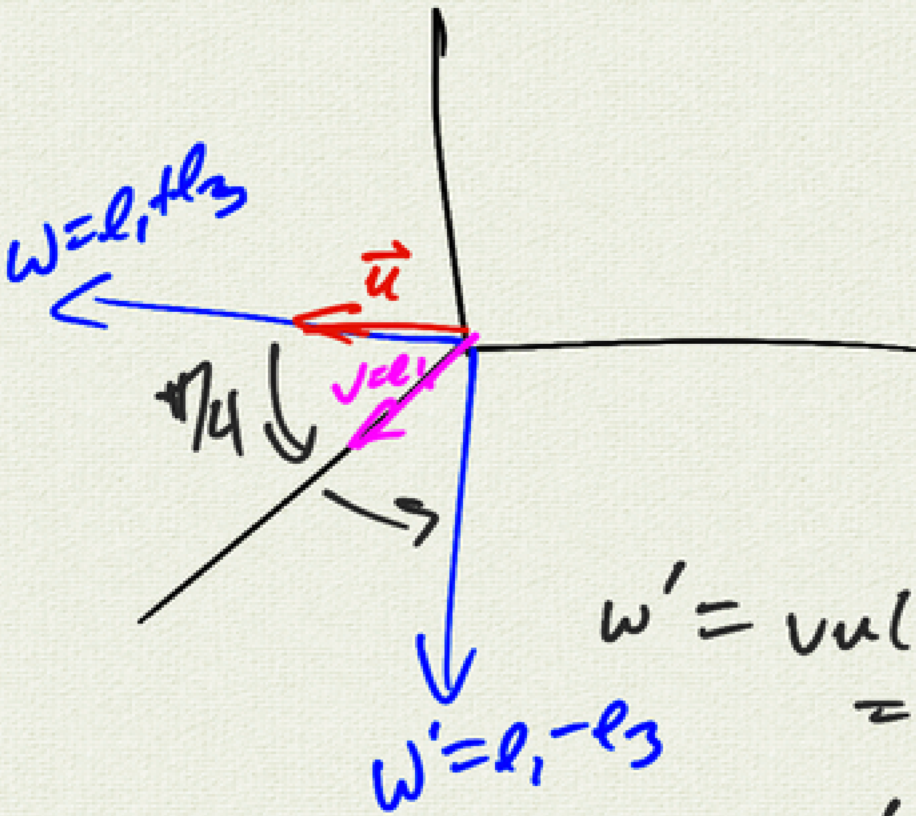


$u = \frac{1}{\sqrt{2}}(e_1 + e_3)$

$v = e_1$

$$\boxed{\underbrace{vu}_{\text{typo}}(w)uv}$$

$$uv = \frac{1}{\sqrt{2}}(e_1+e_3)e_1$$
$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}e_3e_1$$
$$= \underset{\text{dot}}{\frac{1}{\sqrt{2}}} - \underset{\text{wedge}}{\frac{1}{\sqrt{2}}e_1e_3}$$

$$vu = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}e_1e_3$$

$$w' = vu(w)uv$$
$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}e_1e_3\right)(e_1+e_3)\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}e_1e_3\right)$$
$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}e_1e_3\right)\left(\frac{1}{\sqrt{2}}e_1 - \frac{1}{\sqrt{2}}e_3 + \frac{1}{\sqrt{2}}e_3 - \frac{1}{\sqrt{2}}\underbrace{e_3e_1e_3}_{-e_1}\right)$$
$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}e_1e_3\right)\frac{1}{\sqrt{2}}(e_1+e_1)$$
$$= \frac{1}{2}(1+e_1e_3)(2e_1)$$
$$= e_1 + e_1e_3e_1$$
$$\boxed{w' = e_1 - e_3}$$

