

Homework 6.1 Combinatorics
PCHA 2021-22 / Dr. Kessner

No calculator unless absolutely necessary.

1. Evaluate the following by hand:

$${}_3P_1$$

$${}_3P_2$$

$${}_3P_3$$

$${}_4P_1$$

$${}_4P_2$$

$${}_4P_3$$

$${}_4P_4$$

$${}_5P_2$$

$${}_5P_3$$

2. Evaluate the following by hand:

$$\binom{2}{0}$$

$$\binom{2}{1}$$

$$\binom{2}{2}$$

$$\binom{3}{0}$$

$$\binom{3}{1}$$

$$\binom{3}{2}$$

$$\binom{3}{3}$$

$$\binom{5}{2}$$

$$\binom{5}{3}$$

$$\binom{6}{2}$$

$$\binom{6}{4}$$

3. How many pizzas can you make with 12 toppings?
4. How many ways can you pick 1st, 2nd, and 3rd place from 8 contestants? How many ways can you pick a committee of 3 from 8 people?
5. You have 6 dogs and 4 cats. You decide to organize a dog parade for your favorite 5 dogs, so you need to line up 5 dogs (in order of preference), and you need to pick two cats to be official Dog Herders to keep the dogs in line. How many ways can you do this?
6. How many subsets does a set of size 7 have? How many binary sequences are there with length 8?
7. How many license plates have the form #AAA###, i.e. a digit followed by 3 letters, followed by 3 digits?
8. A sequence of DNA can be thought of as a sequence of letters from an alphabet of size 4: $\{A, C, G, T\}$. A *codon* is a sequence of length 3. For example, AAA, ACT, and TTG are all codons. How many possible codons are there?
9. A protein is a sequence of amino acids, of which there are 20 types, so you can think of a protein as a sequence of letters from an alphabet of size 20. How many possible proteins are there of length 10?
10. How many 4 letter words are possible using the English alphabet?

Answers:

1) 3, 6, 6, 4, 12, 24, 24, 20, 60

2) 1, 2, 1, 1, 3, 3, 1, 10, 10, 15, 15

3) 4096

4) ${}_8P_3 = 336, \binom{8}{3} = 56$

5) ${}_6P_5 \cdot \binom{4}{2} = 6! \cdot 6 = 720 \cdot 6$

6) $2^7 = 128, 2^8 = 256$

7) $10 * 26 * 26 * 26 * 10 * 10 * 10$

8) $4^3 = 64$

9) 20^{10}

10) 26^4