No calculator! Have fun!

- 1. Consider the function $f(x, y) = 2x^2 4x + 3y^2 + 12y + 20$.
 - a. Find the equation of the tangent plane to the surface z = f(x, y) at (x, y) = (0, 0).
 - b. A critical point of f is a point (x, y) where both f_x and f_y are either zero or undefined. Find all critical points of f (there is only one for this example).
 - c. At the critical point, find the linear approximation of f.
 - d. Let d^2f be the matrix of 2nd partial derivatives:

$$d^2 f = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}$$

Find d^2f and det d^2f (at the critical point). What does d^2f tell you about the shape of the surface at the critical point?

- e. Complete the square to write the function in the form $f(x, y) = a(x h)^2 + b(y k)^2 + c$. What does this tell you about the surface?
- **2.** Do the same calculations for the function $g(x, y) = -4x^2 + 16x 5y^2 11$.
- **3.** Do the same calculations for the function $h(x, y) = x^2 2x 2y^2 + 8y 9$.