③ $$\begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

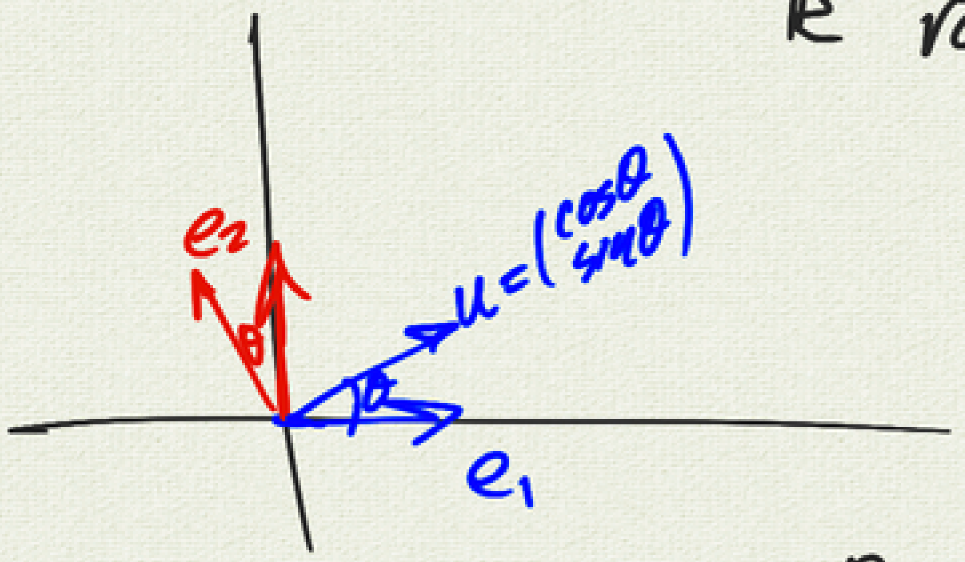
$$x = \frac{\begin{vmatrix} 2 & 0 & 1 \\ 1 & -1 & 1 \\ 0 & -1 & 1 \end{vmatrix}}{\begin{vmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ -1 & -1 & 1 \end{vmatrix}}$$

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$\longleftrightarrow \quad -1\begin{vmatrix} -1 & 1 \\ -1 & 1 \end{vmatrix} - 0\begin{vmatrix} \cdot & \cdot \\ \cdot & \cdot \end{vmatrix} + 1\begin{vmatrix} 0 & -1 \\ -1 & -1 \end{vmatrix}$$

$$z = \frac{\begin{vmatrix} -1 & 0 & 2 \\ 0 & -1 & 1 \\ -1 & -1 & 0 \end{vmatrix}}{\left| \qquad \right|}$$

$\mathbb{R}^2$ rotations

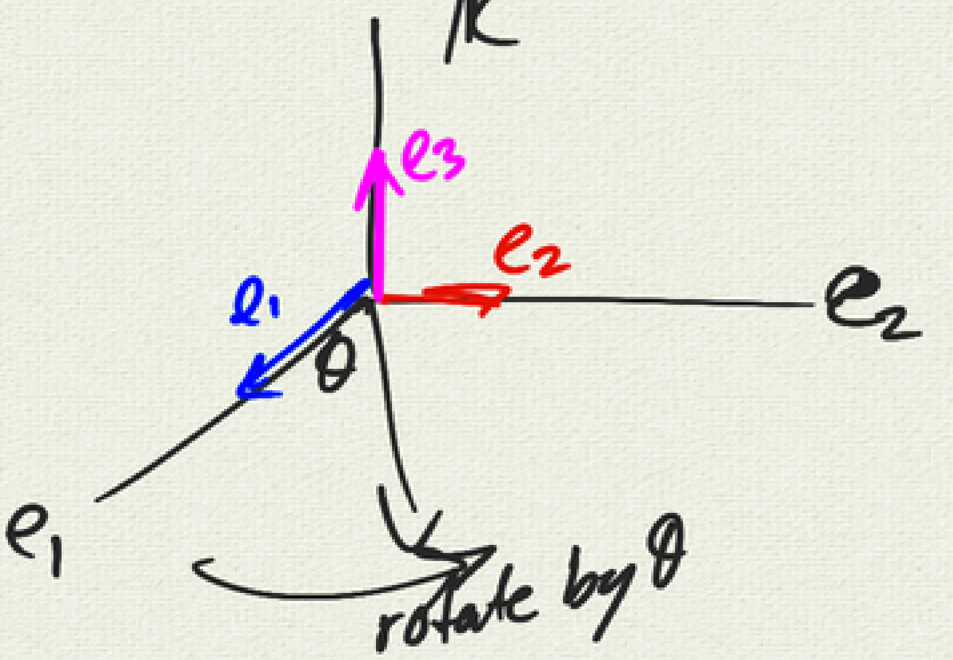


$$\begin{array}{cc} & \begin{array}{cc} u & v \end{array} \\ R_\theta = & \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \end{array}$$

$$R_\theta v = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

rotates v by θ
(matrix multiplication)

