Geometric Algebra HW 4 (Geometric Product in $\mathbb{R}^3$)
MultiV 2021-22 / Dr. Kessner

1. Let $w = e_1 + e_3$. Let $w' = (e_2 e_1) w (e_1 e_2)$

   Show that $w' = -e_1 + e_3$.

   Draw w and w'. Verify that w' is the result of reflecting w in e_1, and then e_2. Also verify that this equivalent to rotation by π in the $e_1 e_2$ plane.

2. Let $w = e_1 + e_3$. Let $u = \frac{w}{\sqrt{2}}$ and $v = e_1$. Note that u and v are unit vectors in the $e_1 e_3$ plane, and the angle between the two vectors is $\frac{\pi}{4}$.

   Let $w' = (vu) w (uv)$.

   Show that $w' = e_1 - e_3$.

   Draw w and w'. Verify that w' is the result of rotating w by $\frac{\pi}{2}$ in the $e_1 e_3$ plane .

3. Let $w = e_1 + e_3$. Find two vectors u and v to represent rotation by $-\frac{\pi}{4}$ in the $e_2 e_3$ plane. (Clockwise $45°$ if you're on the positive e_1 axis looking at the origin). Let $w' = (vu) w (uv)$.

   Show that $w' = e_1 + e_2$.