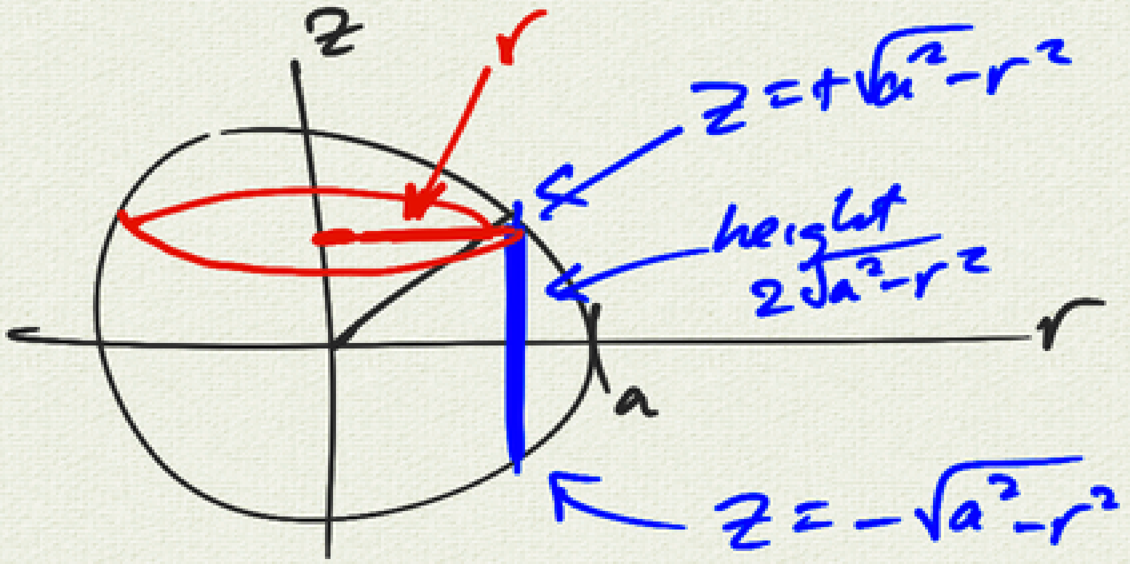
$$=\pi\left[a^2z-\frac{z^3}{3}\right]_{-a}^{a}$$

$$=2\pi\left(a^3-\frac{a^3}{3}\right)$$

$$=\frac{4\pi}{3}a^3$$



$$V=\int_0^a 2\pi\int_{-\sqrt{a^2-r^2}}^{\sqrt{a^2-r^2}} dz\, r\, dr$$



$$=\int_0^a 2\pi\, 2\sqrt{a^2-r^2}\, r\, dr$$

$$=\int_0^a \underbrace{2\pi r}_{\text{radius}}\cdot \underbrace{2\sqrt{a^2-r^2}}_{\text{height of shell}}\, dr$$

