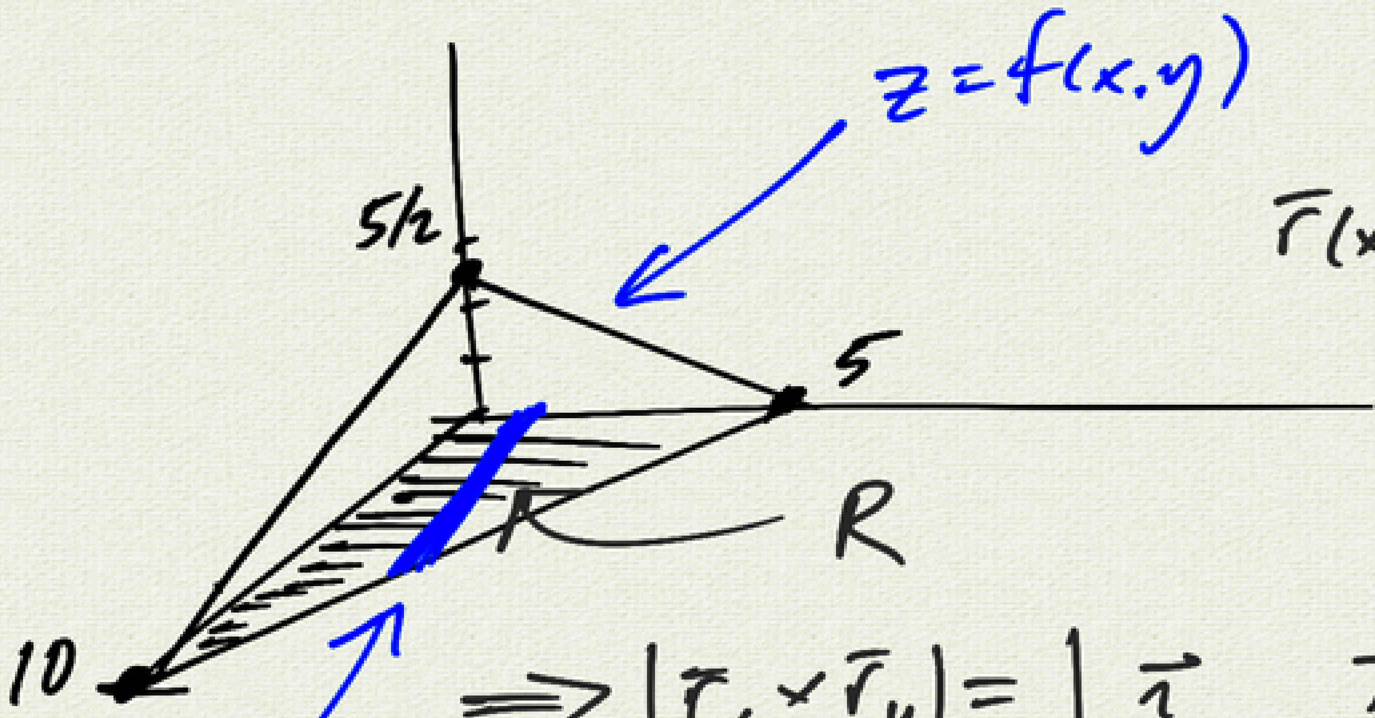
Guichard 16.6

(3) area of $x+2y+4z=10$ in first octant

$$4z = 10 - x - 2y$$
$$z = \frac{5}{2} - \frac{x}{4} - \frac{y}{2}$$



$x+2y=10$

$x=10-2y$

$$\bar{r}(x,y) = \begin{pmatrix} x \\ y \\ \frac{5}{2} - \frac{x}{4} - \frac{y}{2} \end{pmatrix}$$

$$\Longrightarrow |\bar{r}_x \times \bar{r}_y| = \left| \begin{vmatrix} \vec{\imath} & \vec{\jmath} & k \\ 1 & 0 & -1/4 \\ 0 & 1 & -\frac{1}{2} \end{vmatrix} \right|$$

