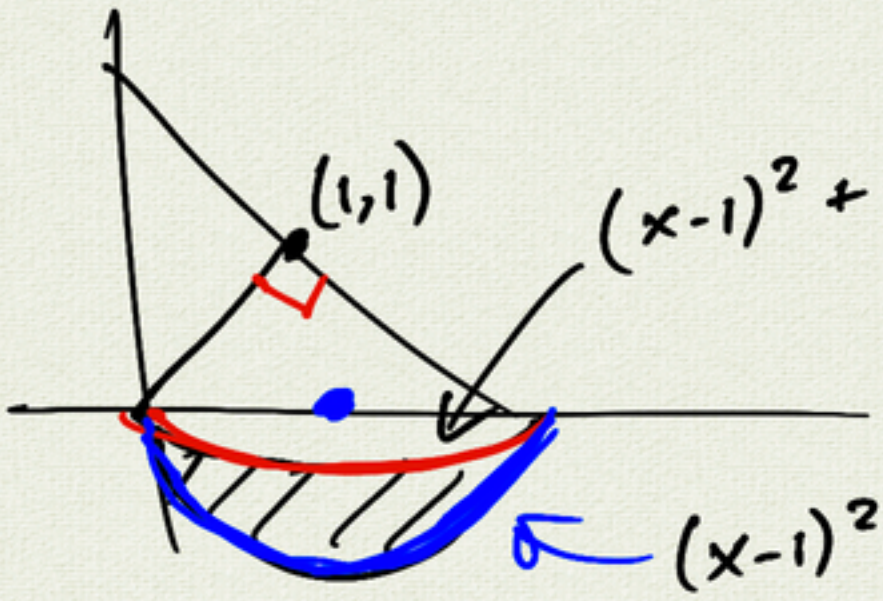
5.3 More more double integrals



$(x-1)^2 + (y-1)^2 = 2$

$(y-1)^2 = 2 - (x-1)^2$

$y = 1 \pm \sqrt{2-(x-1)^2}$

$(x-1)^2 + y^2 = 1$

$y^2 = 1 - (x-1)^2$

$y = \pm\sqrt{1-(x-1)^2}$

$$A = \int_0^2 \int_{-\sqrt{1-(x-1)^2}}^{1-\sqrt{2-(x-1)^2}} 1 \; dy \; dx$$

$$= \int_0^2 \left(1 - \sqrt{2-(x-1)^2} + \sqrt{1-(x-1)^2}\right) dx$$

$$= \int_0^2 dx - \boxed{\int_0^2 \sqrt{2-(x-1)^2}\,dx} + \int_0^2 \sqrt{1-(x-1)^2}\; dx$$

$$= 2 - \frac{\pi}{2} + \frac{\pi}{2}$$

$$\int_0^2 \sqrt{2-(x-1)^2}\,dx$$

$\boxed{x-1 = \sqrt{2}\sin\theta}$ ⟸

$dx = \sqrt{2}\cos\theta\, d\theta$

$x = 2 \Rightarrow x-1 = 1$

$\sqrt{2}\sin\theta = 1$

$\sin\theta = 1/\sqrt{2} = \sqrt{2}/2$

$\theta = \pi/4$

$x = 0 \Rightarrow x-1 = -1$

$\sqrt{2}\sin\theta = -1$

$\sin\theta = -\sqrt{2}/2$

$\theta = -\pi/4$

$$= \int_{-\pi/4}^{\pi/4} \left(\sqrt{2}\cos\theta\right)\left(\sqrt{2}\cos\theta\, d\theta\right)$$