$$=4\pi\int_0^a r\sqrt{a^2-r^2}\, dr$$

$$=4\pi\int_{a^2}^0 \sqrt{u}\ \frac{du}{(-2)}$$

$$=2\pi\int_0^{a^2}\sqrt{u}\, du$$

$$=2\pi\left(\frac{2}{3}u^{3/2}\right)\Big|_0^{a^2}$$

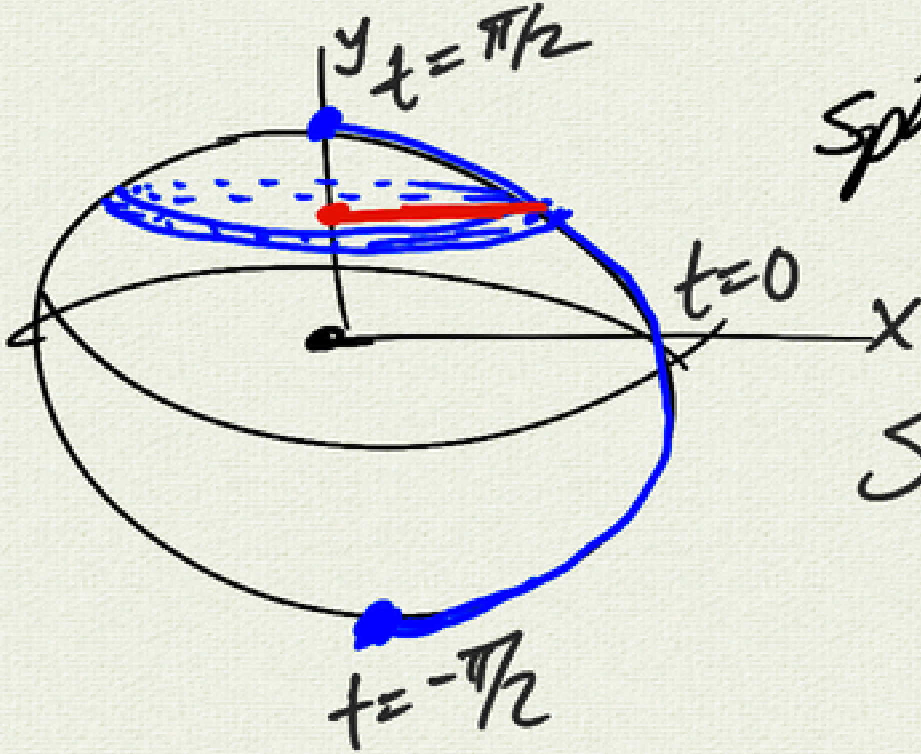
$$=\frac{4}{3}\pi a^3$$

$u=a^2-r^2$

$du=-2r\,dr$

$r=0 \Rightarrow u=a^2$

$r=a \Rightarrow u=0$



Sphere surface area

$$SA = \int_C 2\pi x \, ds$$

$$\vec{r}(t) = \begin{pmatrix} a\cos t \\ a\sin t \end{pmatrix}$$

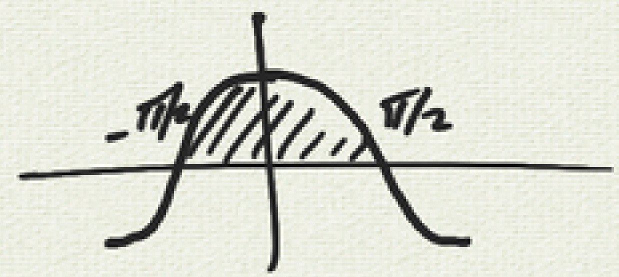
$$\vec{r}\,'(t) = \begin{pmatrix} -a\sin t \\ a\cos t \end{pmatrix}$$