$$= \left| \left\langle \frac{1}{4}, +\frac{1}{2}, 1 \right\rangle \right|$$

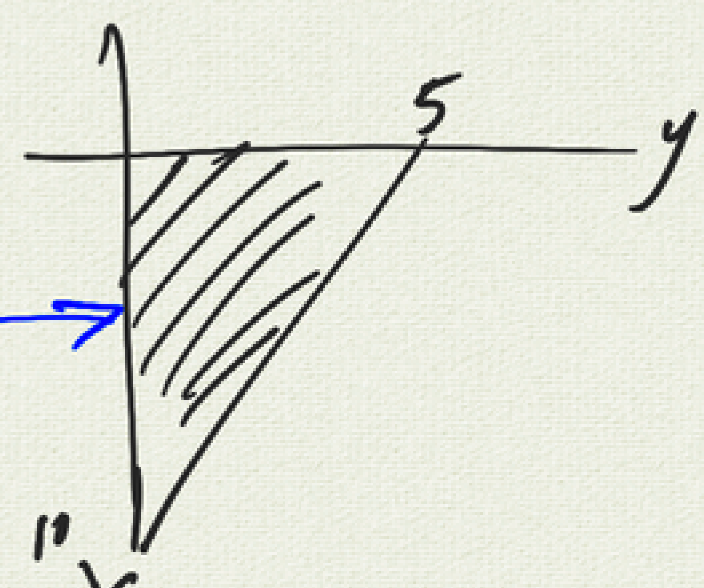
$$= \sqrt{1 + \frac{1}{4} + \frac{1}{16}}$$

$$= \frac{\sqrt{21}}{4}$$

$$SA = \iint_R |\bar{r}_x \times \bar{r}_y| \, dx\,dy$$

$$= \int_0^5 \int_0^{10-2y} \frac{\sqrt{21}}{4} \, dx\,dy$$

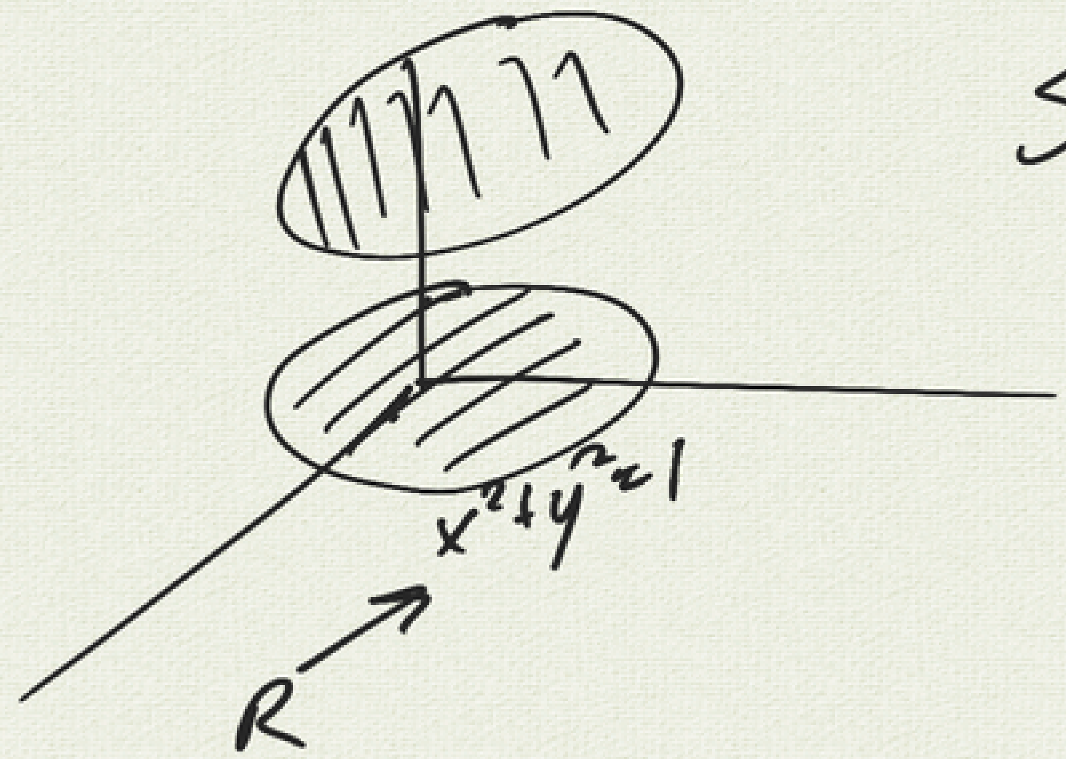
$$= \frac{\sqrt{21}}{4} \underbrace{\iint_R dx\,dy}_{\text{area of } \Delta = 25}$$