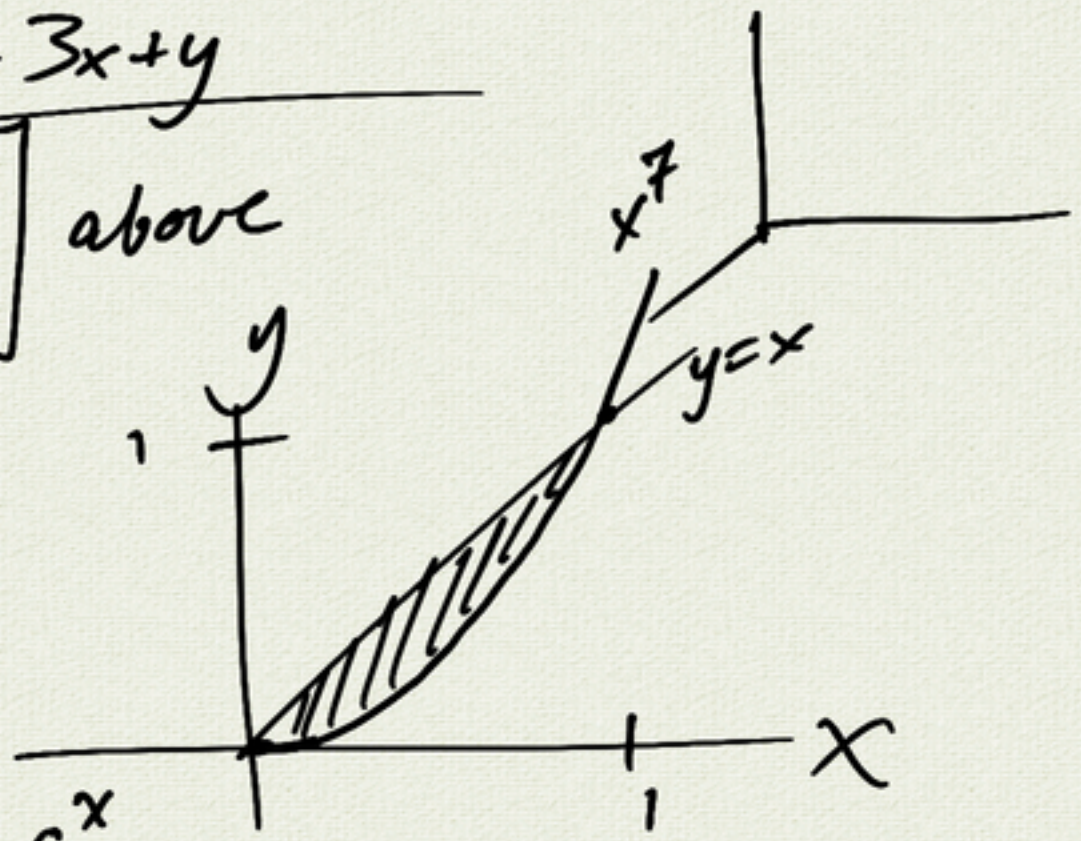
dx

$$= 2\int_{-\pi/4}^{\pi/4} \cos^2\theta\, d\theta$$

$$= \pi/2$$

(103) plane $z = 3x + y$ — below

between $y = x^7$ and $y = x$ — above



$$V = \int_0^1 \int_{x^7}^{x} (3x + y)\, dy\, dx$$

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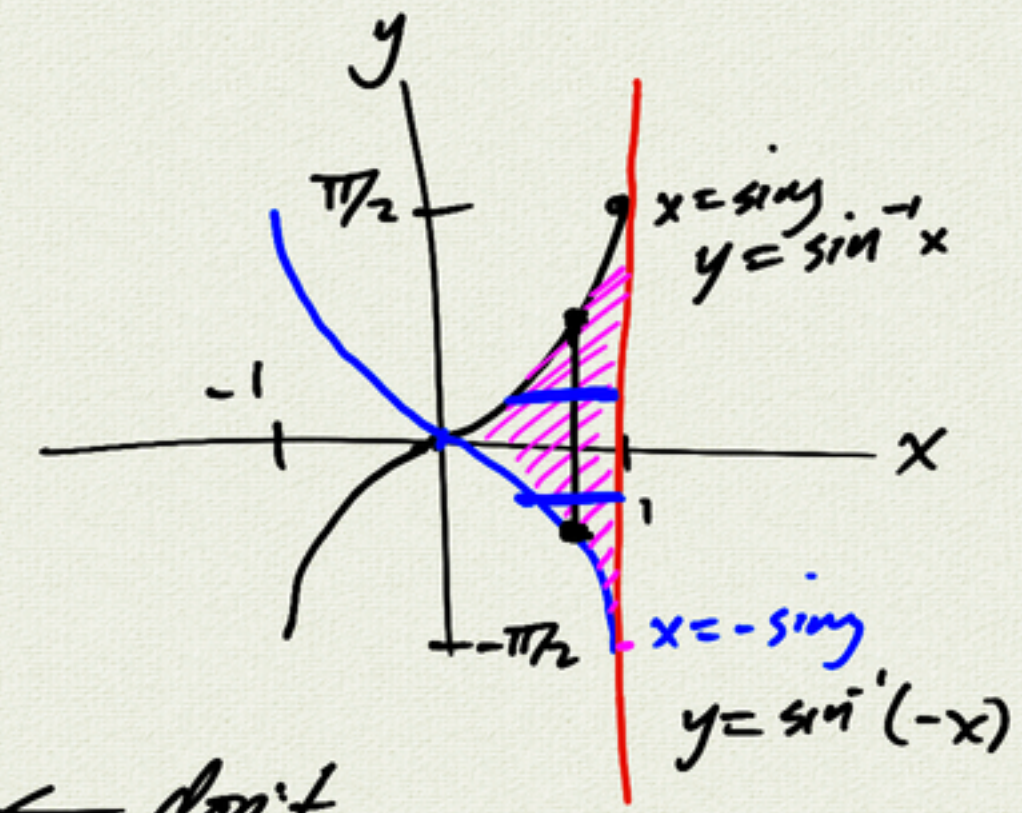
(105) Volume under $z = x^3$ $f(x,y)$

