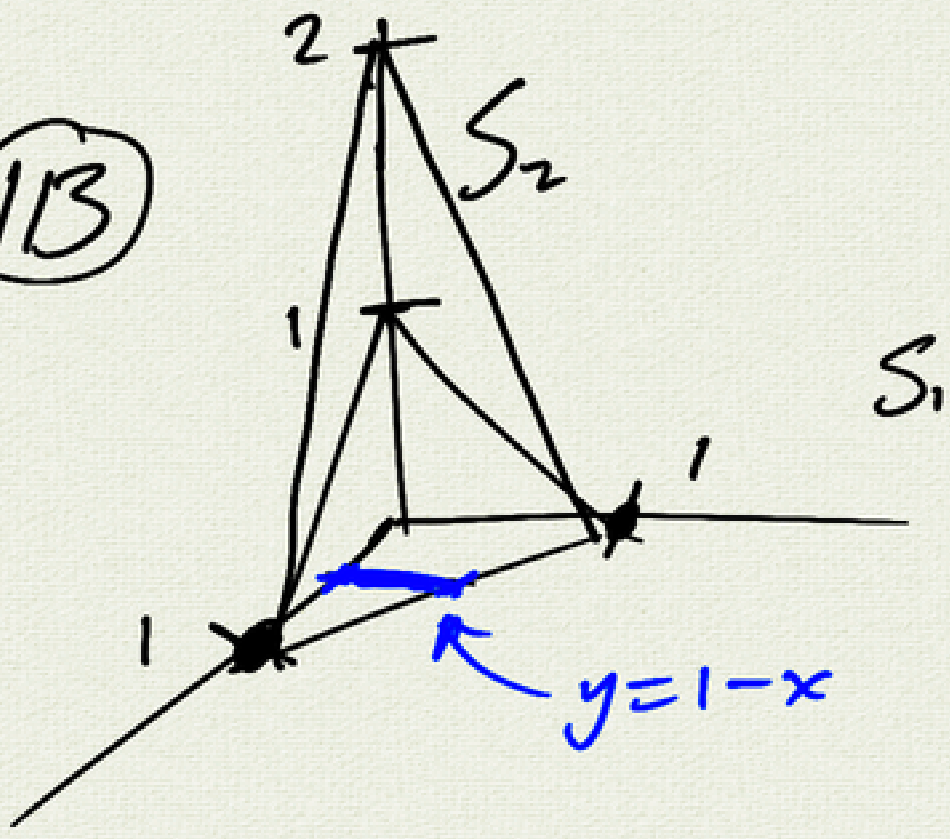
(113)



$z=1-x-y$

S_1: $x+y+z=1$

S_2: $2x+2y+z=2$

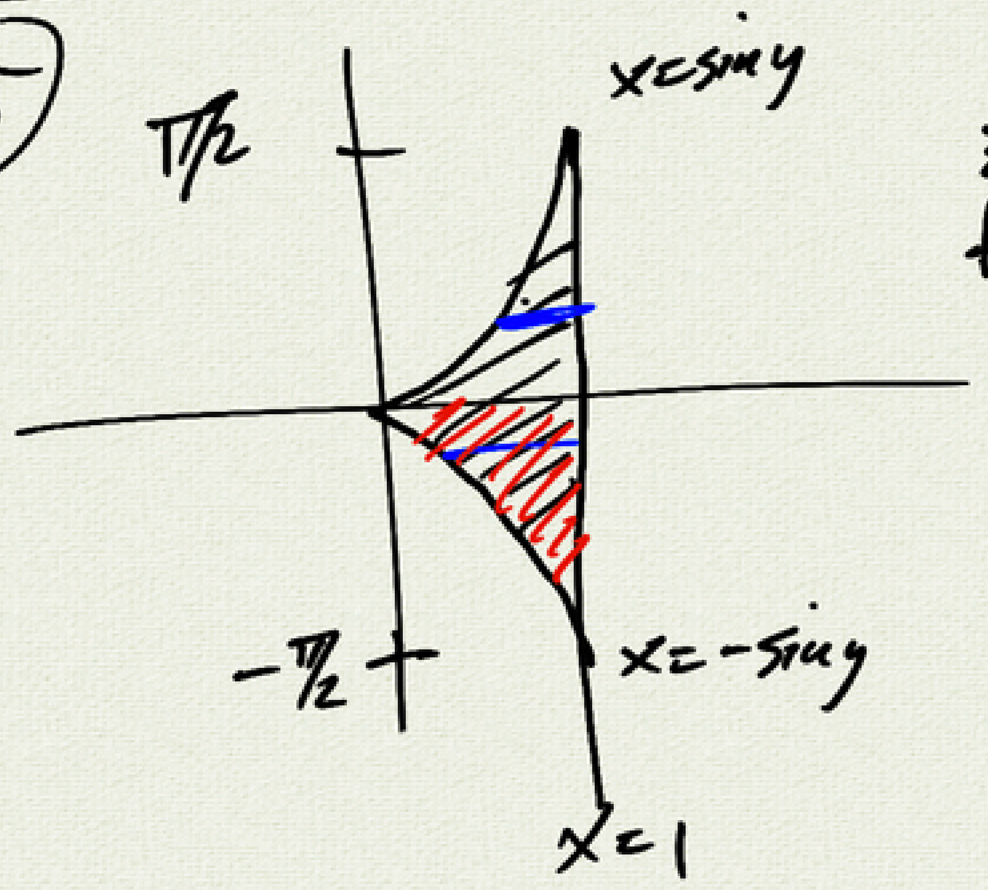
$z=2-2x-2y$

$=2(1-x-y)$

$$V_1=\int_0^1\left[\int_0^{1-x}(1-x-y)\,dy\right]dx$$

---

(105)