④ $2x+4y+z=0$

inside $x^2+y^2=1$

$|\bar r_x \times \bar r_y| = \sqrt{21}$



$$SA = \iint_R |\bar r_x \times \bar r_y| \, \underbrace{dx\,dy}$$

$$= \iint_R \sqrt{21}\,dx\,dy$$

$$= \sqrt{21}\underbrace{\iint_R dx\,dy}_{\pi}$$

$$\iint_R dx\,dy = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy\,dx$$

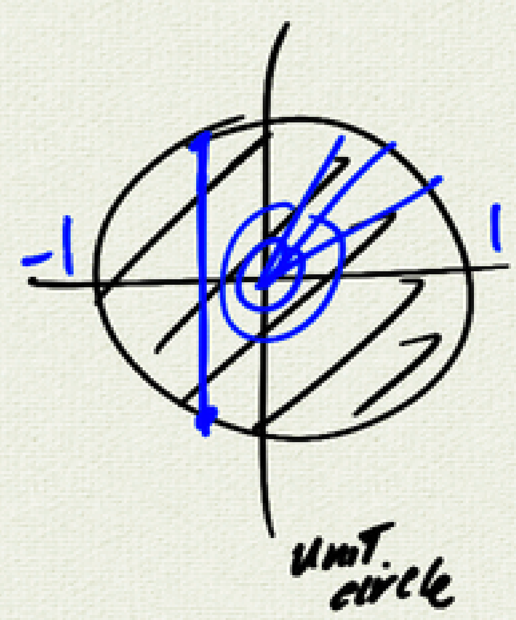
$$= \int_{-1}^{1} 2\sqrt{1-x^2}\,dx \Leftarrow \text{trig sub.}$$

$$= \pi$$

or: $$\iint_R \underbrace{dx\,dy}_{dA} = \int_0^{2\pi}\int_0^1 1\cdot \boxed{\underbrace{r\,dr\,d\theta}_{dA}}$$

$$= \int_0^{2\pi} \frac{r^2}{2}\Big|_0^1 d\theta$$

$$= \pi$$

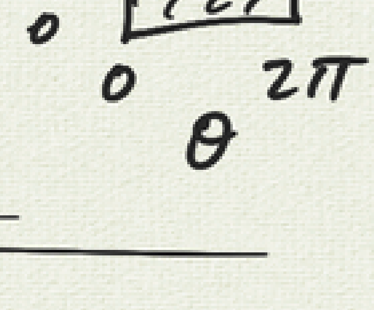


$\bar r(r,\theta) \Longrightarrow |\bar r_r \times \bar r_\theta|$

$$SA = \int_0^{2\pi}\int_0^1 \boxed{|\bar r_r \times \bar r_\theta|}\,dr\,d\theta$$

