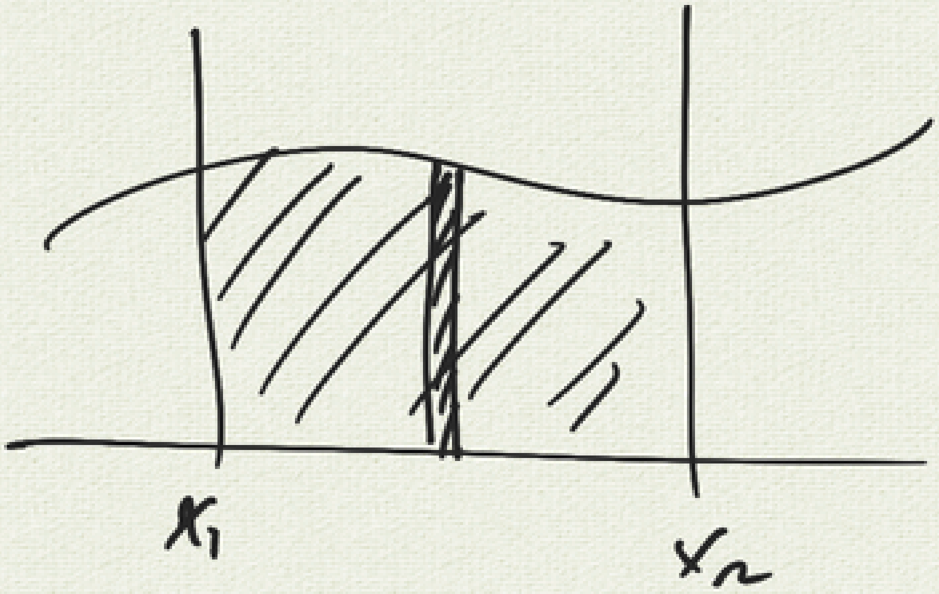
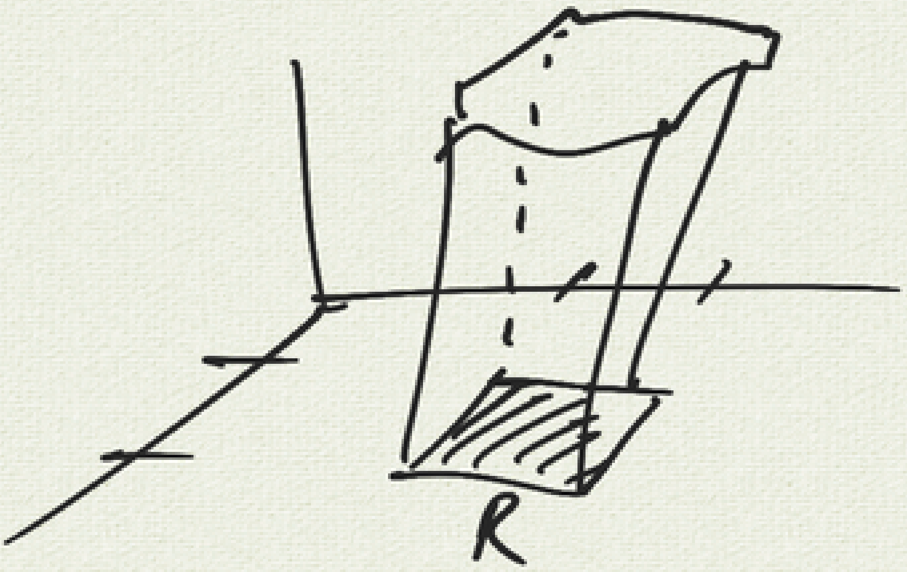


