(17) $$\int_{\ln 2}^{\ln 3}\left[\int_0^1 e^{x+y}\,dy\right]dx$$

$$= \int_{\ln 2}^{\ln 3}\left[\int_0^1 e^x e^y\,dy\right]dx$$

$$= \int_{\ln 2}^{\ln 3} e^x\left[\int_0^1 e^y\,dy\right]dx$$

$$e-1$$

$$= (e-1)\int_{\ln 2}^{\ln 3} e^x\,dx$$

$$= e-1$$

5.2 More double integrals

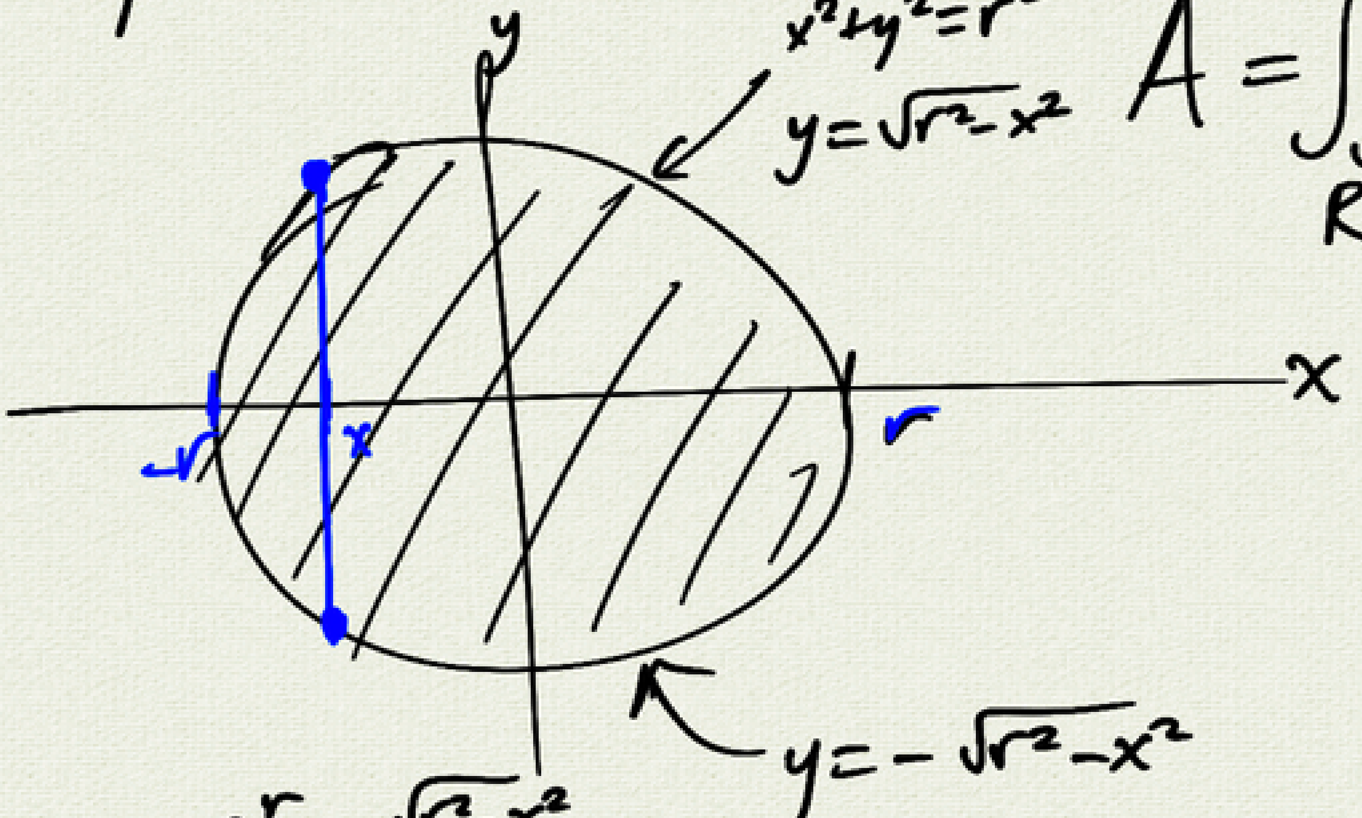
example: area of circle radius r

$x^2+y^2=r^2$

$y=\sqrt{r^2-x^2}$

$$A=\iint_R 1\, dA$$

R ← region



$y=-\sqrt{r^2-x^2}$

$$A=\int_{-r}^{r}\int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} 1\, dy\, dx$$

