MultiV - Unit 1

3D Distance Formula: $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$
3D Sphere Formula: $r^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$

Magnitude of a Vector: $|v| = \sqrt{(x^2 + y^2)}$
Direction of a Vector: $\tan(\varnothing) = y/x$
Unit Vector in Direction of Other Vector: $u = v / |v|$
Dot Product: $u \cdot v = < x_1 x_2 + y_1 y_2 >$

If the dot product of two vectors is 0, the vectors are orthogonal

Magnitude²: $|u|^2 = u \cdot u = x^2 + y^2$
Law of Cosines Math: $u \cdot v = |u||v| \cos(\varnothing) \rightarrow \cos(\varnothing) = (u \cdot v) / |u||v|$

$|$ Proj$_u$ (v) $|$ = $v \cdot (u / |u|)$
Proj$_u$ (v) = $|\text{proj}_u(v)| \times (u / |u|)$

Matrices: $\begin{vmatrix} a & c \\ b & d \end{vmatrix} \times \begin{vmatrix} x \\ y \end{vmatrix} = \begin{vmatrix} e \\ f \end{vmatrix}$
Inverse Matrices: $A \times A^{-1} = I$
Determinant: ad-bc (area of parallelogram)

If A^{-1} exists, $\det A \neq 0$

Cross Product: $u \times v = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} i - \begin{vmatrix} u_1 & u_3 \\ v_2 & v_3 \end{vmatrix} j + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} k$

Finding Parametric Equations from Two Points:
1) Set up vertical vectors on left side of = with x, y, and possibly z variables
2) Use first point as first vertical vector on right side of =
3) Find vector between two points, multiply by t for final part of vector equation
4) Vertically separate to get parametric equations by variable

Find Distance from a Point to a Line:
1) Find equation of line
2) Find vector from line to point using a random point on the line
3) Project the vector you found from point to line onto the line
   - If given parametric form of line, use the given start point and then the number attached to t is the actual vector of the line
4) Find the rejection, which is the shortest vector from the line to the point
   - Rejection = (vector from line to point) - (projection of vector onto line)
5) The magnitude of the rejection vector is the distance from the line to the point

Equation of Plane: $ax + by + cz = d$
Position of Point r on Plane: $n \cdot (r - p) = 0$

- n (normal) represented in equation by vertical (a, b, c)
- p (given point) represented by actual values
- r (point we are finding) represented by vertical (x, y, z)

Scalar Form of Point on Plane: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Finding Equation of a Plane from Three Points:
1) Given the three points, find two vectors between the points
2) Find the cross product of these two vectors
    - This new vector is n (normal), perpendicular to the plane we are finding
3) Pick a random point that we know for sure is on the plane
    - Good idea is to use one of the starting given points
4) Use the equation $n \cdot (r - p) = 0$ with (a, b, c) as r values → solve for standard form of plane

Find Distance from Plane to Origin (or any point):
1) Take the coefficients from the equation of the plane to create the normal vector
2) Find a point on the plane by setting y and z equal to 0, solve for equation
3) Find vector between given point and random point on plane
4) Project this vector onto the normal

Going from Rectangular (x, y, z) to Cylindrical (r, theta, z):
1) Solve for r with the equation $r^2 = x^2 + y^2$
2) Find theta by $\tan(\varnothing) = y/x$

Going from Cylindrical (r, theta, z) to Spherical (rho, theta, phi):
1) Find rho by $r^2 + z^2 = rho^2$
2) Find phi by $z = rho \cos(phi)$