

Geometric Algebra HW 3 (Wedge Product in \mathbb{R}^3)
MultiV 2021-22 / Dr. Kessner

1. For each of the following sets of vectors, find the following: $u \wedge v$, $u \times v$, $u \wedge v \wedge w$, and $(u \times v) \cdot w$.

a. $u = 3e_1, v = 2e_2, w = 5e_3$

b. $u = 3e_1 + e_2, v = 2e_1 + 2e_2, w = 5e_3$

c. $u = 3e_1 + e_2, v = 2e_1 + 2e_2, w = 7e_1 + 11e_2 + 5e_3$

2. Use the wedge product representation of the plane

$$(r - r_0) \wedge u \wedge v = 0$$

to solve the following problems.

a. Find the standard equation of the plane through the points $\begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 10 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix}$.

Also find the distance from the plane to the origin.

b. Find the standard equation of the plane through the points $\begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$.

Also find the distance from the plane to $\begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$.

Answers:

- 1a. $u \wedge v = 6e_1e_2, u \times v = 6e_3, u \wedge v \wedge w = 30 e_1e_2e_3, (u \times v) \cdot w = 30.$
- 1b. $u \wedge v = 4e_1e_2, u \times v = 4e_3, u \wedge v \wedge w = 20 e_1e_2e_3, (u \times v) \cdot w = 20.$
- 1c. $u \wedge v = 4e_1e_2, u \times v = 4e_3, u \wedge v \wedge w = 20 e_1e_2e_3, (u \times v) \cdot w = 20.$
- 2a. $x + y + z = 10, d = \frac{10}{\sqrt{3}}$
- 2b. $x + z = 4, d = 2\sqrt{2}$