$$|\vec{r}\,'(t)| = a$$

$$= \int_{-\pi/2}^{\pi/2} 2\pi \big(\underbrace{a\cos t}_{x}\big) \cdot a\,dt$$

$$ds = |\vec{r}\,'(t)|\,dt$$

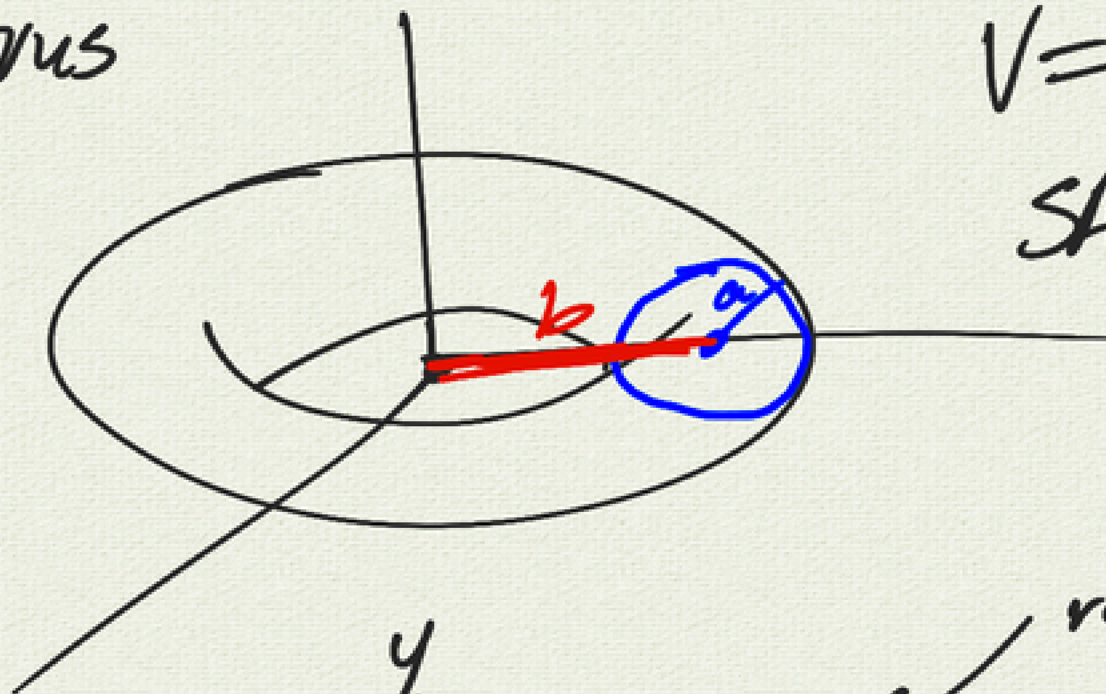
$$= 2\pi a^2 \underbrace{\int_{-\pi/2}^{\pi/2} \cos t \, dt}_{2}$$