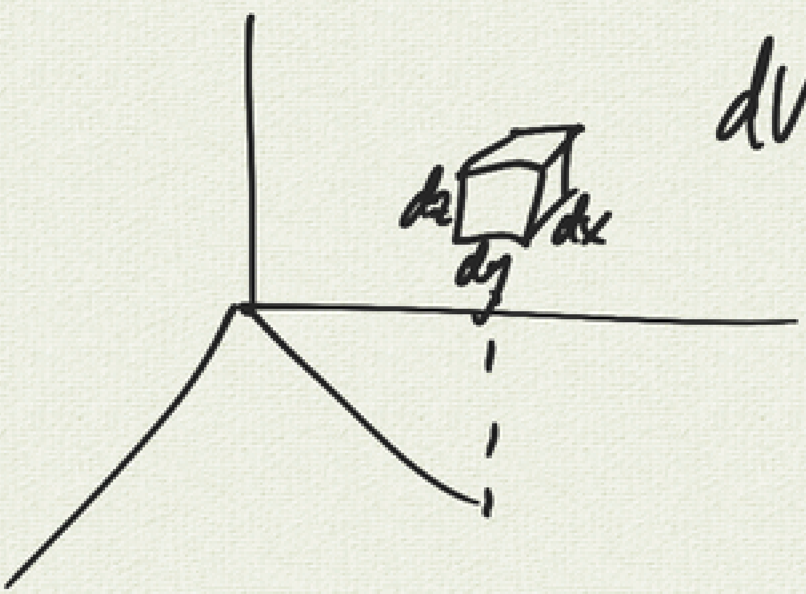
$y = f(x)$

$$A = \int_{x_1}^{x_2} f(x)\,dx$$



$z = f(x,y)$

$$V = \iint_R f(x,y)\,dA$$



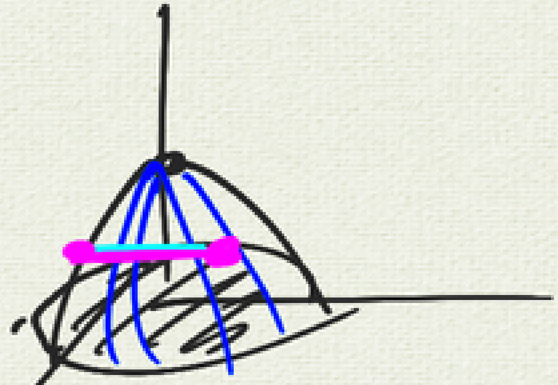
dV

$$\iiint_V f(x,y,z)\,dV$$

197 $\quad -1 \le x \le 1$

$-\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$

$0 \le z \le 1-x^2-y^2$



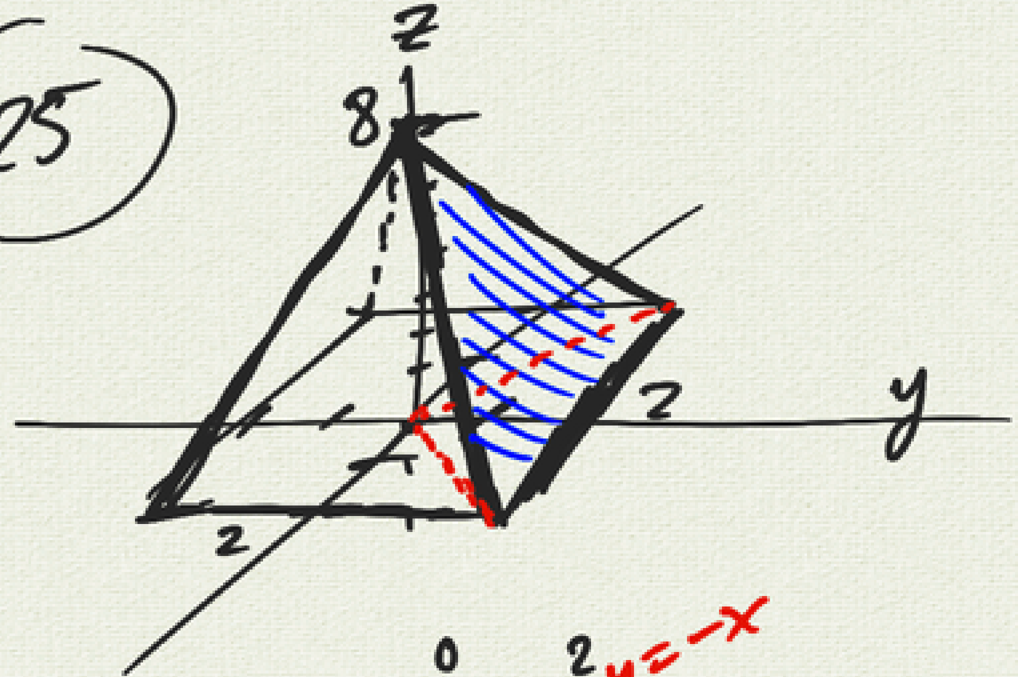
$$\iiint y \, dV = \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{0}^{1-x^2-y^2} y \, dz \, dy \, dx$$

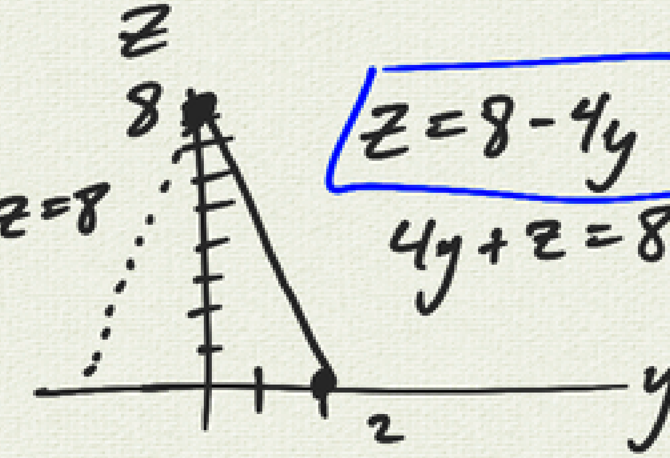
$$= \int_{-1}^{1} \underbrace{\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} y(1-x^2-y^2) \, dy}_{0} \, dx$$

$$\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (1-x^2) y \, dy + \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (-y^3) \, dy$$

$$\underbrace{(1-x^2)\boxed{y^2}\Big|_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}}_{0} \qquad \underbrace{\Big[-\tfrac{1}{4}\boxed{y^4}\Big]_{\sqrt{1-y^2}}^{\sqrt{1-x^2}}}_{0}$$

