Unit 5 HW (Change of Variables)
MultiV 2021-22 / Dr. Kessner

1. Find the area of the parallelogram with vertices $(0, 0), (1, 2), (2, 1), (3, 3)$ in two different ways:

a. Integration (express the area as a sum of 3 double integrals).

b. Change variables using $x = 2u + v$ and $y = u + 2v$, and integrate over the unit square $\{(u, v) \mid 0 \leq u \leq 1, 0 \leq v \leq 1\}$.

Note that this change-of-variables is a linear transformation, and can be expressed as a matrix multiplication. Make sure you understand why you are integrating over the unit square.

2. Consider the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

a. Express the volume of the ellipsoid as a triple integral, and then look away in disgust.

b. Apply the change of variables $x = au$, $y = bv$, $z = cw$ to reduce this problem to computing the volume of the unit sphere. Then use spherical coordinates to compute that volume.

3. This exercise is preparation for next class. Figuring out what calculation to do is the main part of the exercise.

a. What is the average value of $\sin(x)$ on $[0, \pi]$? (In other words, what is the average y value of the points on the curve $y = \sin x$ between 0 and π?)

b. What is the average y value of all the points in the 2-dimensional region between the x-axis and $y = \sin(x)$, for $x \in [0, \pi]$?