$$= 4\pi a^2$$

5.12 Pappus' Theorem

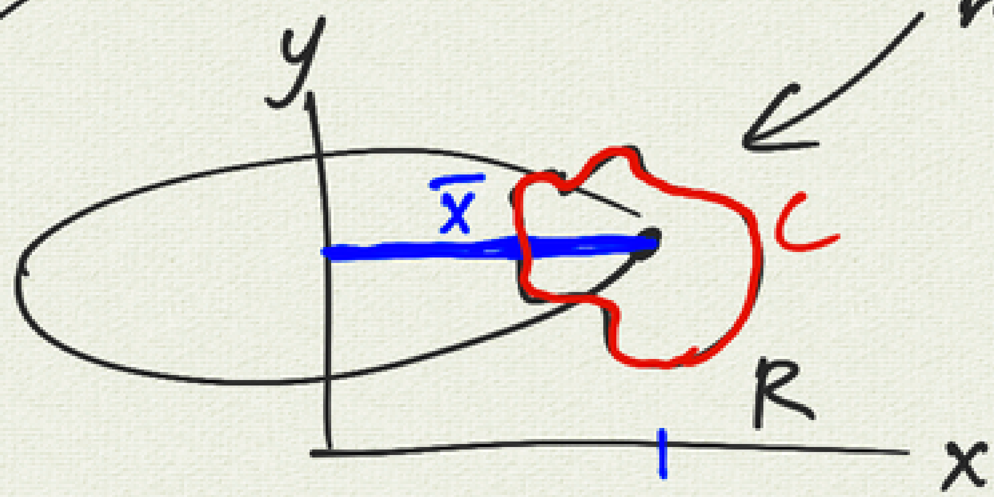
torus



$V = (\pi a^2)(2\pi b)$

$SA = (2\pi a)(2\pi b)$

revolve: toroid



$A = \iint_R dA$ area

$L = \int_C ds$ arc length

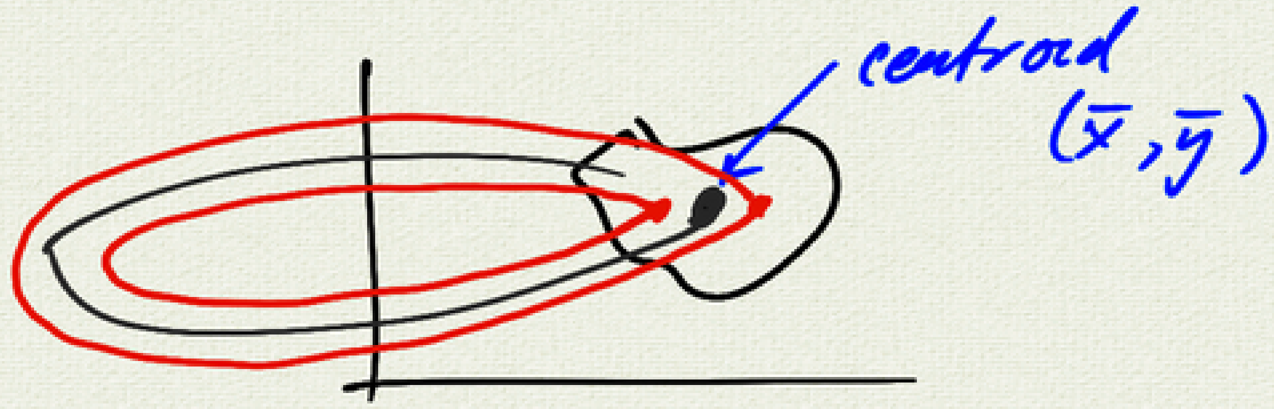
$\Rightarrow \boxed{V = (2\pi \bar{x})A}$

Pappus

$\bar{x}_{curve} = \int_C x\,ds \big/ L$

$$\boxed{SA = (2\pi \bar{x}_{curve})L}$$

$\left(= \int 2\pi x\,ds\right)$



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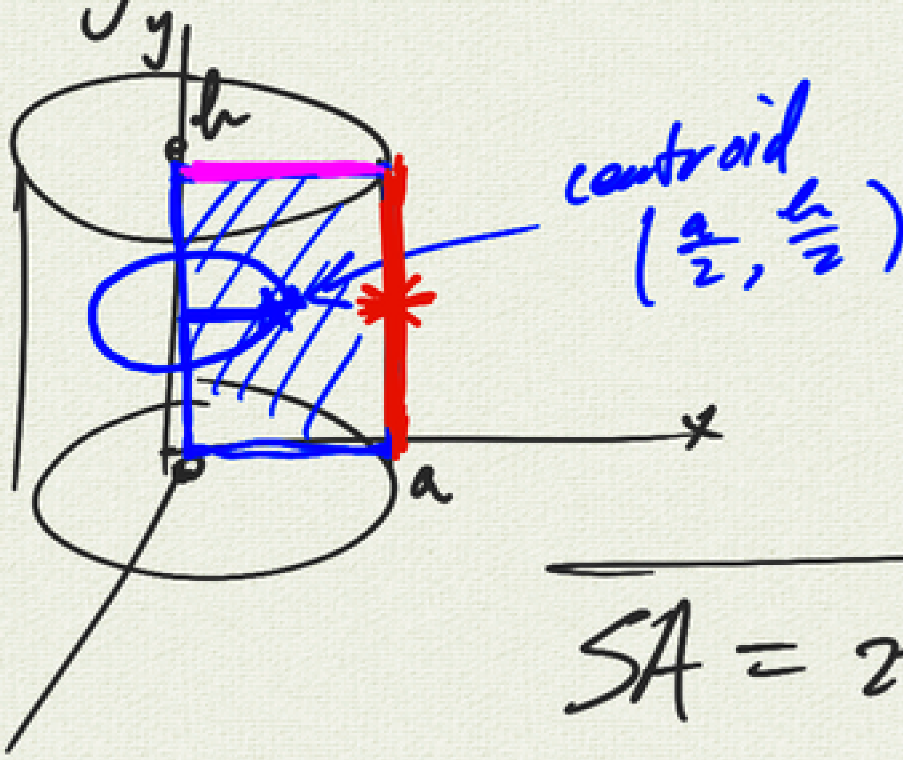
cylinder