225

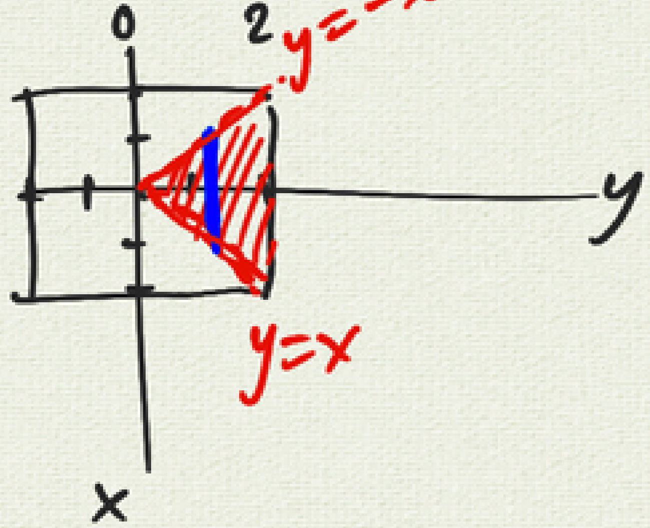




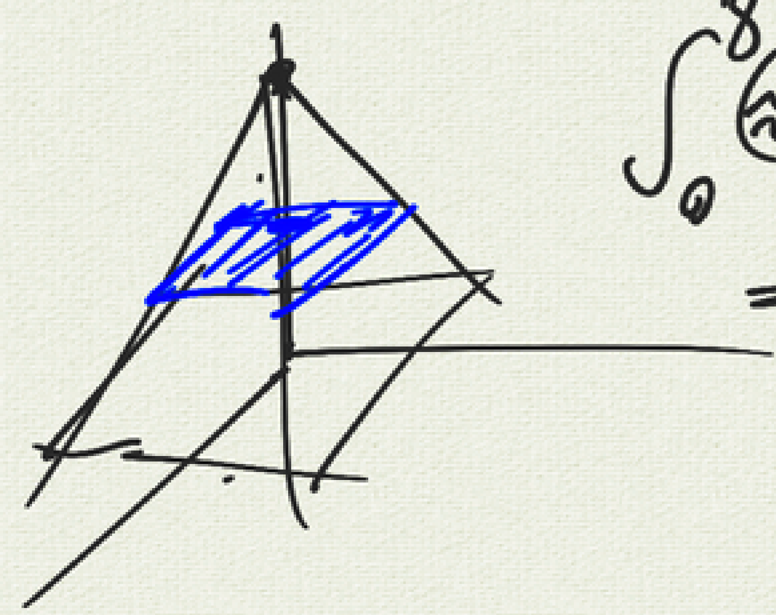
$-4y+z=8$

$\boxed{z=8-4y}$

$4y+z=8$



$$V = 4\int_0^2 \int_{-y}^{y} \int_0^{8-4y} 1 \, dz\, dx\, dy$$



$$\int_0^8 \left(\text{[illegible]}\, z^2\right) dz$$

$\Longrightarrow \frac{1}{3}.$

pyramid volume

5.8 Cylindrical & Spherical Coordinates
(for triple integrals)

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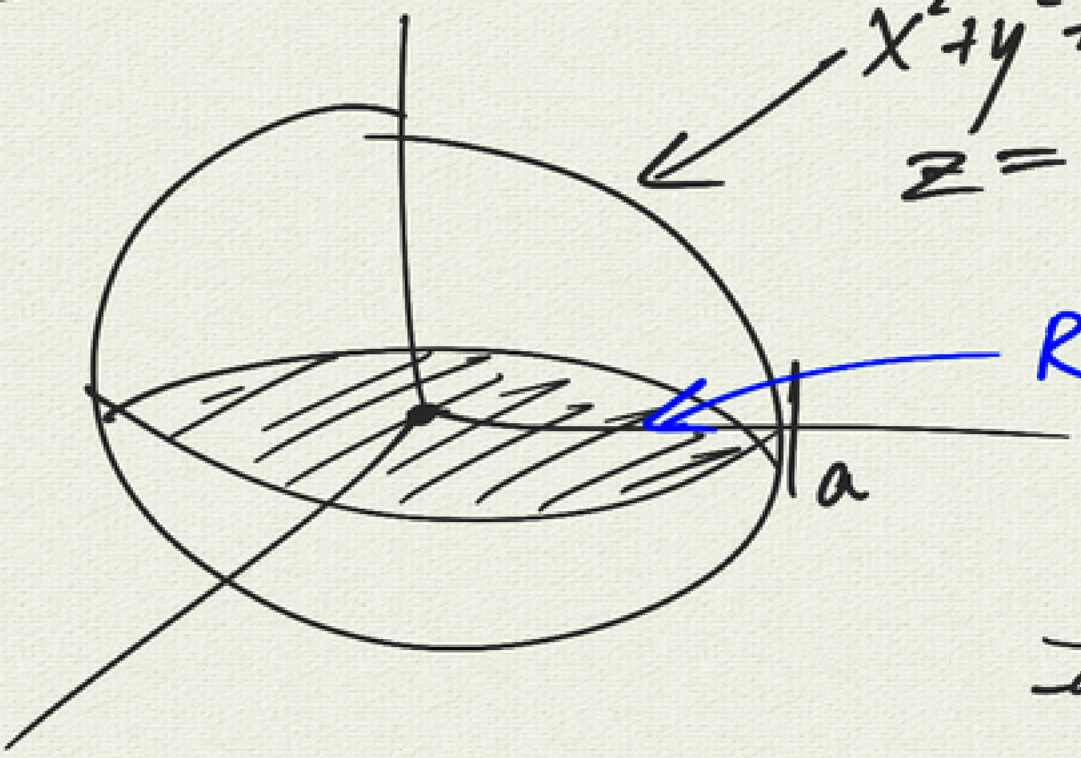