$\sin^2\theta+\cos^2\theta=1$

"$\sqrt{1-x^2}$"

$$=\int_{-r}^{r} 2\sqrt{r^2-x^2}\, dx$$

trig sub

$x=r\sin\theta$

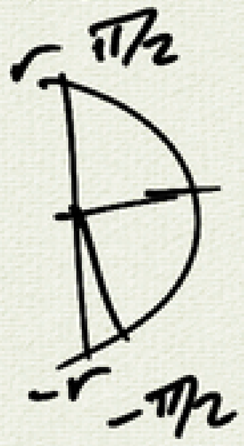
$dx=r\cos\theta\, d\theta$

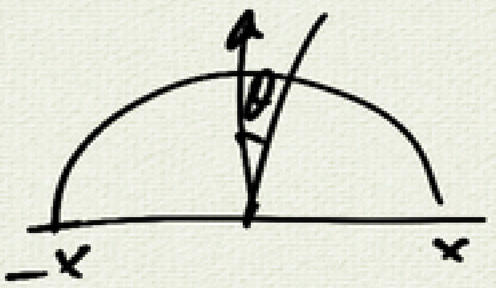
try $u=r^2-x^2$

$\Rightarrow du=-2x\,dx$

no u-sub

$$=2\int_{-\pi/2}^{\pi/2}\underbrace{(r\cos\theta)}_{\sqrt{r^2-x^2}}\underbrace{(r\cos\theta\, d\theta)}_{dx}$$





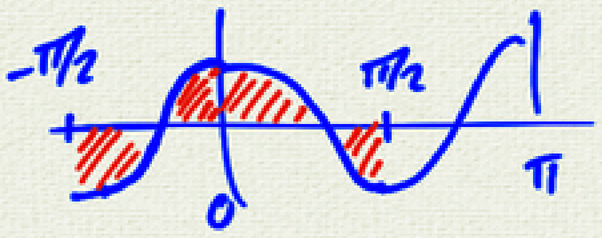
$$=2\int_{-\pi/2}^{\pi/2} r^2\cos^2\theta\, d\theta$$

$$=r^2\int_{-\pi/2}^{\pi/2}(1+\cos 2\theta)\, d\theta$$

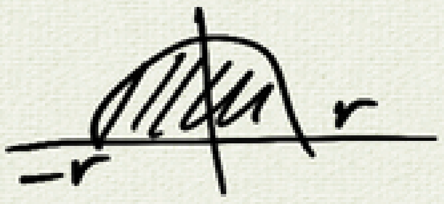
power-reducing: $\cos^2\theta=\frac{1}{2}(1+\cos 2\theta)$

$$=\pi r^2$$

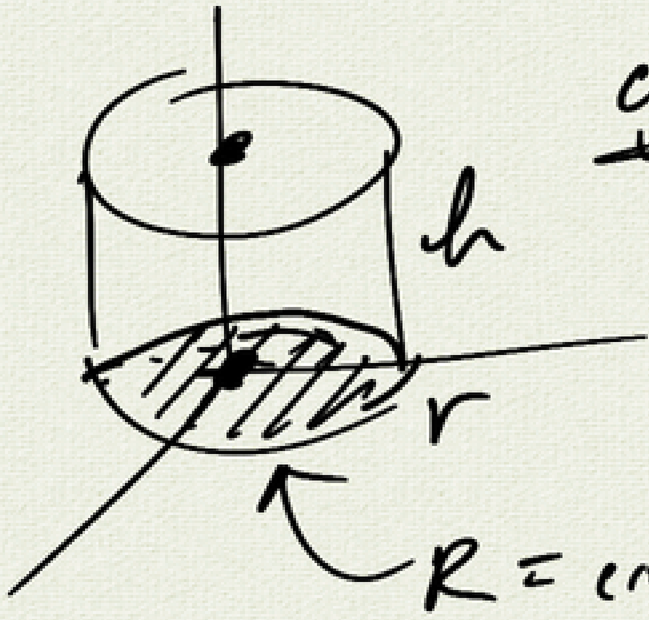
check $\int_{-\pi/2}^{\pi/2}\cos 2\theta\, d\theta=0$



observation: $2\int_{-r}^{r}\sqrt{r^2-x^2}\, dx=\pi r^2$



cylinder $\quad f(x,y) = h$



R = circle radius r

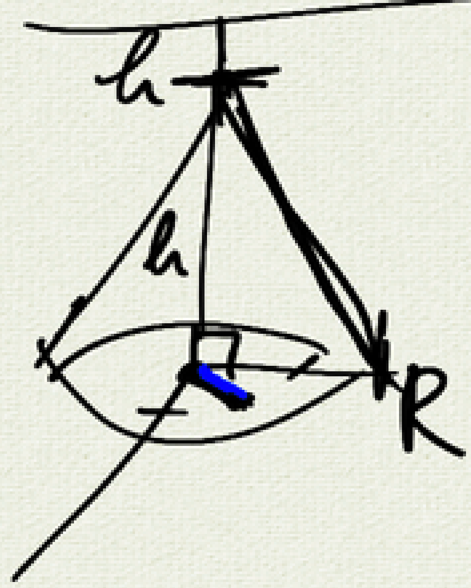
$$V = \iint_R f(x,y)\,dA$$

$$= \iint_R h\,dA$$

$$= \int_{-r}^{r}\int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} h\,dy\,dx$$

