Unit 1 Group Work
MultiV 2021-22 / Dr. Kessner

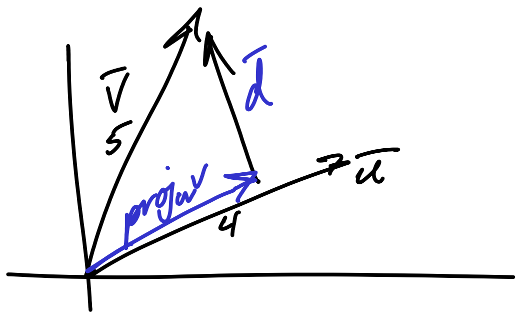
No calculator! Have fun!

1. Let

$$\vec{u} = \left\langle 2\sqrt{3}, 2 \right\rangle = 4\left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$$

$$\vec{v} = \left\langle \frac{5}{2}, \frac{5\sqrt{3}}{2} \right\rangle = 5\left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$$

Express $\vec{v}$ as the sum of a vector with the same direction as $\vec{u}$ and a vector orthogonal to $\vec{u}$.



$$|\text{proj}_u v| = \bar{v} \cdot \frac{\bar{u}}{|\bar{u}|}$$
$$= 5\left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle \cdot \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$$
$$= 5\left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} \right)$$
$$= \frac{5\sqrt{3}}{2}$$

$$\text{proj}_u v = \frac{5\sqrt{3}}{2}\left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$$
$$= \left\langle \frac{15}{4}, \frac{5\sqrt{3}}{4} \right\rangle$$

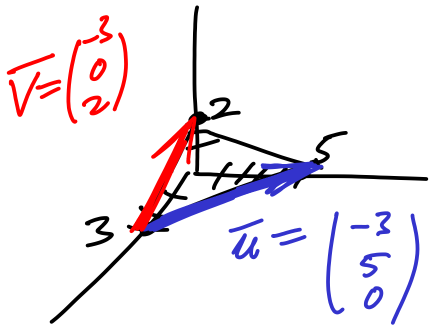
$$\bar{d} = v - \text{proj}_u v$$
$$= \left\langle \frac{5}{2}, \frac{5\sqrt{3}}{2} \right\rangle - \left\langle \frac{15}{4}, \frac{5\sqrt{3}}{4} \right\rangle$$
$$= \left\langle -\frac{5}{4}, \frac{5\sqrt{3}}{4} \right\rangle$$

check:

$$\text{proj}_u v + \bar{d}$$
$$= \left\langle \frac{15}{4}, \frac{5\sqrt{3}}{4} \right\rangle + \left\langle \frac{-5}{4}, \frac{5\sqrt{3}}{4} \right\rangle$$
$$= \left\langle \frac{5}{2}, \frac{5\sqrt{3}}{2} \right\rangle = \bar{v} \checkmark$$

$\bar{d} \perp \bar{u}$: $$\bar{d} \cdot \frac{\bar{u}}{|\bar{u}|} = \left\langle -\frac{5}{4}, \frac{5\sqrt{3}}{4} \right\rangle \cdot \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$$
$$= -\frac{5\sqrt{3}}{8} + \frac{5\sqrt{3}}{8}$$
$$= 0 \checkmark$$

2. Find the equation of the plane through the points: $(3, 0, 0)$, $(0, 5, 0)$, and $(0, 0, 2)$. You must use vectors to obtain your equation. Once you have your equation, verify your intercepts. Also calculate the distance from the plane to the origin.



$$\bar{n} = \bar{u} \times \bar{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 5 & 0 \\ -3 & 0 & 2 \end{vmatrix}$$

$$= 10\vec{i} + 6\vec{j} + 15\vec{k}$$

let $\bar{p} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$

plane $\vec{n} \cdot (\bar{r} - p) = 0$

$$10(x-3) + 6(y-0) + 15(z-0) = 0$$

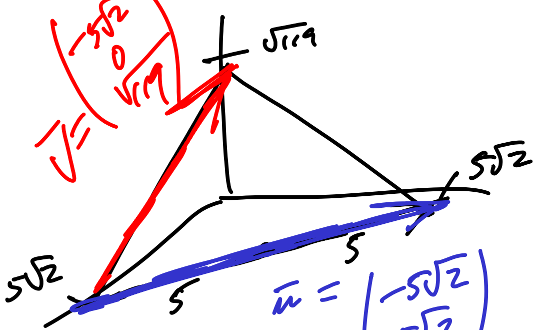
$$10x + 6y + 15z = 30$$

distance $d = \left| \bar{p} \cdot \frac{\bar{n}}{|\bar{n}|} \right|$

$$= \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 10 \\ 6 \\ 15 \end{pmatrix} \frac{1}{\sqrt{361}}$$

$$= \frac{30}{\sqrt{361}}$$

3. Find parametric equations for the line through $\langle 5\sqrt{2}, 0, 0\rangle$ and $\langle 0, 5\sqrt{2}, 0\rangle$. Find the distance from the point $\langle 0, 0, \sqrt{119}\rangle$ to the line.