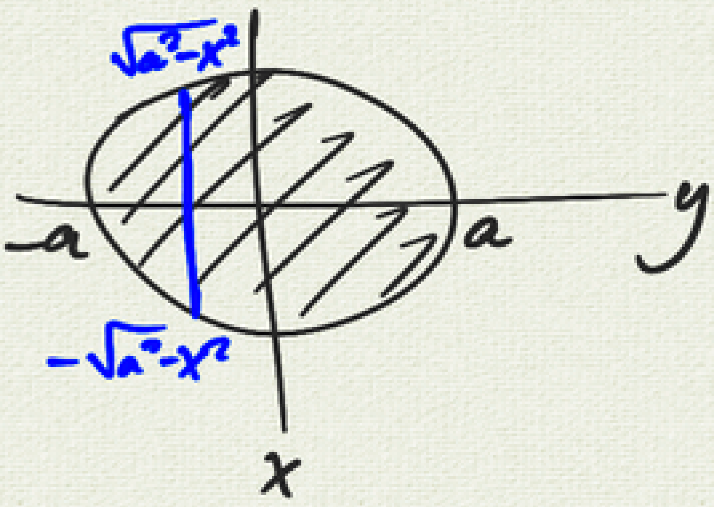
sphere volume



$x^2+y^2+z^2=a^2$

$z=\pm\sqrt{a^2-x^2-y^2}$

$R:\ x^2+y^2=a^2$



$$V=\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}\int_{-\sqrt{a^2-x^2-y}}^{\sqrt{a^2-x^2-y^2}} 1 \; dz\,dy\,dx$$

$$=\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} 2\sqrt{a^2-x^2-y^2}\;\underbrace{dy\,dx}_{dA}$$

$\Rightarrow$ $\boxed{\text{polar}}$

$$=\int_{0}^{2\pi}\int_{0}^{a} 2\sqrt{a^2-r^2}\;\underbrace{r\,dr\,d\theta}_{dA}$$