$$A = ah$$

$$V = 2\pi \underbrace{\left(\frac{a}{2}\right)}_{\text{radius}} \cdot \underbrace{ah}_{A}$$

$$= \pi a^2 h$$

$$SA = 2\pi (a) h$$

$$\text{top: } 2\pi\left(\frac{a}{2}\right)\cdot a = \pi a^2$$

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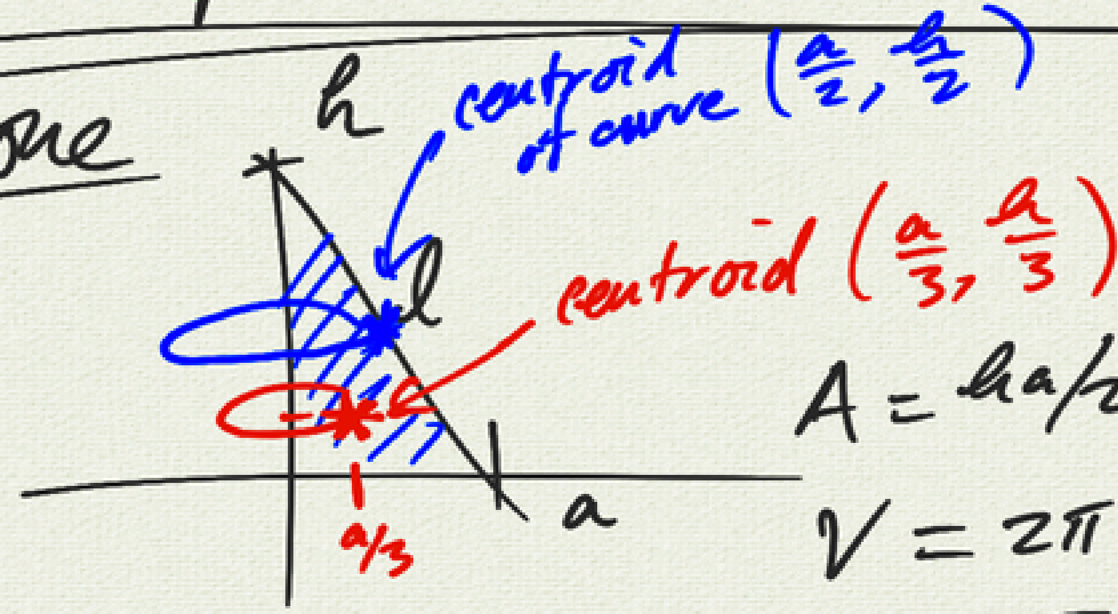
cone