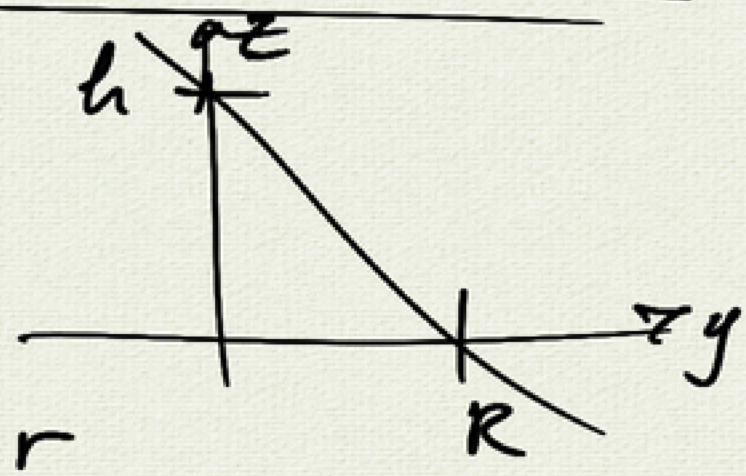
$$= 2h\int_{-r}^{r}\sqrt{r^2-x^2}\,dx$$

$$= h\cdot\pi r^2$$



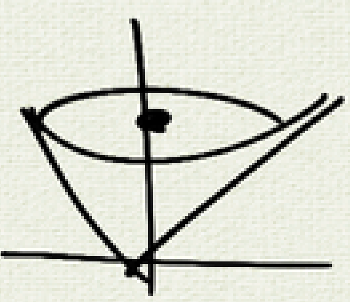
$V = \frac{1}{3}\pi R^2 h$

cone



$$\Rightarrow \quad z = h - \frac{h}{R}r$$

$$z = h - \frac{h}{R}\sqrt{x^2+y^2}$$

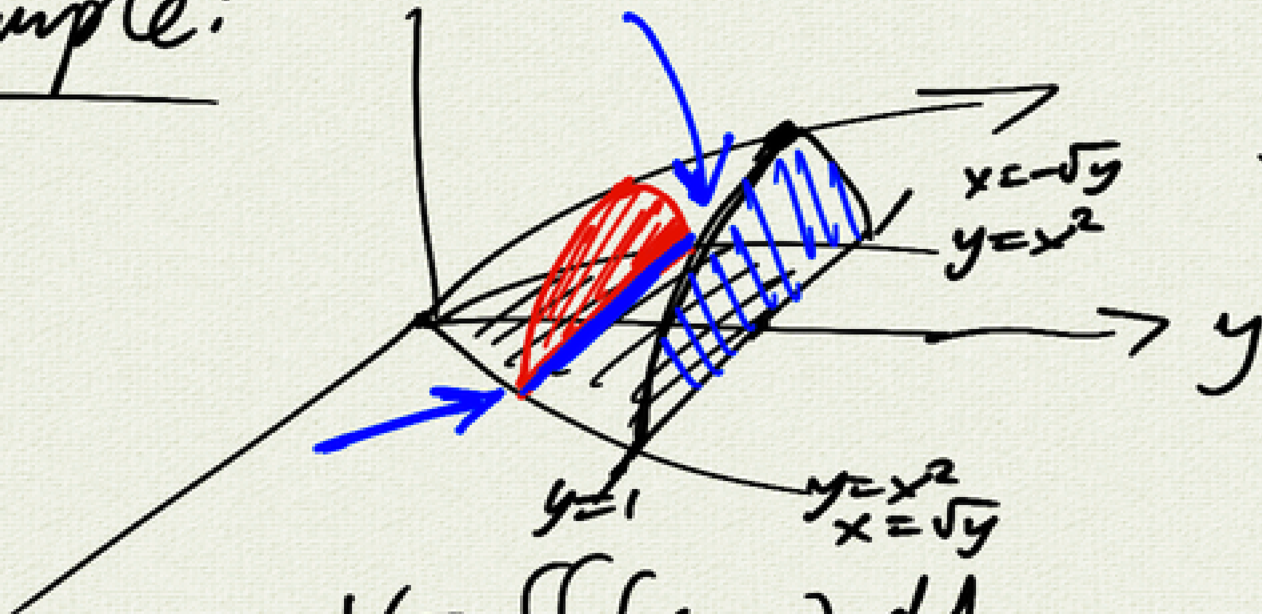


$z = r$
$= \sqrt{x^2+y^2}$

$$V = \int_{-r}^{r}\int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} \left(h - \frac{h}{R}\sqrt{x^2+y^2}\right) dy\,dx$$