$$\bar{v} = \begin{pmatrix} -5\sqrt{2} \\ 0 \\ \sqrt{119} \end{pmatrix} \qquad \bar{u} = \begin{pmatrix} -5\sqrt{2} \\ 5\sqrt{2} \\ 0 \end{pmatrix}$$

$$|\text{proj}_u v| = \bar{v} \cdot \frac{\bar{u}}{|\bar{u}|}$$

$$= \begin{pmatrix} -5\sqrt{2} \\ 0 \\ \sqrt{119} \end{pmatrix} \cdot \begin{pmatrix} -5\sqrt{2} \\ 5\sqrt{2} \\ 0 \end{pmatrix} \frac{1}{10}$$

$$= 5$$

$$\text{proj}_u v = 5 \cdot \frac{\bar{u}}{|\bar{u}|} = \frac{5\sqrt{2}}{2}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

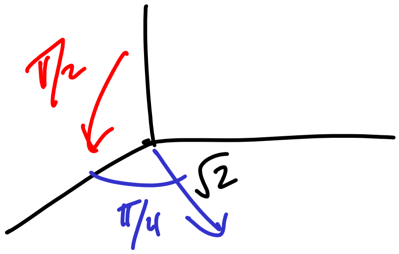
$$\bar{d} = \bar{v} - \text{proj}_u v = \begin{pmatrix} -5\sqrt{2} \\ 0 \\ \sqrt{119} \end{pmatrix} - \frac{5\sqrt{2}}{2}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -5\sqrt{2}/2 \\ -5\sqrt{2}/2 \\ \sqrt{119} \end{pmatrix}$$

$$d^2 = 2\left(\frac{5\sqrt{2}}{2}\right)^2 + \left(\sqrt{119}\right)^2 = 2 \cdot \frac{25}{2} + 119 = 144$$

$$d = 12$$

4. Express the following vectors in both cylindrical and spherical coordinates.

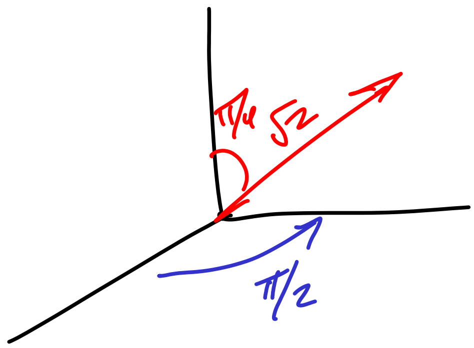
a. $\mathbf{i} + \mathbf{j}$



$$(\rho, \theta, \varphi) = \left(\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{2}\right)$$

$$(r, \theta, z) = \left(\sqrt{2}, \frac{\pi}{4}, 0\right)$$

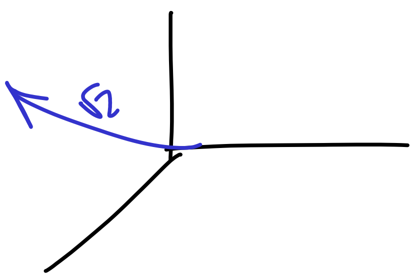
b. $\mathbf{j} + \mathbf{k}$



$$(\rho, \theta, \varphi) = \left(\sqrt{2}, \frac{\pi}{2}, \frac{\pi}{4}\right)$$

$$(r, \theta, z) = \left(1, \frac{\pi}{2}, 1\right)$$

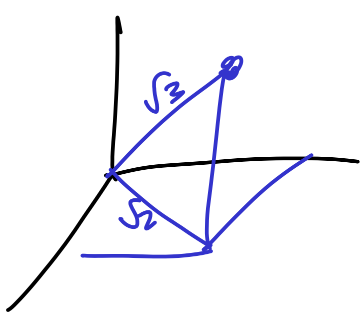
c. $\mathbf{k} + \mathbf{i}$



$$(\rho, \theta, \varphi) = \left(\sqrt{2}, 0, \pi/4\right)$$

$$(r, \theta, z) = (1, 0, 1)$$

d. $\mathbf{i} + \mathbf{j} + \mathbf{k}$



$$(r, \theta, z) = \left(\sqrt{2}, \frac{\pi}{4}, 1\right)$$

$$\rho \cos\varphi = z$$

$$\to \cos\varphi = \frac{1}{\sqrt{3}}$$

$$\varphi = \cos^{-1}\frac{1}{\sqrt{3}}$$

$$(\rho, \theta, \varphi) = \left(\sqrt{3}, \pi/4, \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)$$