$$A = ha/2$$

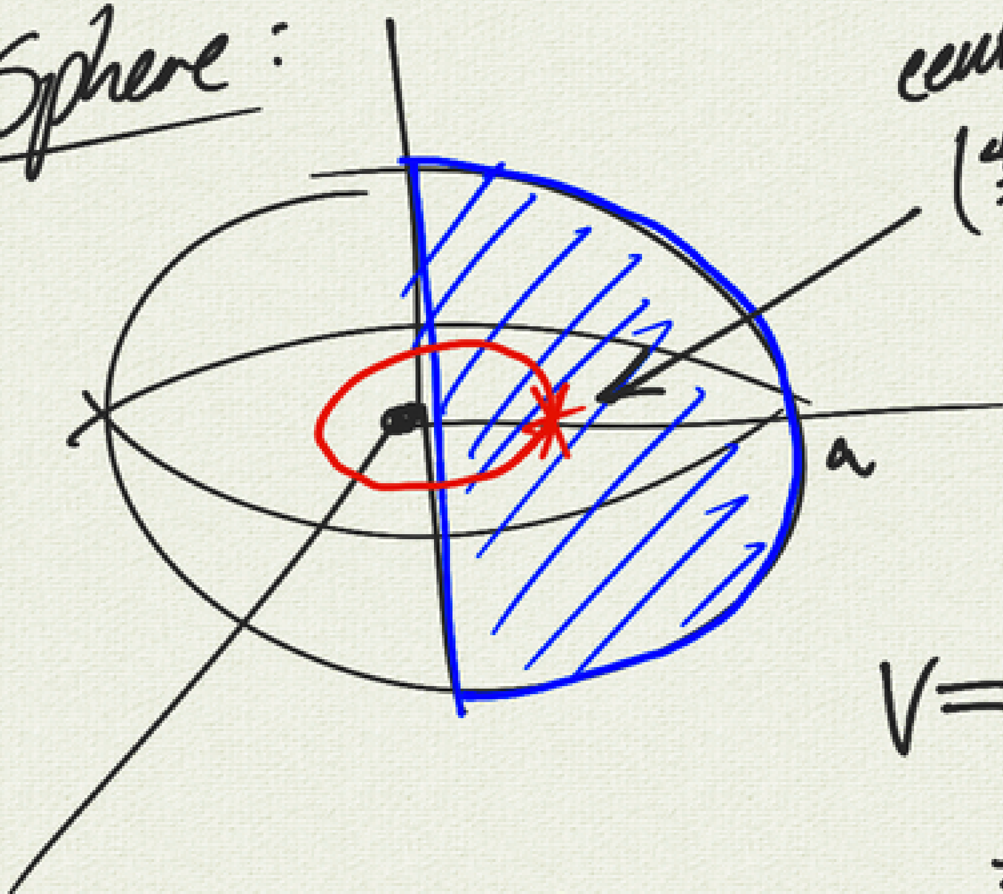
$$V = 2\pi\left(\frac{a}{3}\right)\left(\frac{ha}{2}\right)$$

$$= \frac{\pi a^2 h}{3}$$

$$SA = 2\pi\left(\frac{a}{2}\right)(l)$$

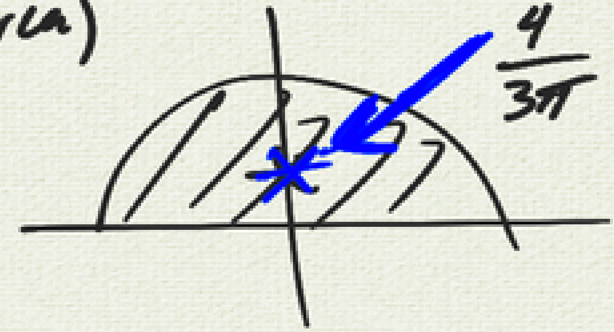
$$= \pi a l$$

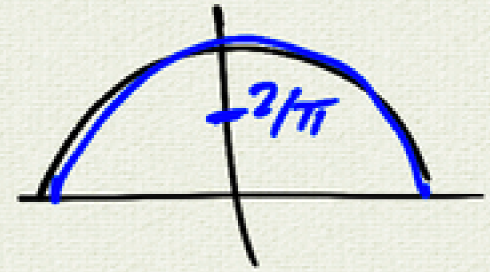
Sphere:



centroid (area)

$\left(\frac{4a}{3\pi}, 0\right)$





$$V = 2\pi\left(\frac{4a}{3\pi}\right)\left(\frac{\pi a^2}{2}\right)$$

$$= \frac{4}{3}\pi a^3$$

$$SA = 2\pi \underbrace{\left(\frac{2a}{\pi}\right)}_{\text{centroid of curve}} (\pi a)$$

$$= 4\pi a^2$$