$\Rightarrow$ wait for polar

example:



$y = x^2 + z^2$

$z = \sqrt{y - x^2}$

$f(x,y) = \sqrt{y-x^2}$

$$V = \iint_R f(x,y)\, dA$$

$$= \int_0^1 \boxed{\int_{-\sqrt{y}}^{\sqrt{y}} \sqrt{y-x^2}\, dx}\, dy$$

$r = \sqrt{y}$

$\Rightarrow \frac{\pi y}{2}$

$$\int_{-r}^{r} \sqrt{r^2 - x^2}\, dx = \frac{\pi r^2}{2}$$

$$= \int_0^1 \frac{\pi y}{2}\, dy$$

$$= \frac{\pi}{2}\left[\frac{y^2}{2}\right]_0^1$$

$$= \frac{\pi}{4}$$

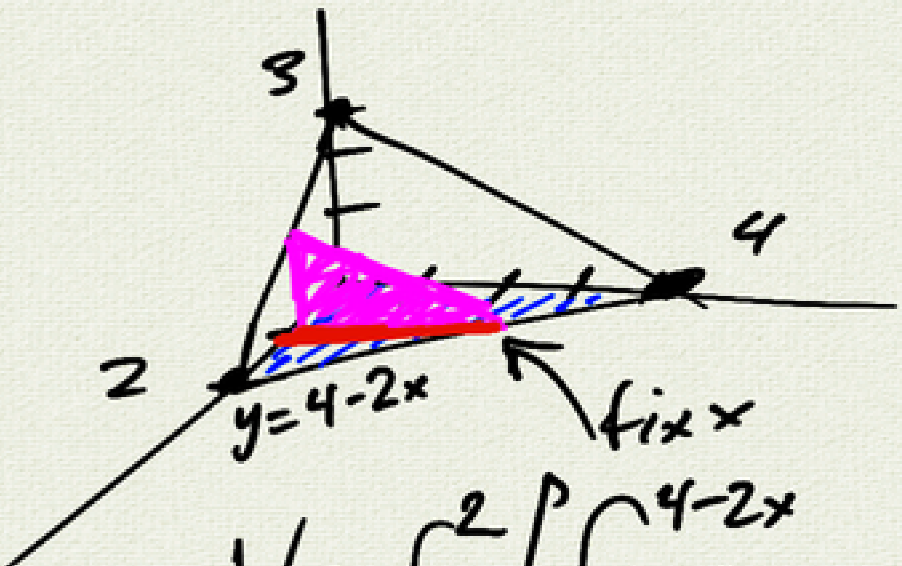
example:

$$6x+3y+4z=12$$

formula: $V=\frac{1}{3}Bh$

$$=\frac{1}{3}(4)(3)$$

$$=4$$



$$V=\int_0^2\left[\int_0^{4-2x} f(x,y)\,dy\right]dx$$

$$6x+3y+4z=12$$

$$z=3-\frac{3}{2}x-\frac{3}{4}y$$

$$=\int_0^2\int_0^{4-2x}\left(3-\frac{3}{2}x-\frac{3}{4}y\right)dy\,dx$$

$$=\int_0^2\left[\left(3-\frac{3}{2}x\right)y-\frac{3}{4}\left(\frac{y^2}{2}\right)\right]_0^{4-2x}dx$$

$$=\int_0^2\left[\left(3-\frac{3}{2}x\right)(4-2x)-\frac{3}{8}(4-2x)^2\right]dx$$

$$=\int_0^2\left[12-12x+3x^2-\frac{3}{8}\left(16-16x+4x^2\right)\right]dx$$

$$\left[12-12x+3x^2-6+6x-\frac{3}{2}x^2\right]$$

$$=\int_0^2\left(6-6x+\frac{3}{2}x^2\right)dx$$

$$=\left[6x-\frac{6x^2}{2}+\frac{3}{2}\frac{x^3}{3}\right]_0^2$$

$$=12-12+4$$

$$=4$$