$z=x^3$
$f(x,y)$

$$V=\int_{-\pi/2}^{0}\int_{-\sin y}^{1}x^3\,dx\,dy+\int_0^{\pi/2}\int_{\sin y}^{1}x^3\,dx\,dy$$

$$=2\int_0^{\pi/2}\int_{\sin y}^{1}x^3\,dx\,dy$$

$$=2\int_0^{\pi/2}\left[\frac{x^4}{4}\right]_{\sin y}^{1}dy$$

$$=\frac{1}{2}\int_0^{\pi/2}(1-\sin^4 y)\,dy$$

$$=\frac{1}{2}\int_0^{\pi/2}1\,dy-\underbrace{\frac{1}{2}\int_0^{\pi/2}\sin^4 y\,dy}$$

$$=\frac{\pi}{4}-\underbrace{\boxed{}}$$

$$\frac{1}{2}\int_0^{\pi/2}\sin^4 y\,dy$$

$$=\frac{1}{8}\int_0^{\pi/2}(1-\cos 2y)^2\,dy$$

$$=\frac{1}{8}\int_0^{\pi/2}(1-2\cos 2y+\cos^2 2y)\,dy$$

$$=\frac{\pi}{16}-\underbrace{\frac{1}{4}\int_0^{\pi/2}\cos 2y\,dy}_{0}+\frac{1}{8}\int_0^{\pi/2}\cos^2 2y\,dy$$

$$\frac{1}{16}\int_0^{\pi/2}(\underbrace{1}_{\frac{\pi}{32}}+\underbrace{\cos 4y}_{0})\,dy$$

$$=\frac{3\pi}{32}$$

$\sin^2\theta=\dfrac{1-\cos 2\theta}{2}$ $\cos^2\theta=\dfrac{1+\cos 2\theta}{2}$