$\mathbb{R}^3$

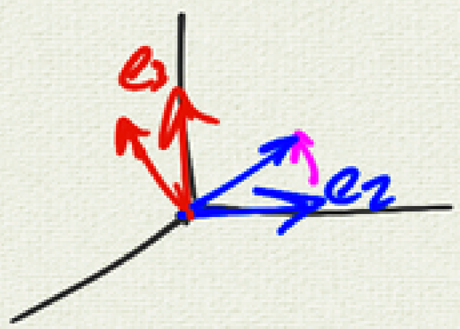


$$\begin{array}{cc} & \begin{array}{ccc} e_1 & e_2 & e_3 \end{array} \\ R_\theta = & \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

rotation by θ in e_1e_2 plane
xy

---

rotate in e_2e_3 plane
yz



$$R_\theta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

---

rotate in e_1e_3 plane:
xz

$$R_\theta = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$$

---

notation: R_x = rotation around x-axis by π

yz plane

$e_2 e_3$

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\pi & -\sin\pi \\ 0 & \sin\pi & \cos\pi \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$R_x \begin{pmatrix} w_x \\ w_y \\ w_z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} w_x \\ w_y \\ w_z \end{pmatrix} = \boxed{\begin{pmatrix} w_x \\ -w_y \\ -w_z \end{pmatrix}}$$

rotation by π around y-axis in $e_1 e_3$ plane xz

$$R_y = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad R_z = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_z R_y R_x = I$$

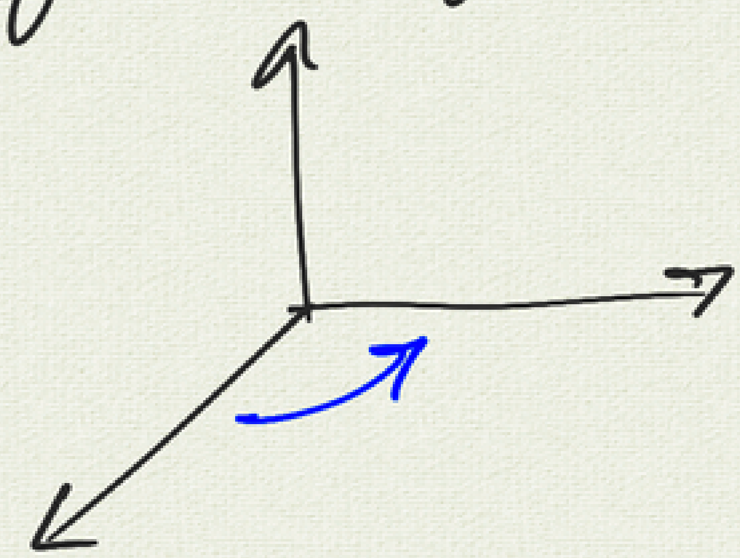
$$R_x R_y = R_z$$

flip y, z

flip x, z

flip x, y

geometric algebra:



rotation in e_1e_2 plane by π

need u, v

$(vu)\, w\, (uv) =$ w rotated by π in e_1e_2 plane

$(2\theta = \pi)$

$\theta = \pi/2$

$\Rightarrow$ let $u = e_1$, $v = e_2$

$$\Rightarrow w' = (e_2e_1)(w_x e_1 + w_y e_2 + w_z e_3)(e_1e_2)$$

$$= w_x \underbrace{e_2e_1e_1e_1e_2}_{-e_1}$$

$$+ w_y \underbrace{e_2e_1e_2e_1e_2}_{-e_2}$$

$$+ w_z \underbrace{e_2e_1e_3e_1e_2}_{e_3}$$

$$= -w_x e_1 - w_y e_2 + w_z e_3 = \begin{pmatrix} -w_x \\ -w_y \\ w_z \end{pmatrix}$$

$R_z \longleftrightarrow e_1e_2$

rotation in e_1e_2 plane

rotation around z-axis

$w' = (e_2e_1)\, w\, (e_1e_2)$

$w' = R_z w$

$R_y \longleftrightarrow e_3e_1$

$R_x \longleftrightarrow e_2e_3$

$R_x R_y = R_z$

$(e_1 \underbrace{e_2)(e_2}_{} e_3) = (e_1e_3)$

rotation $\cdot$ rotation = rotation