$\sqrt{21}\,r$

no r

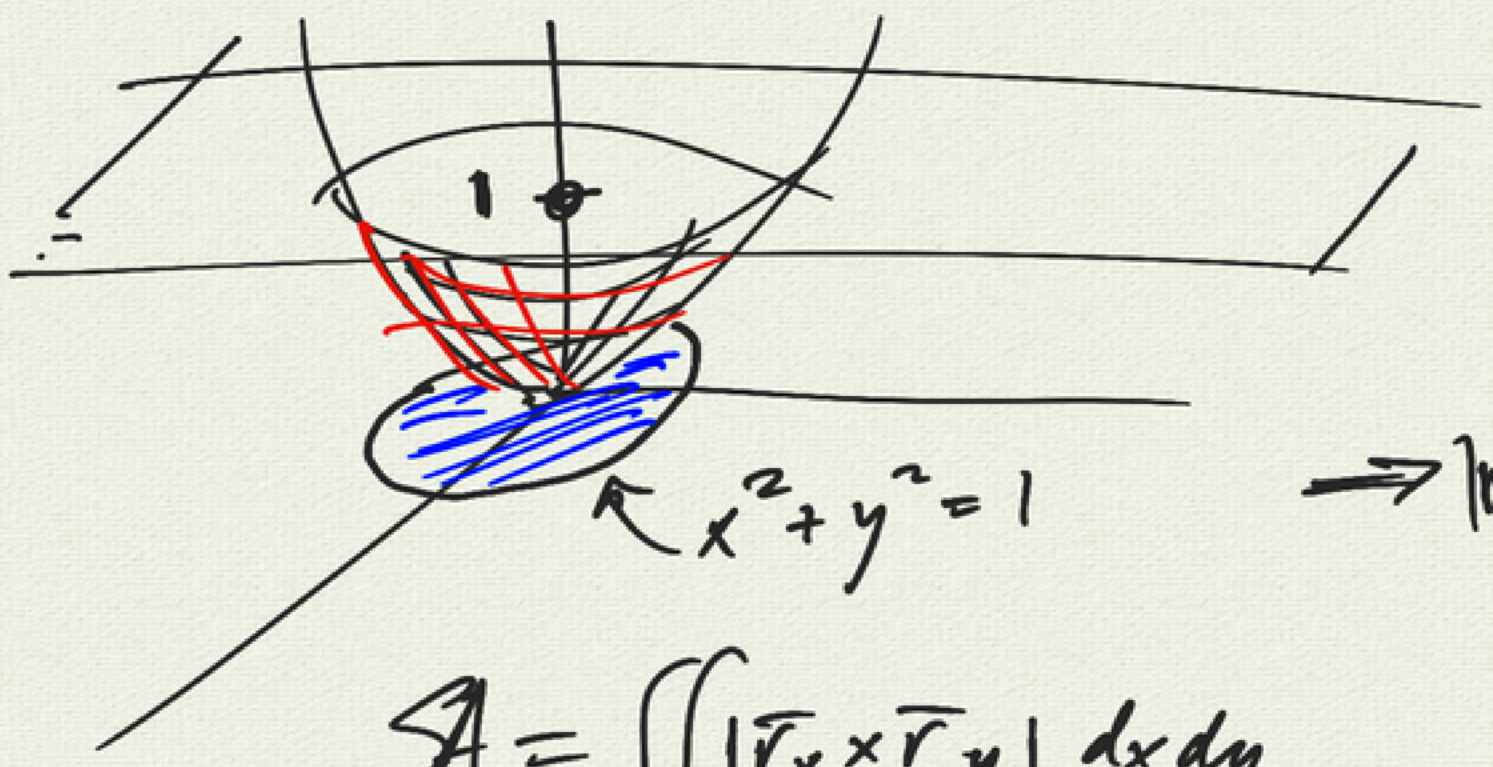


$\bar r(u,v) \Longrightarrow SA = \iint |\bar r_u \times \bar r_v|\,du\,dv$

paraboloid

$$\begin{pmatrix} x \\ y \\ f(x,y) \end{pmatrix}$$

(5) area of $\boxed{z = x^2 + y^2}$ below $z = 1$



$x^2 + y^2 = 1$

$$\bar{r}(x,y) = \begin{pmatrix} x \\ y \\ x^2+y^2 \end{pmatrix} \Rightarrow |\bar{r}_x \times \bar{r}_y| = \begin{array}{c} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 2x \\ 0 & 1 & 2y \end{vmatrix} \end{array} \begin{array}{l} \\ f_x \\ f_y \end{array}$$