$=\frac{1}{2}(1-\cos 2\theta)$

$\sin^4\theta=(\sin^2\theta)^2$

$=\left[\frac{1}{2}(1-\cos 2\theta)\right]^2$

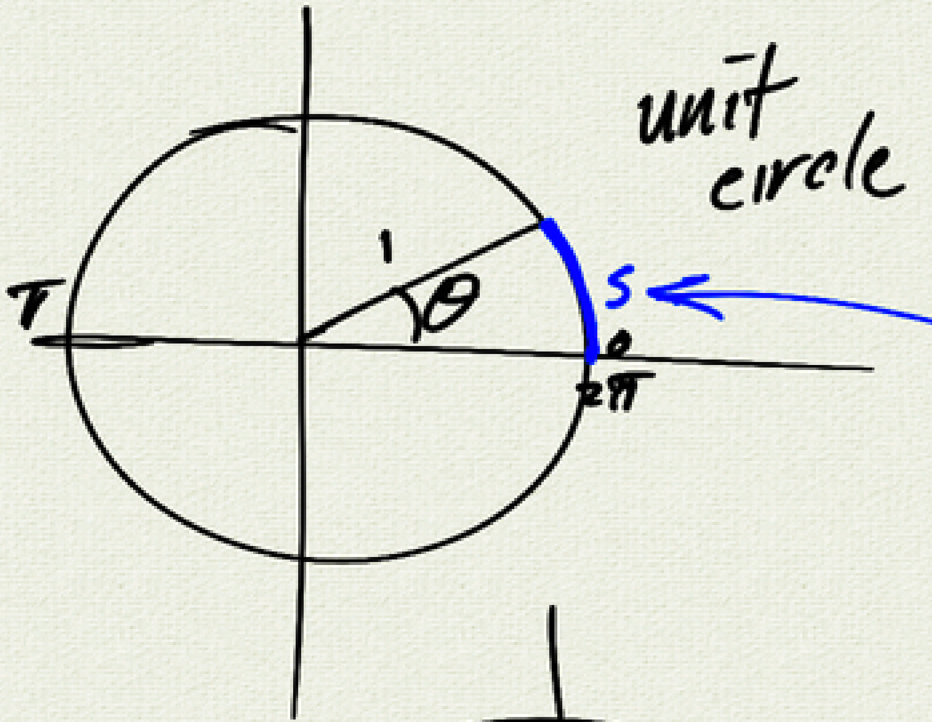
$=\frac{1}{4}(1-\cos 2\theta)^2$

$\cos^2 2y=\dfrac{1+\cos 2(2y)}{2}$

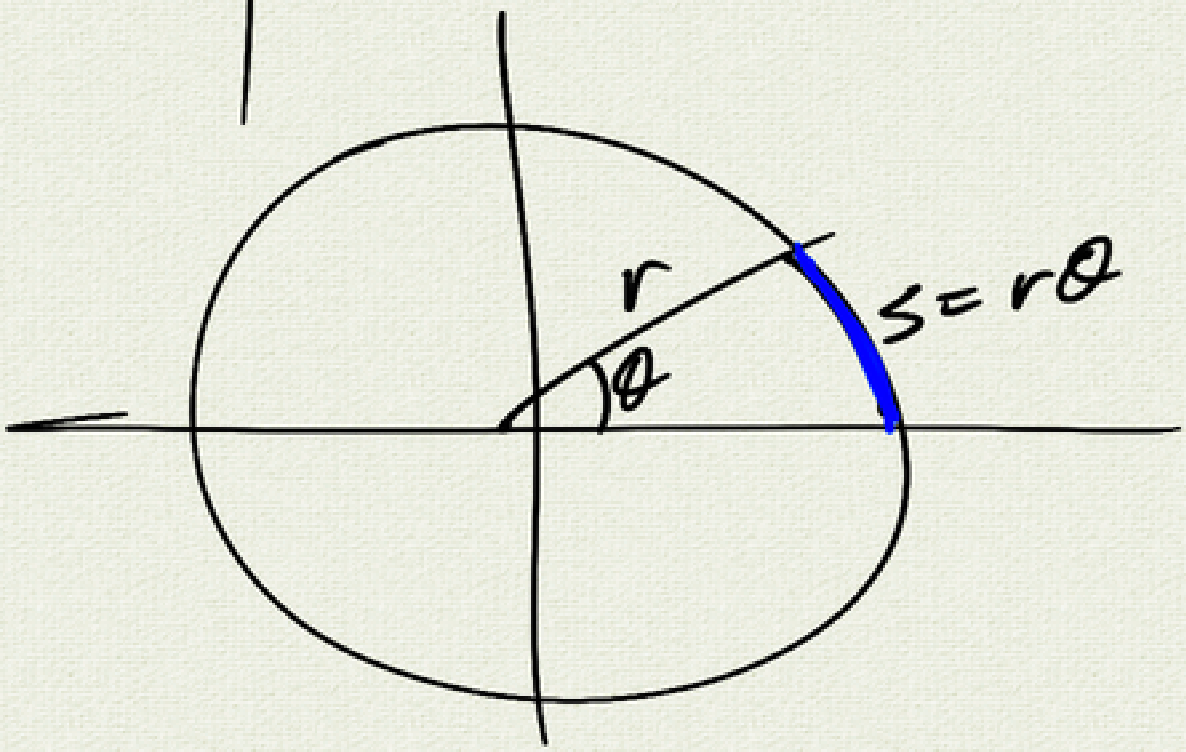
$=\frac{1}{2}(1+\cos 4y)$

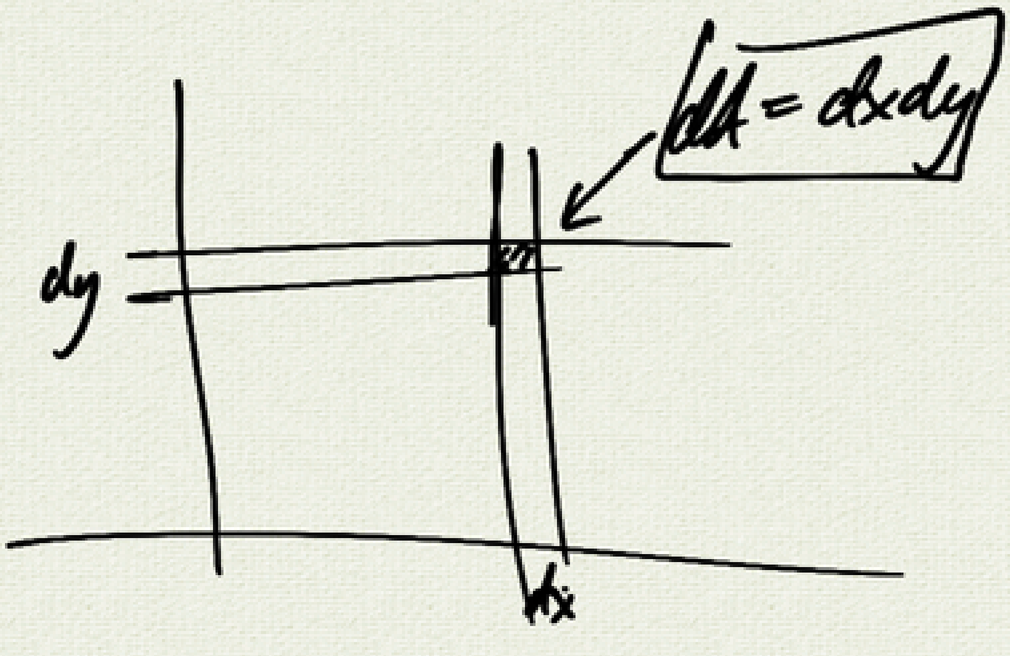
$\frac{d}{dx}\left(+\frac{1}{4}\sin 4y\right)=\cos 4y$