$\boxed{\text{u sub}}$

$u=a^2-r^2$ | $r=0 \Rightarrow u=a^2$

$du=-2r\,dr$ | $r=a \Rightarrow u=0$

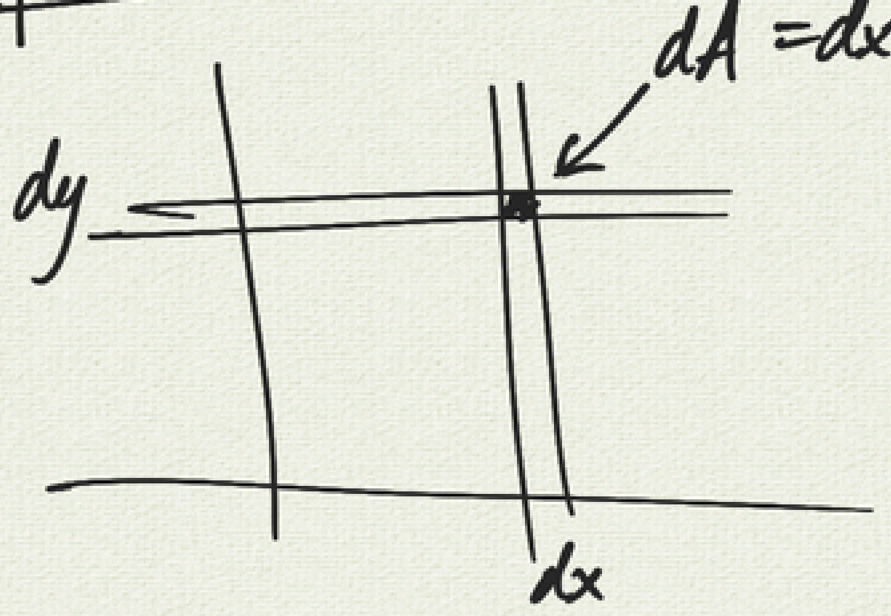
$$2\pi\int_{a^2}^{0} -\sqrt{u}\;du$$

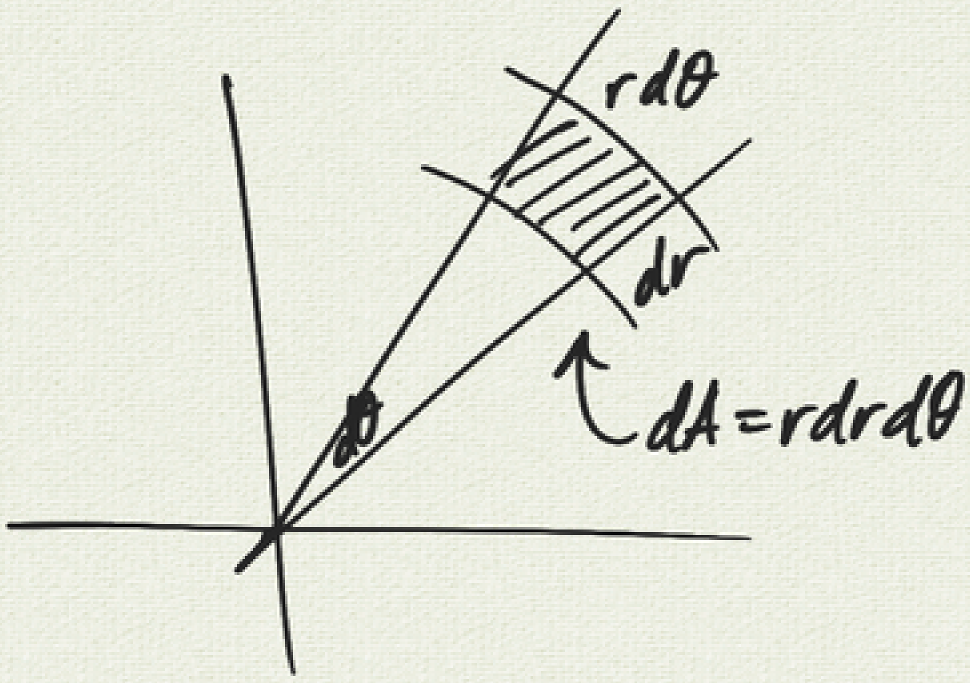
$$=2\pi\int_{0}^{a^2}\sqrt{u}\;du$$

$$=2\pi\,\frac{2}{3}(a^2)^{3/2}$$

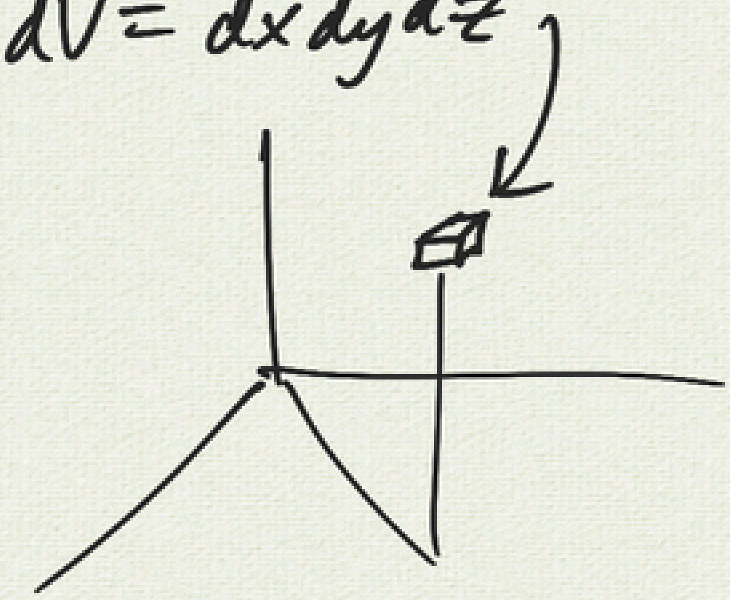
$$=\frac{4}{3}\pi a^3$$

polar:

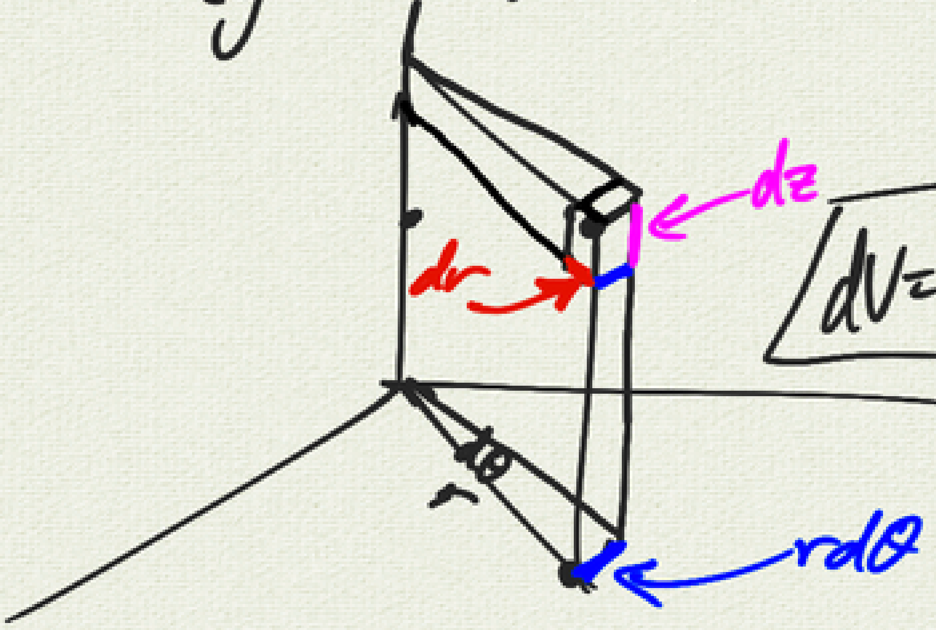




$dV = dx\,dy\,dz$

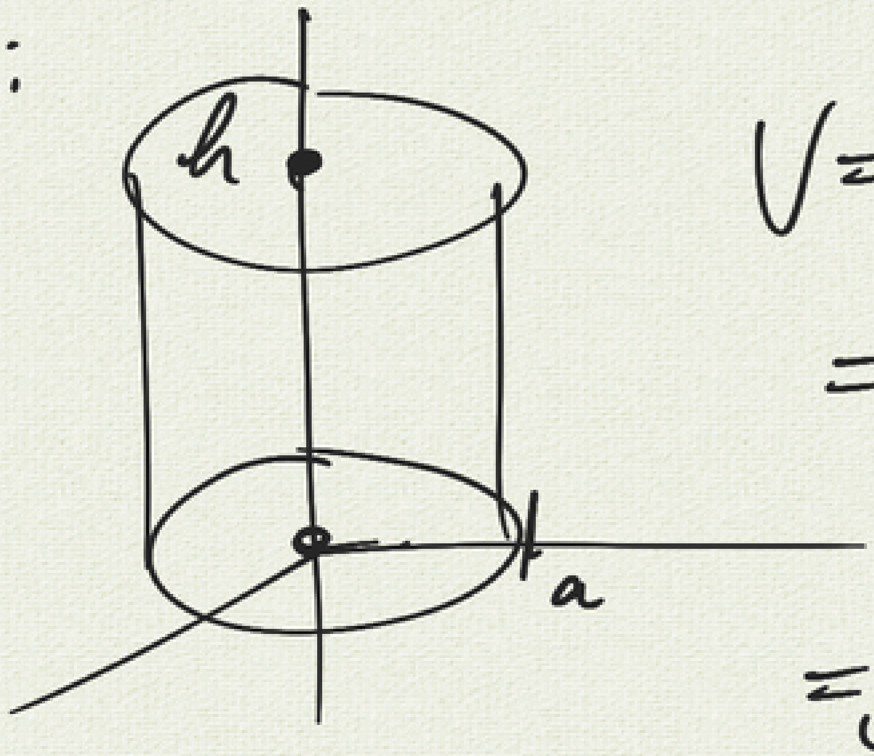


cylindrical: (r, θ, z)



$$\boxed{dV = r\,dr\,d\theta\,dz}$$

example: cylinder volume



$$V = \iiint \underbrace{1\,dV}_{dx\,dy\,dz}$$

$$= \int_0^{2\pi}\int_0^{a}\int_0^{h} \boxed{1\underbrace{r}_{r}\,dz\,dr\,d\theta}$$