$$\boxed{= \sqrt{1 + f_x^2 + f_y^2}}$$

$$\Rightarrow |r_x \times r_y| = \sqrt{1 + 4x^2 + 4y^2}$$

$$SA = \iint_D |\bar{r}_x \times \bar{r}_y| \, dx\,dy$$

$$= \iint_D \sqrt{1 + 4x^2 + 4y^2} \, dx\,dy$$

$$= \iint_D \sqrt{1 + 4r^2} \, r\,dr\,d\theta \Leftarrow \text{u-sub}$$

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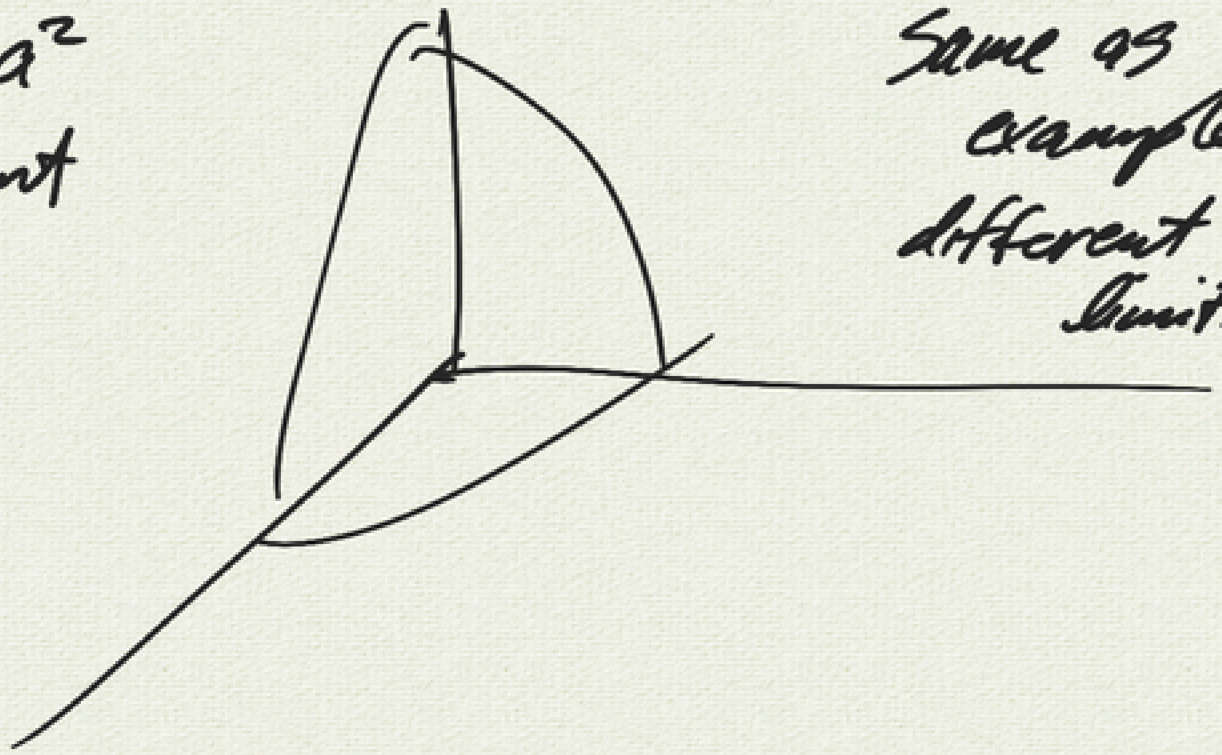
(7) $x^2 + y^2 + z^2 = a^2$
in first octant



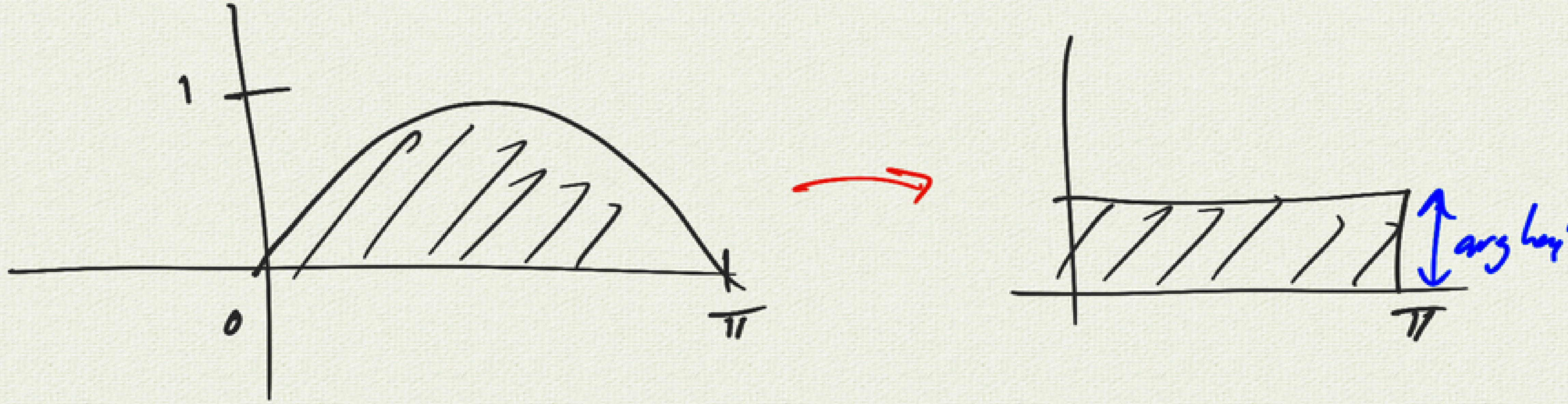
Same as example, different limits

5.7 Triple integrals

warm up:

$$f(x) = \sin x$$

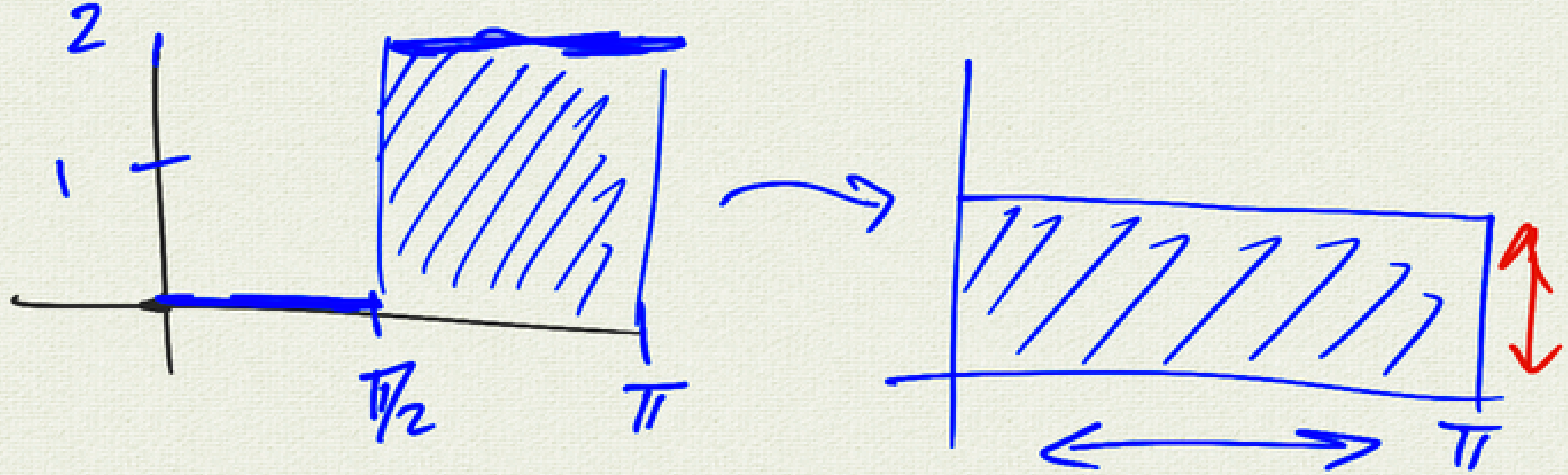
what is the average value of f on $[0, \pi]$