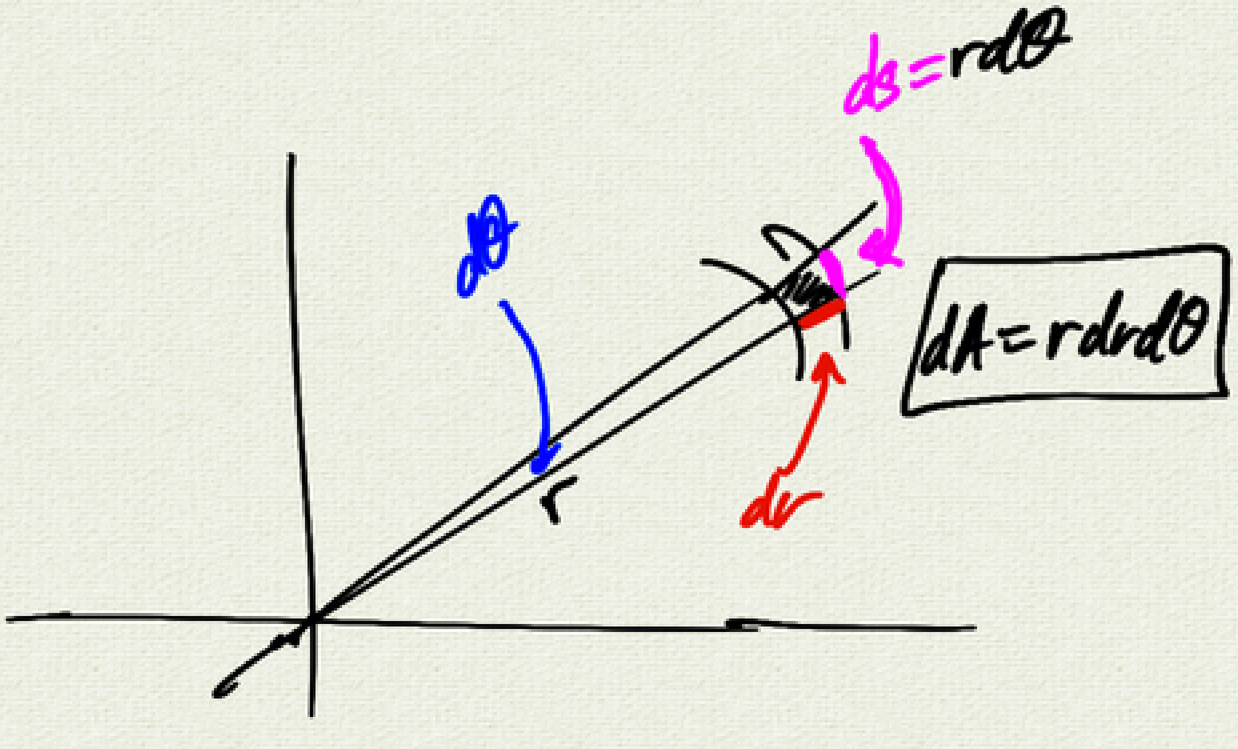
5.4 Polar Coordinates (double integration)



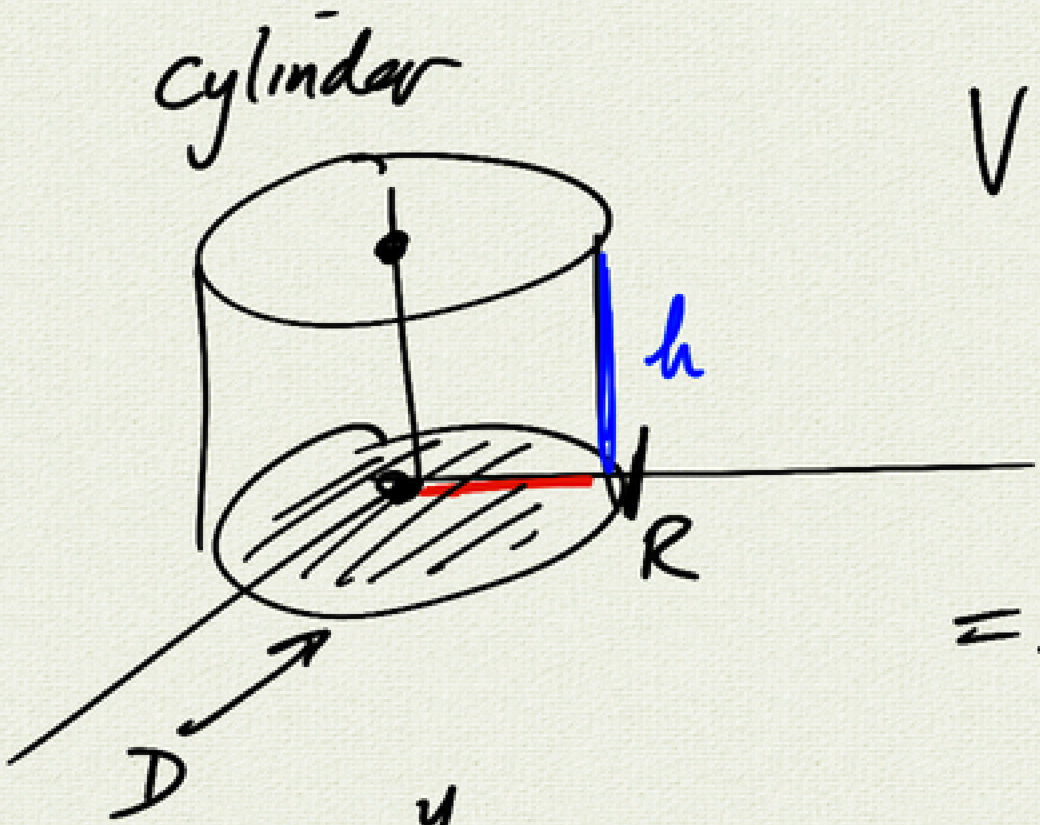
$$s = 2\pi \cdot \left(\frac{\theta}{2\pi}\right) = \theta$$