region: $x = \sin y$

$x = -\sin y$

$x = 1$

$-\pi/2 \le y \le \pi/2$



$$V = \int_0^1 \int_{\sin^{-1}(-x)}^{\sin^{-1}(x)} x^3\, dy\, dx$$ ⟸ don't do this

$$= \int_{-\pi/2}^{0} \int_{-\sin y}^{1} x^3\, dx\, dy + \int_0^{\pi/2} \int_{\sin y}^{1} x^3\, dx\, dy$$

power reduction formulas

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

double angle

$\Rightarrow \sin(u+v) = \sin u\cos v + \cos u\sin v$

$\cos(u+v) = \cos u\cos v - \sin u\sin v$

sum formulas

$(\cos^2\theta = 1-\sin^2\theta)$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

$= 2\cos^2\theta - 1$

$\cos 2\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta = 1 - \cos 2\theta$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$ power reducing

$\cos 2\theta = 2\cos^2\theta - 1$

$2\cos^2\theta = 1 + \cos 2\theta$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\sin^4\theta = (\sin^2\theta)^2$

$= \left(\frac{1-\cos 2\theta}{2}\right)^2$

$= \frac{1}{4}(1 - 2\cos 2\theta + \cos^2 2\theta)$

↖ use power reducing again

(111) region:

$x+y=1$

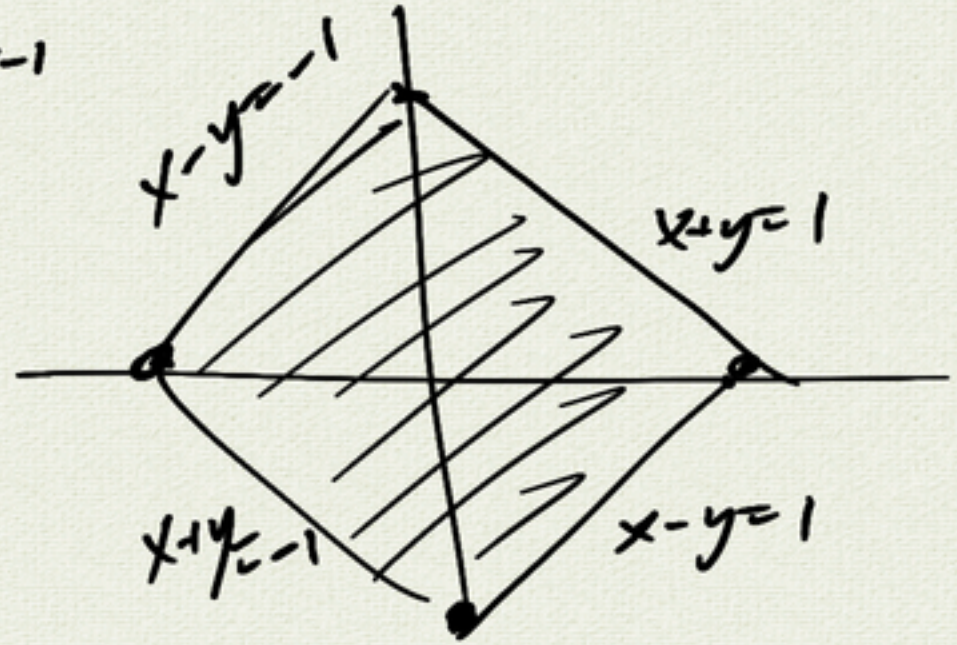
$x-y=1$ ← $y=x-1$

$x+y=-1$

$x-y=-1$

$z=0$

$z=1$



$V=2$