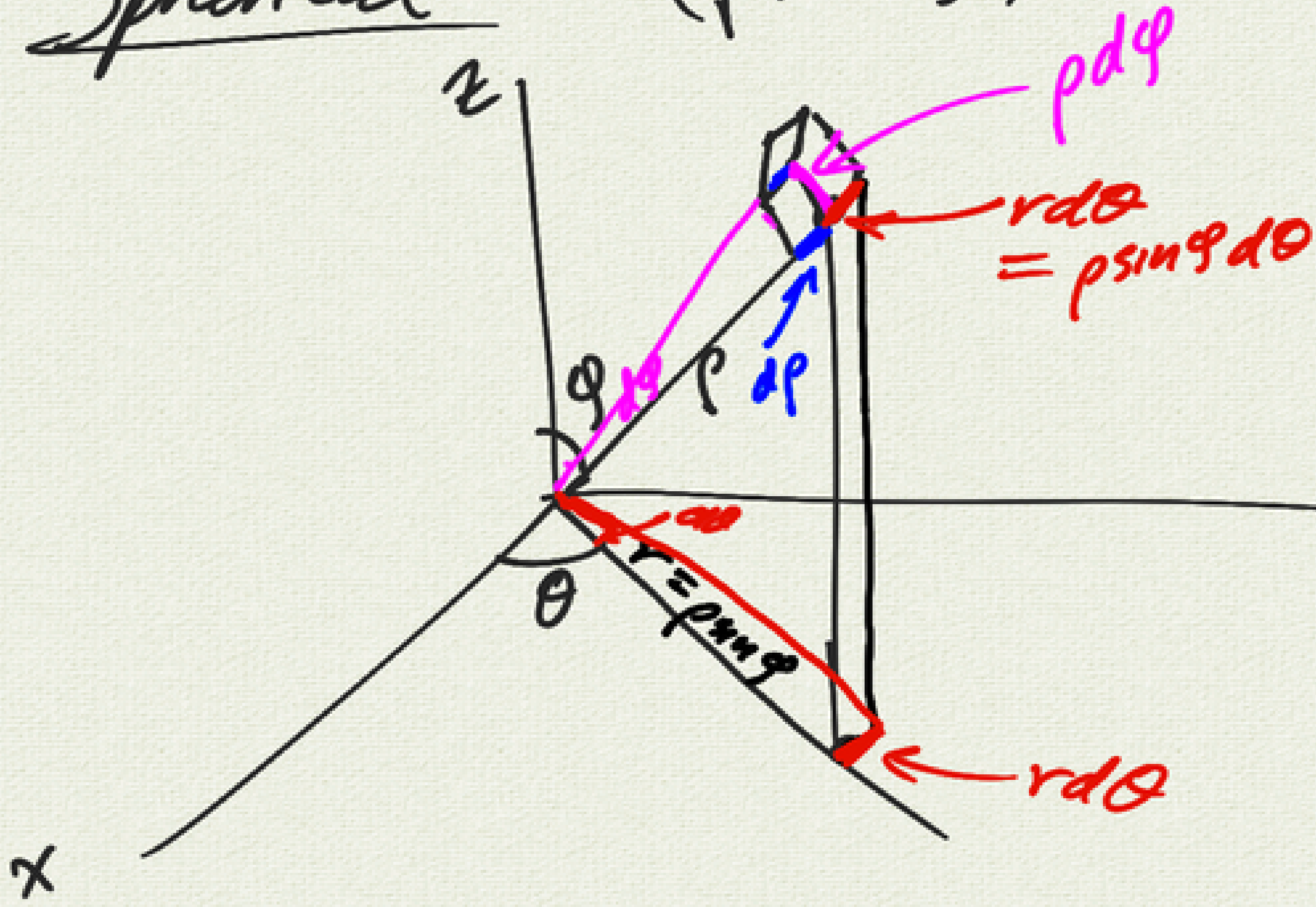
$$= \int_0^{2\pi}\int_0^{a} r h\,dr\,d\theta$$

$$= h\int_0^{2\pi}\left(\int_0^{a} r\,dr\right)d\theta$$

$$= h\int_0^{2\pi}\left(\frac{a^2}{2}\right)d\theta$$

$$= \pi a^2 h$$

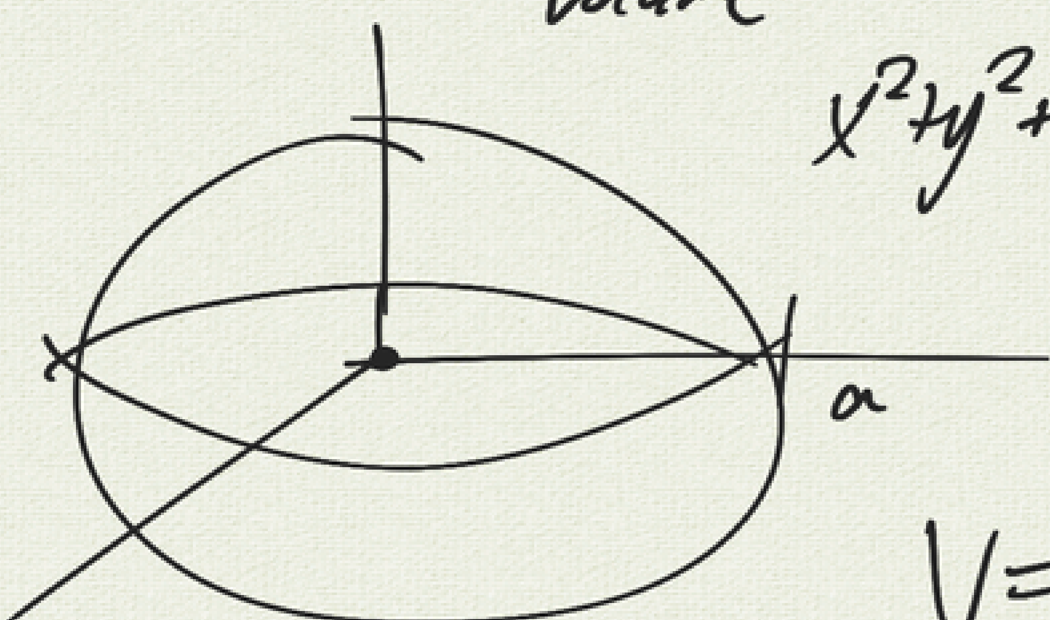
spherical (ρ, θ, φ)