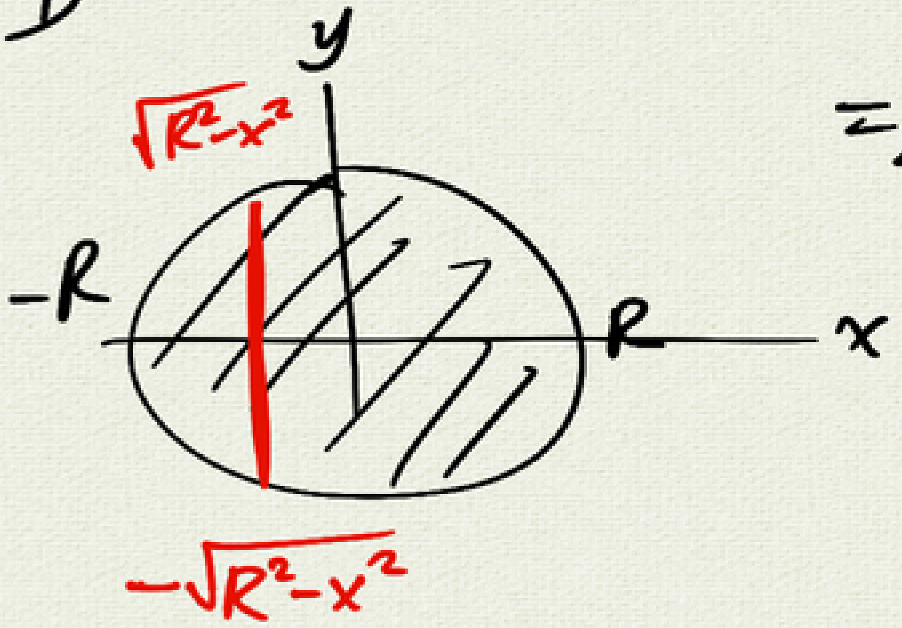
cylinder



$$V=\iint_D h\,dA$$

$$=h\iint_D dA$$

$$=h\int_{-R}^{R}\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} dy\,dx$$



$$=2h\int_{-R}^{R}\sqrt{R^2-x^2}\,dx$$

(trig sub)

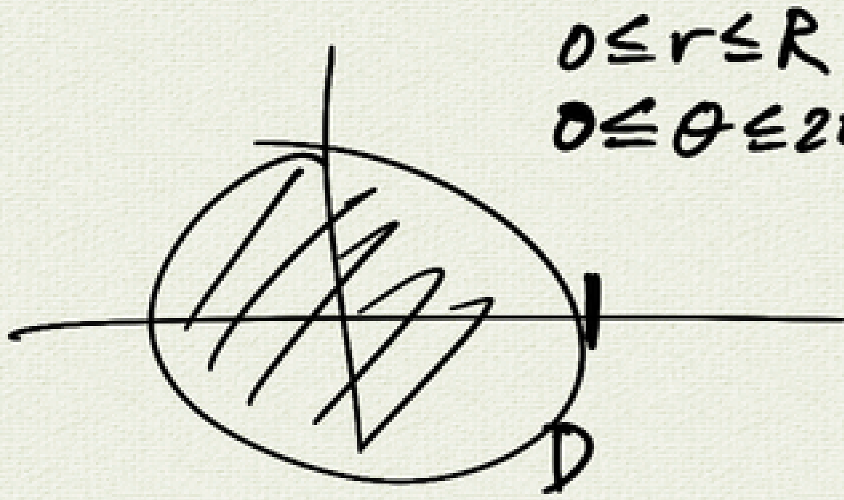
$$\Rightarrow \pi R^2 h$$

---

polar:

$0\le r\le R$

$0\le\theta\le 2\pi$



$$V=\iint_D h\,dA$$

$$=\int_0^{2\pi}\int_0^R h\,r\,dr\,d\theta$$

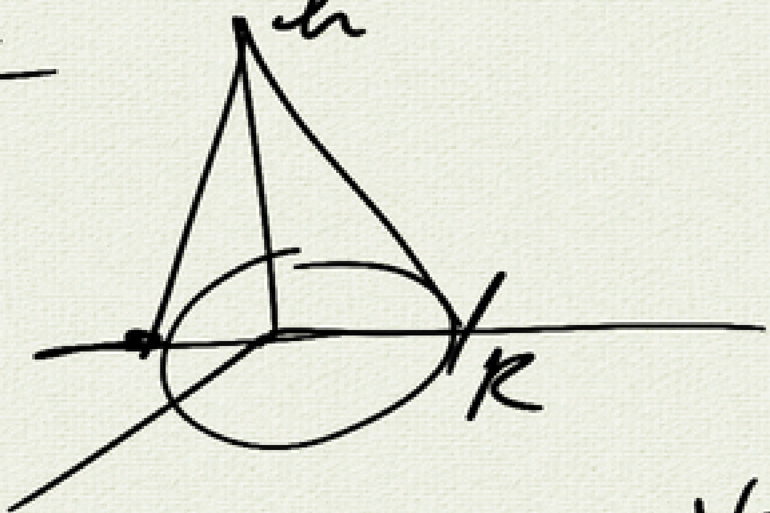
$$=h\int_0^{2\pi}\left[\frac{r^2}{2}\right]_0^R d\theta$$

$$=h\int_0^{2\pi}\frac{R^2}{2}\,d\theta$$

$$=h\frac{R^2}{2}\cdot 2\pi$$

$$=\pi R^2 h$$

cone



$$z = h - \frac{h}{R} r \Leftarrow$$

$$= h - \frac{h}{R}\sqrt{x^2+y^2}$$

$$V = \iint_D \left(h - \underbrace{\frac{h}{R}\sqrt{x^2+y^2}\,}_{?}\right) dx\,dy$$

polar:

