## Semester 2 Warmup

MultiV 2021-22 / Dr. Kessner

## No calculator! Have fun!

1. Consider the function $f(x, y)=2 x^{2}-4 x+3 y^{2}+12 y+20$.
a. Find the equation of the tangent plane to the surface $z=f(x, y)$ at $(x, y)=(0,0)$.
b. A critical point of $f$ is a point $(x, y)$ where both $f_{x}$ and $f_{y}$ are either zero or undefined. Find all critical points of $f$ (there is only one for this example).
c. At the critical point, find the linear approximation of $f$.
d. Let $d^{2} f$ be the matrix of 2 nd partial derivatives:

$$
d^{2} f=\left(\begin{array}{ll}
f_{x x} & f_{x y} \\
f_{y x} & f_{y y}
\end{array}\right)
$$

Find $d^{2} f$ and $\operatorname{det} d^{2} f$ (at the critical point). What does $d^{2} f$ tell you about the shape of the surface at the critical point?
e. Complete the square to write the function in the form $f(x, y)=a(x-h)^{2}+b(y-k)^{2}+c$. What does this tell you about the surface?
2. Do the same calculations for the function $g(x, y)=-4 x^{2}+16 x-5 y^{2}-11$.
3. Do the same calculations for the function $h(x, y)=x^{2}-2 x-2 y^{2}+8 y-9$.

