Unit 3 Group Work Practice
MultiV 2021-22 / Dr. Kessner

Please use your own paper. No calculator! Have fun!

1. Consider the function $p(x, y) = x^2 - y^2$.

a. Draw the level sets $p(x, y) = -4, -1, 0, 1, 4$ in the xy plane.

b. Find the equations of the tangent planes to the surface $z = p(x, y)$ at the following points: $(0, 0), (1, 0), (0, 1), (1, 1)$.

2. Let's think about ants walking on a pringle, i.e. curves on the same surface $z = p(x, y) = x^2 - y^2$. Consider the following parametrically defined curves in $\mathbb{R}^2$:

A) $x(t) = t$, $y(t) = 0$

B) $x(t) = 0$, $y(t) = t$

C) $x(t) = 5$, $y(t) = t$

D) $x(t) = t$, $y(t) = t$ (What happens if $y(t) = -t$?)

E) $x(t) = 5 + t$, $y(t) = t$ (What happens if $y(t) = -t$?)

F) $x(t) = \cos t$, $y(t) = \sin t$

For each of those curves, do the following analysis:

i) Find the composite function $z(t) = p(x(t), y(t))$, and its derivative $z'(t)$.

ii) Find $z'(t)$ using the multivariable chain rule.

iii) Consider the point on the curve when $t = 1$ ($t = \frac{\pi}{2}$ for curve (F)). Find $D_{\mathbf{u}}(p)$ at that point, where $\mathbf{u}$ is the direction of the curve, i.e. in the direction of $\mathbf{r}'(t)$. What is the relation between $D_{\mathbf{u}}(p)$ and $z'(t)$?

iv) Consider the curve in $\mathbb{R}^3$: $\langle x(t), y(t), z(t) \rangle$. Find the tangent vector of the curve. Show that as you move along the curve, the tangent vector is always orthogonal to $\left\langle \frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, -1 \right\rangle$. (In other words, the tangent vector of the curve lies in the tangent plane to the surface at that point. Or, the gradient of a function is orthogonal to its level sets).

Fun fact: Notice that the $\mathbb{R}^3$ curves for (D) and (E) are actually straight lines, and in general, at every point on the pringle there are two straight lines through the point that coincide with the surface. For this reason, the hyperbolic paraboloid is said to be *double ruled*.