Unit 3 Group Work 2
MultiV 2021-22 / Dr. Kessner

Be sure to show all work. No calculator! Have fun!

1. Consider the function $f(x, y) = \sqrt{x^2 + y^2}$.

a. Draw the level sets $f(x, y) = 0, 1, 2, 3$ in the xy plane. What is the surface $z = f(x, y)$?

b. At each of the following points, find the direction of maximum change of f and calculate the directional derivative in that direction. (Do your answers make sense?)

- $\langle x_0, y_0 \rangle = 3 \left\langle \cos \frac{\pi}{6}, \sin \frac{\pi}{6} \right\rangle = \left\langle \frac{3\sqrt{3}}{2}, \frac{3}{2} \right\rangle$.

- $\langle x_0, y_0 \rangle = 2 \left\langle \cos \frac{5\pi}{4}, \sin \frac{5\pi}{4} \right\rangle = \left\langle -\sqrt{2}, -\sqrt{2} \right\rangle$

c. Consider the parametric curve $\langle x(t), y(t)\rangle = \langle 5\cos t, 5\sin t\rangle$. Find the composite function $f(t) = f(x(t), y(t))$. Find the derivative $\frac{df}{dt}$ in two different ways: 1) using standard differentiation, 2) using the multivariable chain rule.

d. Consider the parametric curve $\langle x(t), y(t)\rangle = \langle t\cos t, t\sin t\rangle$. Find the composite function $f(t) = f(x(t), y(t))$. Find the derivative $\frac{df}{dt}$ in two different ways: 1) using standard differentiation, 2) using the multivariable chain rule.

e. For the following points in $\mathbb{R}^3$, verify that the point is on the surface $z = f(x, y)$, and find the tangent plane to the surface there. (Do your answers make sense?)

- (3, 0, 3)
- (0, 5, 5)

f. Now consider this same function in polar coordinates:

$$x = r\cos\theta$$
$$y = r\sin\theta$$

Calculate $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial \theta}$ using the multivariable chain rule (but check your answer by differentiating the composite function). (How is this related to parts (b) and (c)?)

2. Let

$$F(x, y, z) = \frac{x^2}{16} + \frac{y^2}{25}$$

and consider the level surface $F(x, y, z) = 1$. What does the surface look like?

a. For the following points in $\mathbb{R}^3$, verify that the point is on the surface, and find the tangent plane to the surface there. (Do your answers make sense?)

- (4, 0, 0)
- (4, 0, 10)
- (0, 5, 0)
- (0, 5, -10)

b. Verify that the curve $\langle x(t), y(t), z(t) \rangle = \langle 4\cos t, 5\sin t, t \rangle$ lies on the surface. Calculate $\frac{dF}{dt}$ using the chain rule.

c. Verify that the curve $\langle x(t), y(t), z(t)\rangle = \langle 4\cos t, 5\sin t, e^t \sin t\rangle$ lies on the surface. Calculate $\frac{dF}{dt}$ using the chain rule.

d. Verify that the curve $\langle x(t), y(t), z(t)\rangle = \langle 4, 0, e^t \sin t\rangle$ lies on the surface. Calculate $\frac{dF}{dt}$ using the chain rule.

Interpret your calculations (b)-(d) as statements about orthogonality.

3. Let $R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ represent rotation in the plane.

Verify that $R_{\frac{\pi}{2}}^2 = R_\pi$ and $R_\pi^2 = R_{2\pi} = I$.