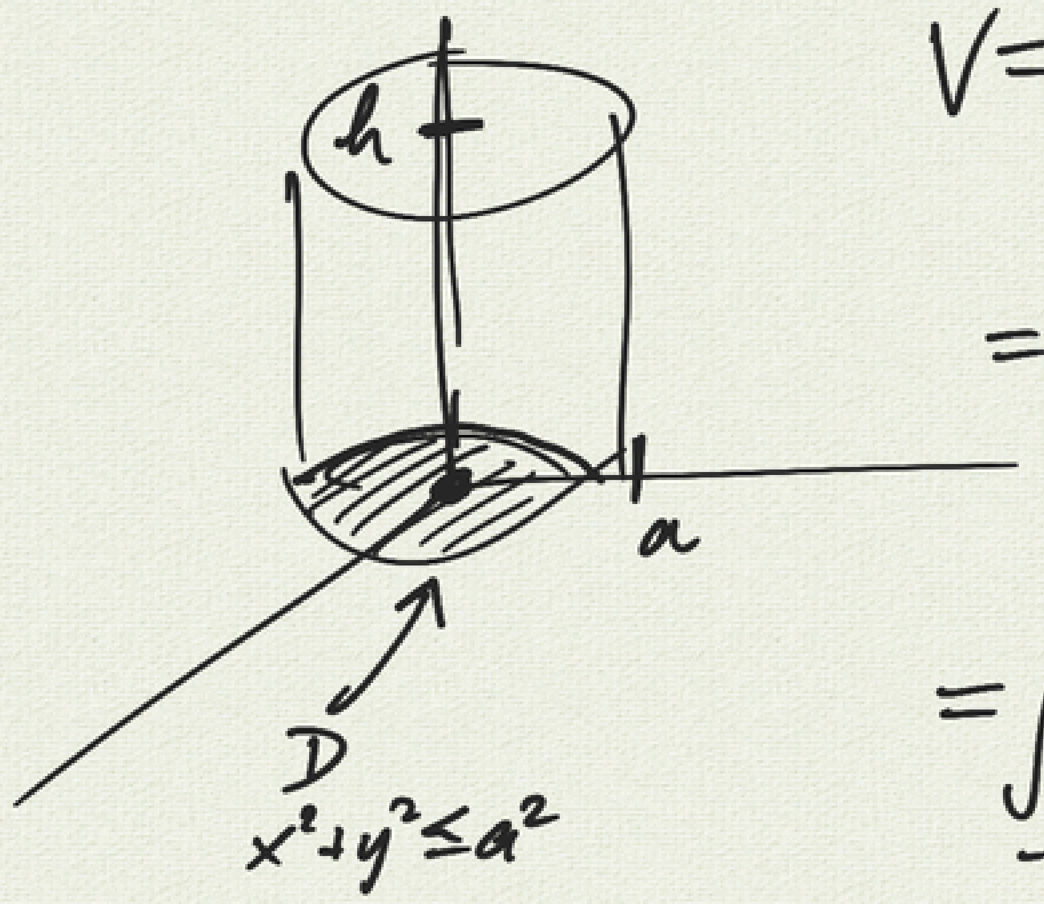
$$= \frac{\int_0^{\pi} \sin x \, dx}{\pi}$$

$$= \frac{\int_0^{\pi} \sin x \, dx}{\int_0^{\pi} dx}$$

$$= \frac{2}{\pi}$$



example: cylinder



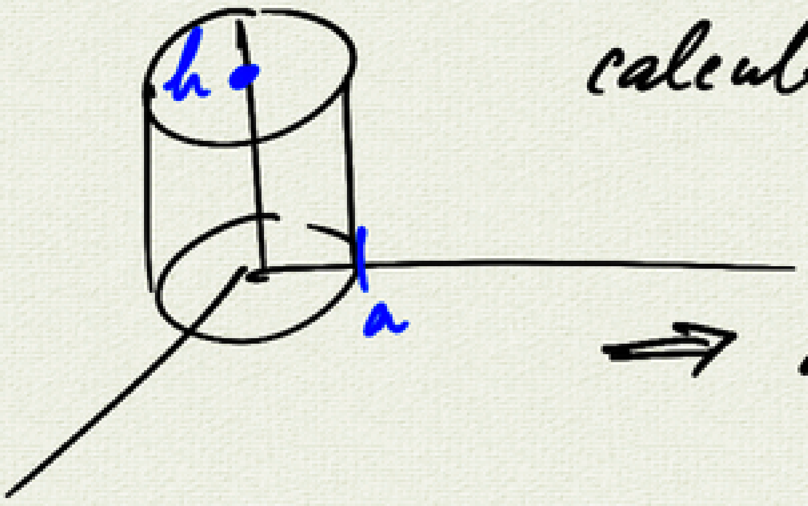
$$V=\iint_D h\; dx\, dy$$

$$=\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} h\; dy\, dx$$

$$=\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}\int_0^h 1\; \underbrace{dz\, dy\, dx}_{dV}$$

---

example: temperature $f(x,y,z)=z$



calculate average temperature in cylinder

$$\Rightarrow \text{avg. temp} = \frac{\iiint_V f(x,y,z)\, dV}{\boxed{\iiint_V dV}}$$ volume $=\pi a^2 h$

$$\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}\int_0^h z\; dz\, dy\, dx$$

