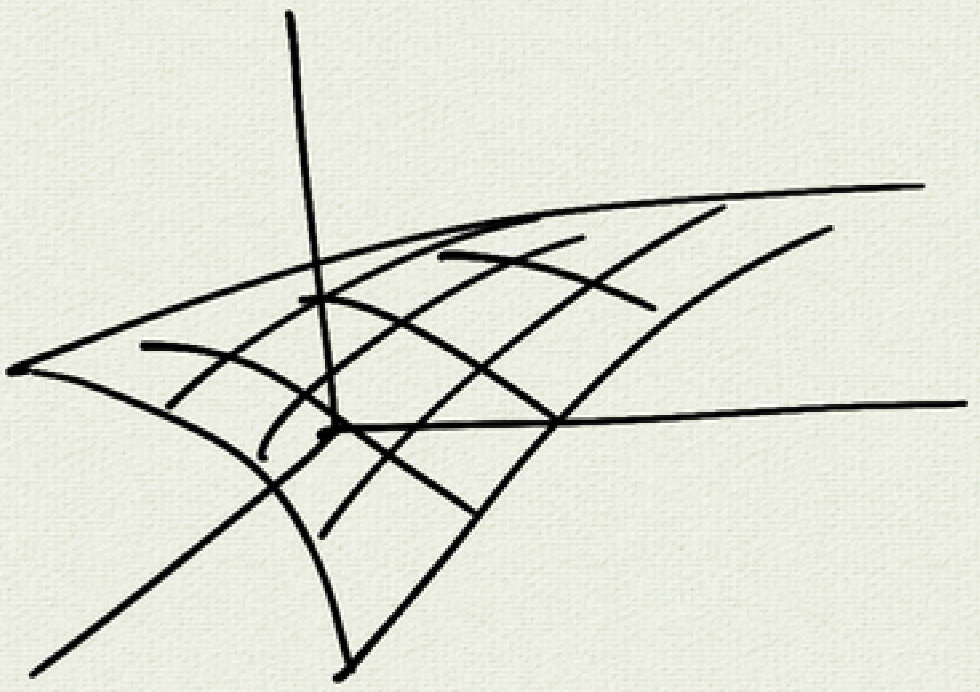
$$z = f(x,y)$$



$$f_x = \frac{\partial f}{\partial x}$$

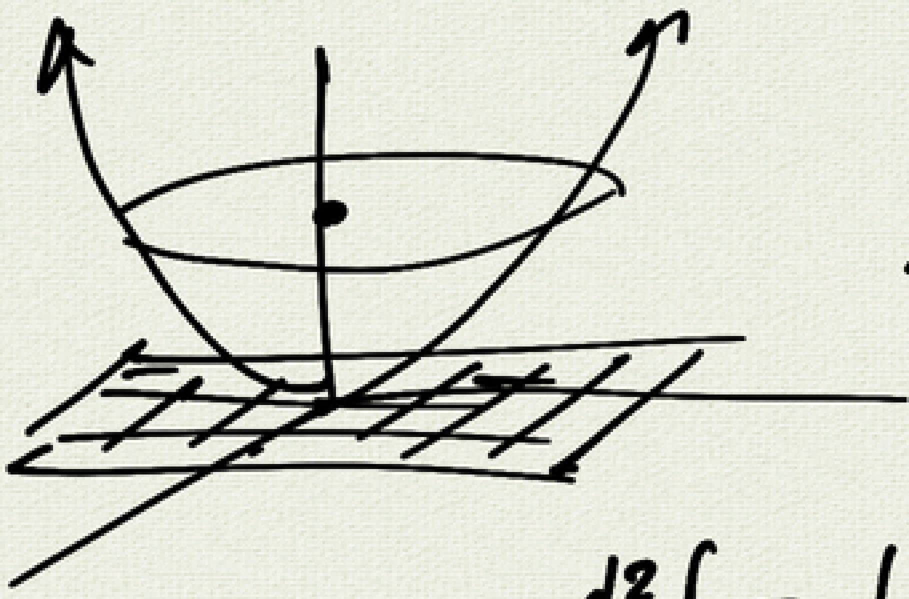
$$f_y = \frac{\partial f}{\partial y}$$

$$f_{xx} = \frac{\partial^2 f}{\partial x^2}$$

$$f_{xy} = \frac{\partial^2 f}{\partial y \, \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$$