$$V = \int_0^{2\pi}\int_0^{R} \left(h - \frac{h}{R} r\right) r\,dr\,d\theta$$

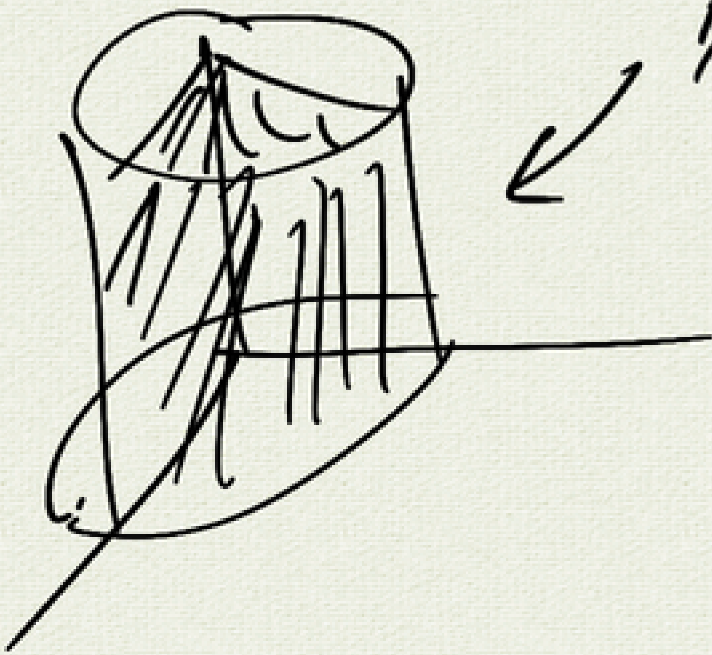
$$= \int_0^{2\pi}\int_0^{R} \left(hr - \frac{h}{R} r^2\right) dr\,d\theta$$

$$= h\int_0^{2\pi} \left[\frac{R^2}{2} - \frac{1}{R}\frac{R^3}{3}\right] d\theta$$

$$= h\int_0^{2\pi} \Big(\underbrace{\frac{R^2}{2} - \frac{R^2}{3}}_{R^2/6}\Big)\, d\theta$$

