

Unit 4 Test
MultiV 2021-22 / Dr. Kessner

No calculator! Remember to verify local minima with a 2nd derivative test where appropriate!
Have fun!

1. a) Find the points where $f(x, y) = 4x + 3y$ is maximized and minimized on the unit circle.

b) For each of these points, find the equation of the line tangent to the unit circle at that point.

2. Find the dimensions that maximize the volume of a cylinder with surface area 600π (lateral area + top + bottom) in two different ways: 1) substitution, and 2) Lagrange multipliers.

3. Let $g(x, y) = -x^2 - y^2 - 2x - 2y - 2$. Find any critical points and identify as a local min/max or saddle point. Complete the square to identify the surface and verify your answer.

4. Let $h(x, y) = \sin x \sin y$.

a) Verify that the following are all critical points: $(\pi/2, \pi/2), (\pi/2, 3\pi/2), (\pi, \pi), (3\pi/2, \pi/2), (3\pi/2, 3\pi/2)$

b) Classify each of the critical points using the 2nd derivative test.

Bonus: Characterize the set of *all* critical points, and specify which are local minima, local maxima, and saddle points. Describe the shape of the graph $z = h(x, y)$.

Bonus: Write the 2nd order approximation for a function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$.

Bonus: Justify the 2nd derivative test for a function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$.

Bonus: Consider the line through (x_0, y_0) that is orthogonal to $\begin{pmatrix} a \\ b \end{pmatrix}$. Show that the point-normal form of the line is equivalent to the slope-point form of the line.

Bonus: Rotate the multiplication table to show that it is a pringle. In other words, rotate the function $m(x, y) = xy$ to remove the xy term and show that it is actually a hyperbolic paraboloid.