6.5 Cramer's Rule

$$ax+cy=e$$
$$bx+dy=f$$

$$\Rightarrow \begin{pmatrix} a & c \\ b & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} e \\ f \end{pmatrix}$$

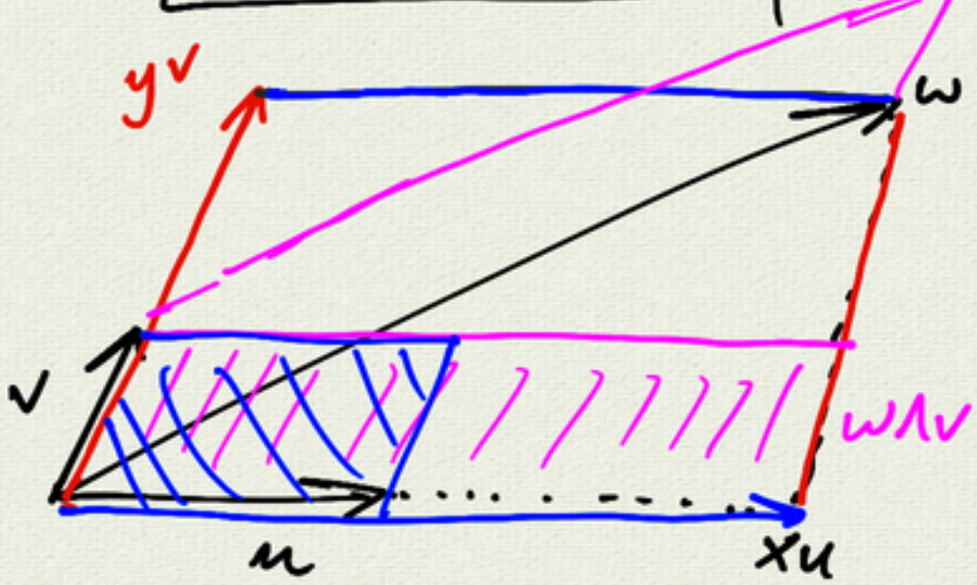
(u v w)

$$\begin{pmatrix} ax+cy \\ bx+dy \end{pmatrix} \Uparrow \text{unknown}$$

$$x\begin{pmatrix} a \\ b \end{pmatrix} + y\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} e \\ f \end{pmatrix}$$

$$\boxed{xu+yv=w}$$

given u, v, w
find x, y



$$xu+yv=w$$
$$(xu+yv)\Lambda v = w\Lambda v$$
$$xu\Lambda v + y\underbrace{v\Lambda v}_{0} = w\Lambda v$$
$$x = \frac{w\Lambda v}{u\Lambda v}$$
$$= \frac{\begin{vmatrix} e & c \\ f & d \end{vmatrix} e_1e_2}{\begin{vmatrix} a & c \\ b & d \end{vmatrix} e_1e_2}$$
$$x = \frac{\begin{vmatrix} e & c \\ f & d \end{vmatrix}}{\begin{vmatrix} a & c \\ b & d \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a & e \\ b & f \end{vmatrix}}{\begin{vmatrix} a & c \\ b & d \end{vmatrix}}$$

$$ax+cy=e$$
$$bx+dy=f$$
$$\begin{pmatrix} a & c \\ b & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} e \\ f \end{pmatrix}$$

example:

$$3x + 2y = 7$$
$$-3x + y = -1$$

$$x = \frac{\begin{vmatrix} 7 & 2 \\ -1 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ -3 & 1 \end{vmatrix}} = \frac{7+2}{3+6} = \frac{9}{9} = 1$$

$$y = \frac{\begin{vmatrix} 3 & 7 \\ -3 & -1 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ -3 & 1 \end{vmatrix}} = \frac{-3+21}{9} = \frac{+18}{9} = 2$$