example:

$$f(x,y) = x^2 + y^2$$



elliptic paraboloid

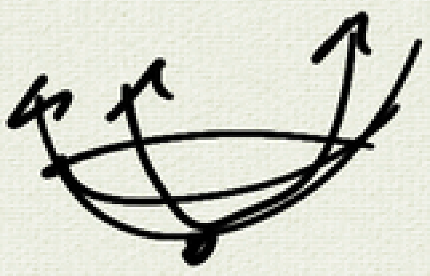
$$f_x = 2x$$

$$f_y = 2y$$

$$d^2 f = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

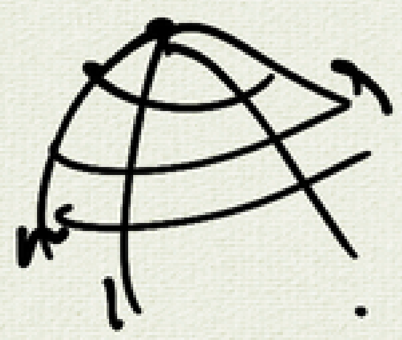
at $(0,0)$: tangent plane $z = 0$

think: $\begin{pmatrix} + & 0 \\ 0 & + \end{pmatrix}$



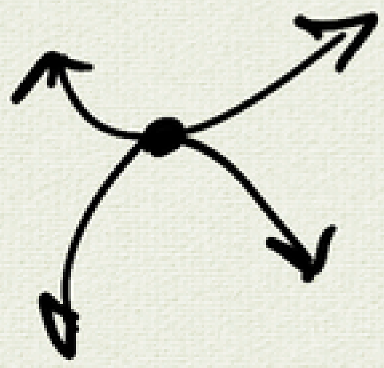
local min

$\begin{pmatrix} - & 0 \\ 0 & - \end{pmatrix}$



local max

$\begin{pmatrix} + & 0 \\ 0 & - \end{pmatrix}$ $\begin{pmatrix} - & 0 \\ 0 & + \end{pmatrix}$



saddle point

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remember how to complete the square:

$$f(x,y) = \underline{\underline{2}}x^2 - 4x + \underline{3}y^2 + 12y + 20$$

$$= 2(x^2 - 2x + 1) + 3(y^2 + 4y + 4) + 20 \; -2 - 12$$

$$= 2(x-1)^2 + 3(y+2)^2 + 6$$

elliptic paraboloid

vertex $(1, -2, 6)$

$$f(x,y) = 2x^2 - 4x + 3y^2 + 12y + 20$$

$$\boxed{\begin{aligned} f_x &= 4x - 4 \\ f_y &= 6y + 12 \end{aligned}} \qquad \begin{aligned} f_{xx} &= 4 & f_{xy} &= 0 \\ f_{yx} &= 0 & f_{yy} &= 6 \end{aligned}$$

critical pts: $f_x = 0 = f_y$

$$\Longrightarrow \quad 4x - 4 = 0 \quad x = 1$$
$$6y + 12 = 0 \quad y = -2$$

$\Longrightarrow$ tangent plane:

$$z = z_0 + \underbrace{f_x}_{0}(x - x_0) + \underbrace{f_y}_{0}(y - y_0)$$

at critical pt

$$z = z_0$$

$$f(x_0, y_0) = f(1,-2) = 6$$

(linear approximation at $(1,-2)$)

$$d^2 f = \begin{pmatrix} 4 & 0 \\ 0 & 6 \end{pmatrix}$$

looks like local min 

at $(0,0)$:

tangent plane

$$z = \underbrace{z_0}_{\substack{f(0,0) \\ = 20}} + f_x(x - \underbrace{x_0}_{0}) + f_y(y - \underbrace{y_0}_{0})$$

$f_x(0,0) = -4$

$f_y(0,0) = +12$

$$z = 20 + (-4)x + 12(y)$$