$$dV = (d\rho)(\rho \sin\varphi \, d\theta)(\rho \, d\varphi)$$

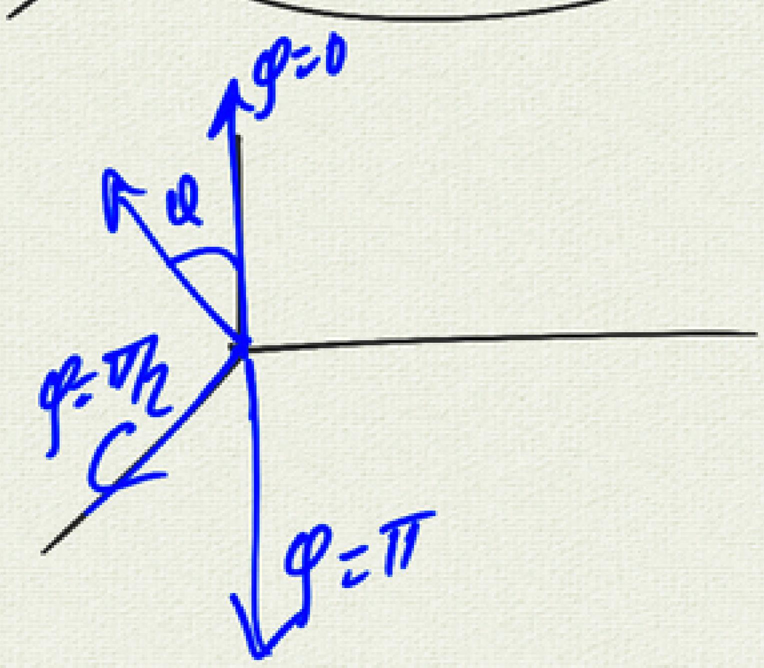
$$\boxed{dV = \boxed{\rho^2 \sin\varphi} \, d\rho \, d\theta \, d\varphi}$$

example: sphere volume



$$x^2 + y^2 + z^2 = a^2$$

$$\rho = a$$

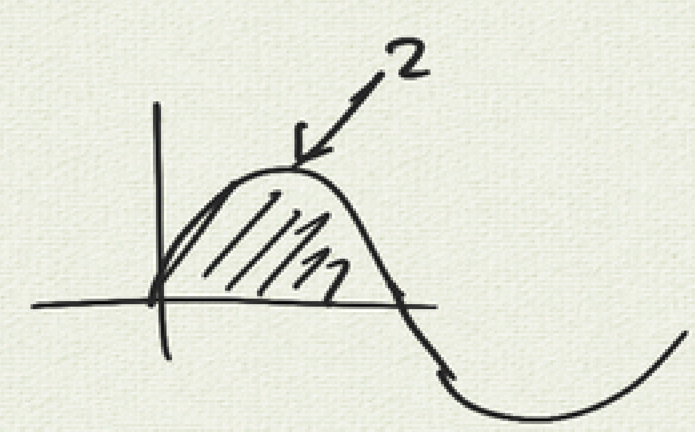


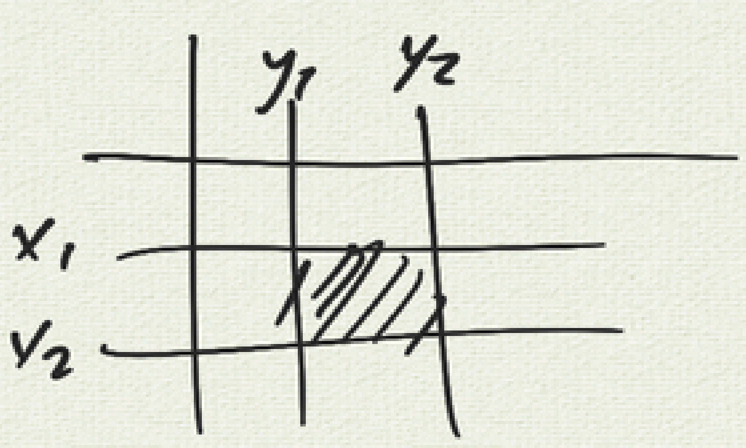
$$V = \iiint 1 \, dV$$

$$= \int_0^{2\pi} \int_0^{\pi} \int_0^{a} 1 \, \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$= \int_0^{2\pi} \int_0^{\pi} \sin\varphi \left(\frac{a^3}{3}\right) d\varphi \, d\theta$$

$$= \frac{a^3}{3} \int_0^{2\pi} \underbrace{\boxed{\int_0^{\pi} \sin\varphi \, d\varphi}}_{2} \, d\theta$$





$$= \frac{2a^3}{3} \int_0^{2\pi} d\theta$$

$$= \frac{4\pi a^3}{3}$$

$dV = dx\,dy\,dz$

$= r\,dr\,d\theta\,dz$ cylindrical

$= \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$ spherical