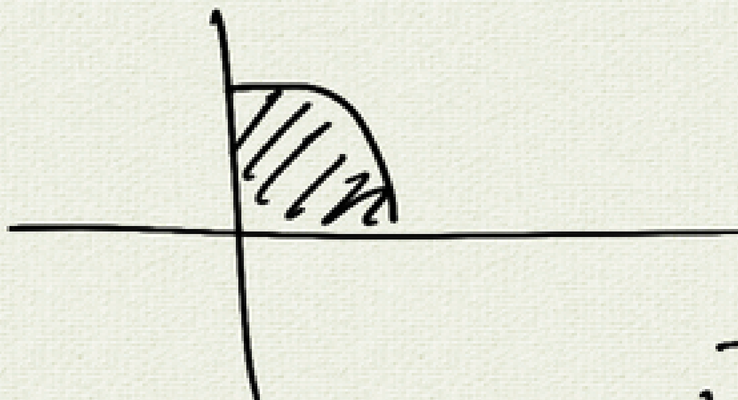
$$= h\frac{R^2}{6}\,2\pi$$

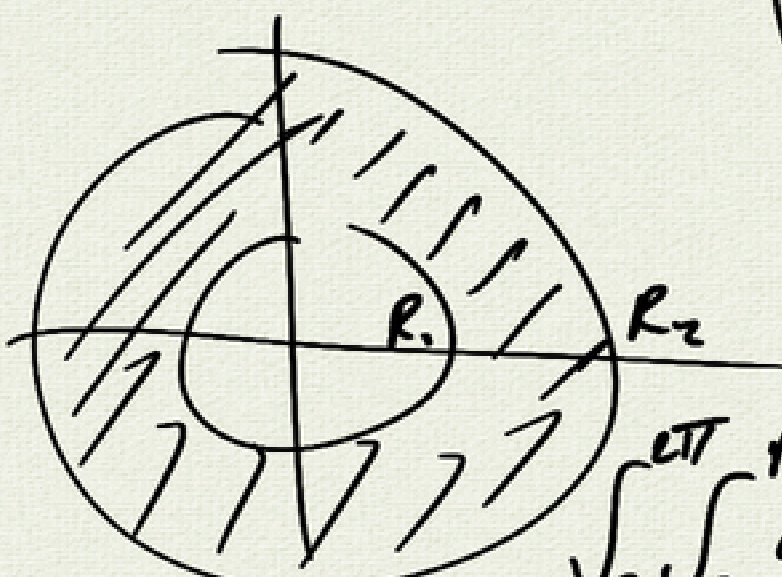
$$= \frac{\pi R^2 h}{3}$$



1/4 cylinder

$$\Rightarrow V = \int_0^{\pi/2} \int_0^R$$



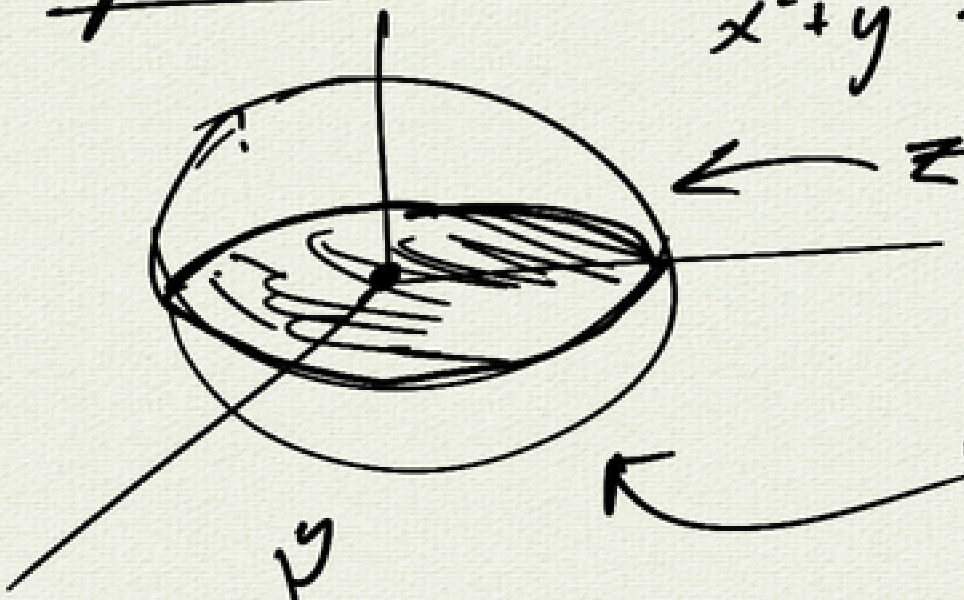


$$\int_0^{2\pi} \int_{R_1}^{R_2} dr\, d\theta$$

$$\int_0^{\pi/2} \int_{R_1}^{R_2} dr\, d\theta$$

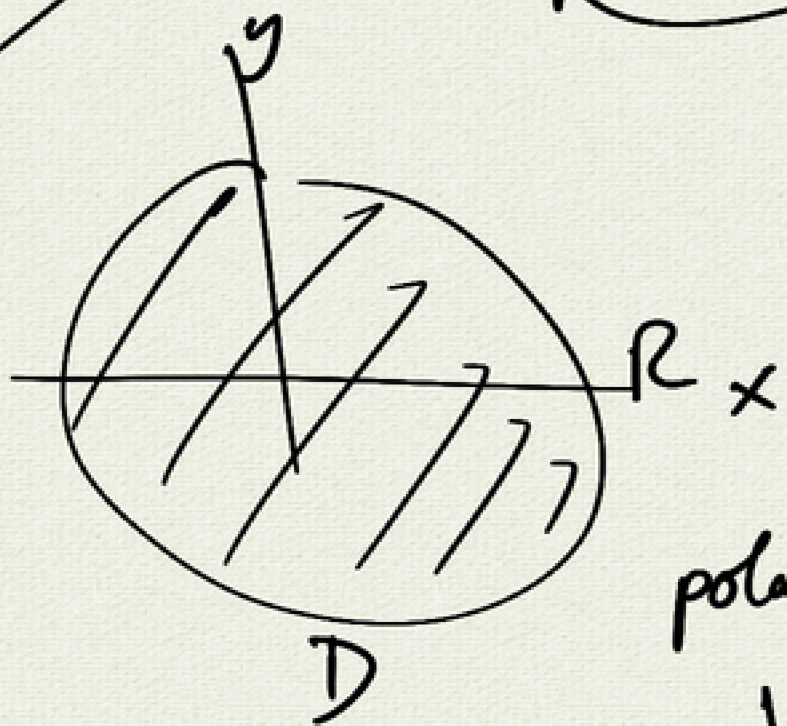
Sphere

$$x^2+y^2+z^2=R^2$$



$$z=+\sqrt{R^2-x^2-y^2}$$

bottom: $-\sqrt{R^2-x^2-y^2}$



$$V=\iint_D 2\underbrace{\sqrt{R^2-x^2-y^2}}_{?}\;\overset{dA}{\boxed{dx\,dy}}$$

$$\boxed{\begin{gathered}x^2+y^2=r^2\\ -(x^2+y^2)=-r^2\end{gathered}}$$

polar:

$$V=\int_0^{2\pi}\int_0^R 2\underbrace{\sqrt{R^2-r^2}}\;\left(\overset{dA}{\boxed{r\,dr\,d\theta}}\right)$$

yay!

$u=R^2-r^2$

$du=-2r\,dr$

$\frac{du}{-2}=r\,dr$

$$=2\int_0^{2\pi}\int_{R^2}^{0}\frac{\sqrt{u}}{-2}\,du\,d\theta$$

$$=\int_0^{2\pi}\int_0^{R^2}\sqrt{u}\,du\,d\theta$$

$$=\int_0^{2\pi}\left[\frac{2}{3}(u)^{3/2}\right]_0^{R^2}d\theta$$

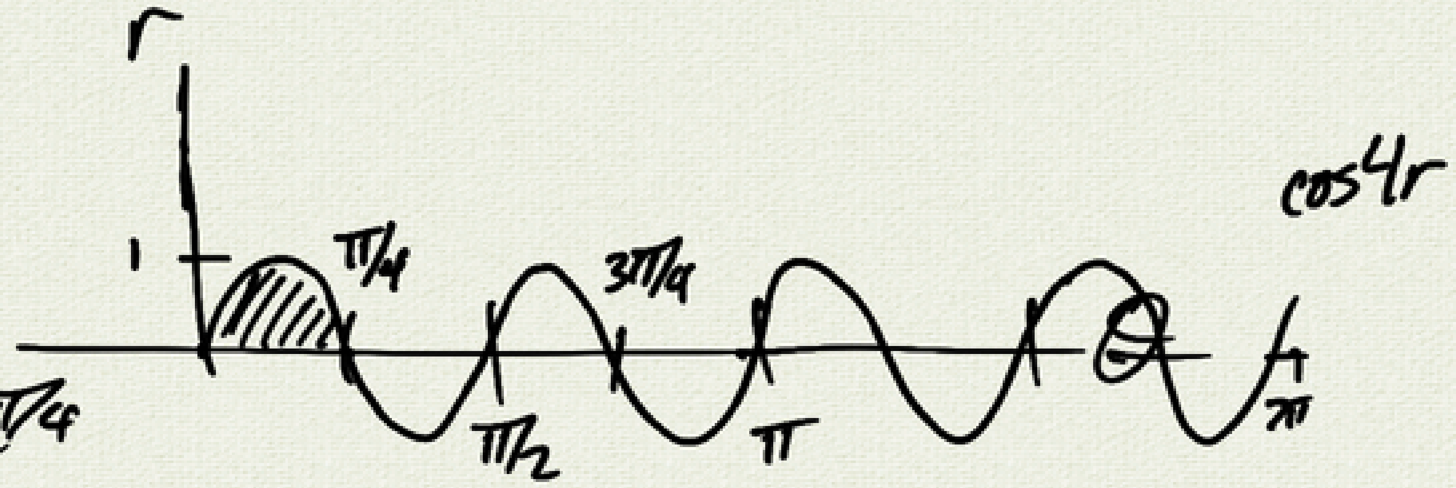
$$\frac{d}{du}\left(\frac{2}{3}u^{3/2}\right)=\frac{2}{3}\cdot\frac{3}{2}u^{1/2}=u^{1/2}$$