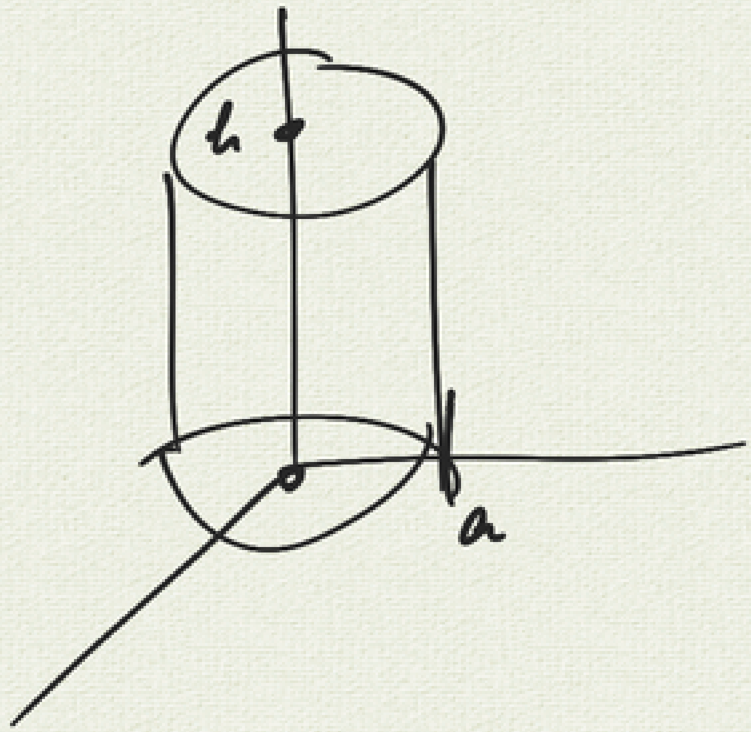
$$=\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \frac{h^2}{2}\, dy\, dx$$

$$=\frac{h^2}{2}\underbrace{\boxed{\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dy\, dx}}_{\pi a^2}$$

$$=\frac{\pi a^2 h^2}{2}$$

$$\Rightarrow \text{avg. temp} = \frac{\left[\frac{\pi a^2 h^2}{2}\right]}{\pi a^2 h} = \frac{h}{2}$$

volume



avg z value $= \frac{h}{2}$

$$\bar{z} = \frac{\iiint z\,dV}{\iiint dV}$$

avg x value

$$\bar{x} = \frac{\iiint x\,dV}{\pi a^2 h}$$

volume

$$\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}\int_{0}^{h} x\,dz\,dy\,dx = \int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} (hx)\,dy\,dx$$

$$= \int_{-a}^{a} hx\,2\sqrt{a^2-x^2}\,dx$$

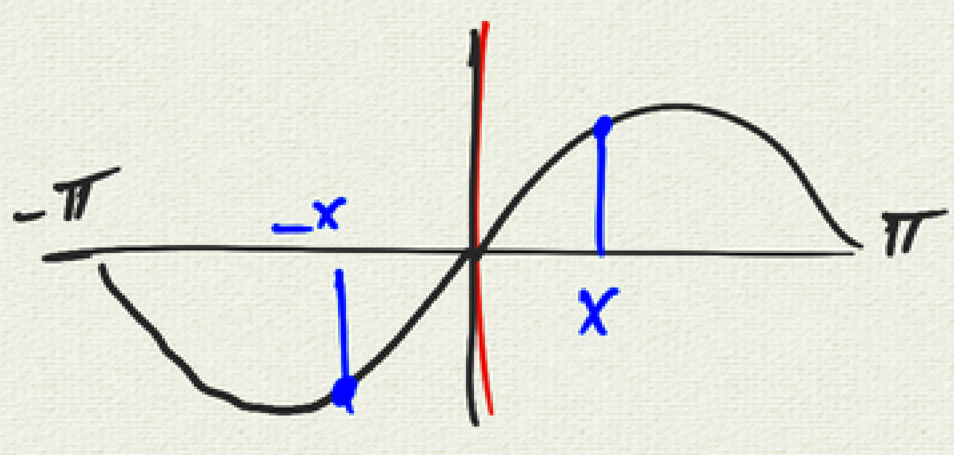
$$= 2h\boxed{\int_{-a}^{a} \boxed{x\sqrt{a^2-x^2}}\,dx}$$

u-sub:
$u = a^2 - x^2$
$du = -2x\,dx$

$$= 2h\int_{0}^{0} \square \,du$$

$$= 0$$

odd



$f(x) = x\sqrt{a^2-x^2}$ odd?

$f(-x) \overset{?}{=} -f(x)$ yes