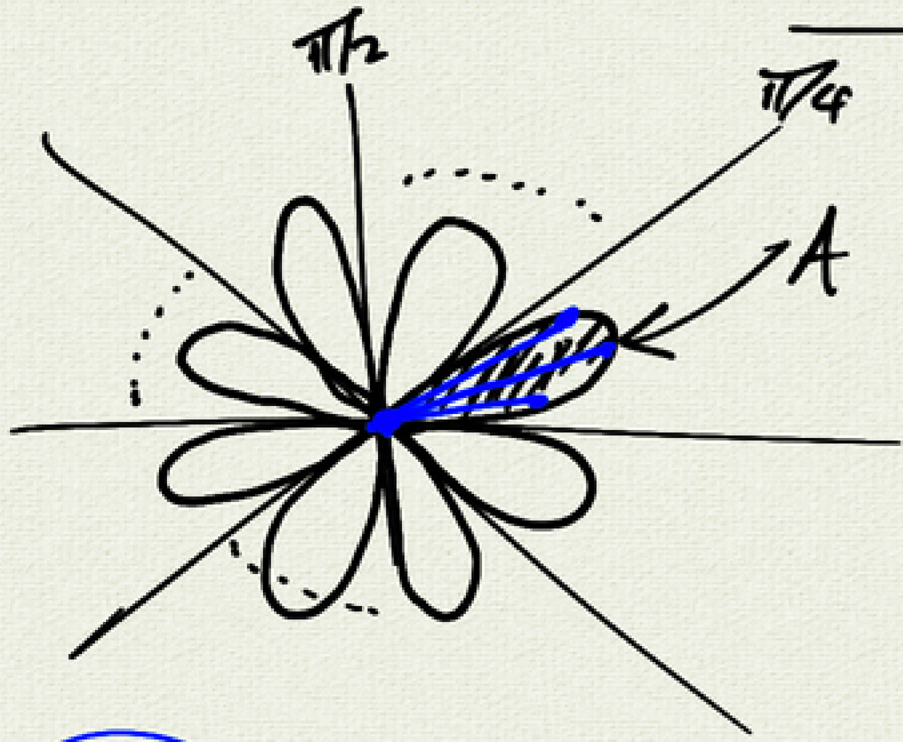
$$=\frac{2}{3}\left[R^3\right]\int_0^{2\pi}d\theta$$

$$\boxed{V_{sphere}=\frac{4\pi}{3}R^3}$$

polar curve

$$\boxed{r=\sin 4\theta}$$





A = area of first petal

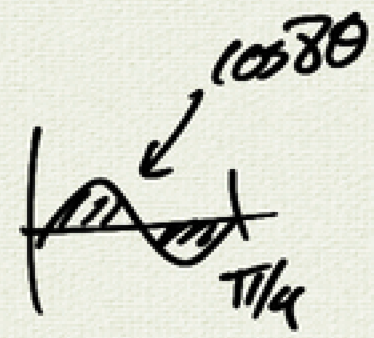
$$A=\iint_R 1\, dA$$

$$=\int_0^{\pi/4}\int_0^{\sin 4\theta} 1 \quad r\, dr\, d\theta$$

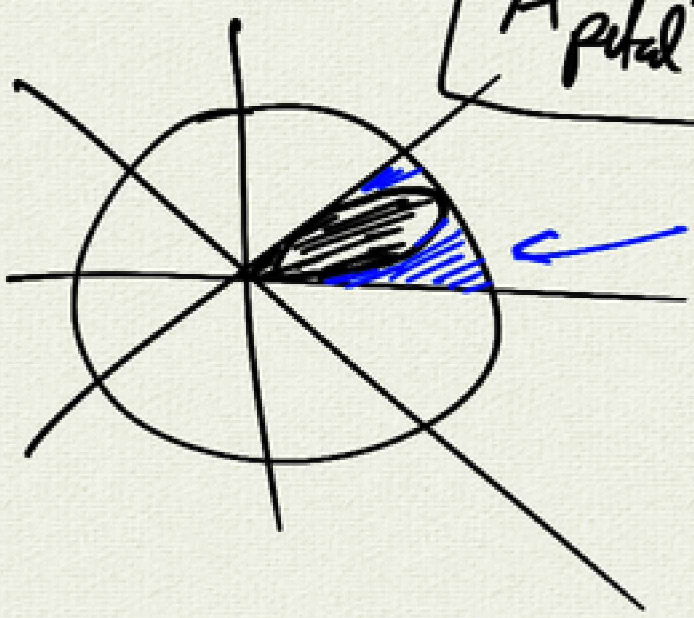
$$A=\int_{\theta_1}^{\theta_2}\frac{r^2}{2}\,d\theta \implies =\int_0^{\pi/4}\frac{(\sin 4\theta)^2}{2}\,d\theta$$

$$=\frac{1}{2}\int_0^{\pi/4}\frac{1}{2}(1-\cos 8\theta)\,d\theta$$

$$=\frac{1}{4}\int_0^{\pi/4}d\theta-\underbrace{\frac{1}{4}\int_0^{\pi/4}\cos 8\theta\,d\theta}_{0}$$



$$\boxed{A_{petal}=\frac{\pi}{16}}$$



$A_{sector}=\dfrac{\pi}{8}$

petal is half the area of the sector