

Classwork 9.1 Chain Rule
PCHA 2022-23 / Dr. Kessner

No calculator. Have fun!

Quotient rule: $(f/g)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

1. Let $f(x) = (2x + 1)^3$. Find $f'(x)$ in two ways (and verify that your answers are the same):

- Use the power and chain rules.
- Expand f and use the power rule.

2. Let $g(x) = x^{-3}$. Find $g'(x)$ in two ways:

- Use the power rule.
- Write $g(x)$ as $\frac{1}{x^3}$ and use the quotient rule.

3. Let $h(x) = \sec x$. Find $h'(x)$ in two ways:

- Write $h(x)$ as $\frac{1}{\cos x}$ and use the quotient rule.
- Write $h(x)$ as $(\cos x)^{-1}$ and use the power and chain rules.

4. Let $k(x) = \cot x$. Find $k'(x)$ in two ways:

- Write $k(x)$ as $\frac{\cos x}{\sin x}$ and use the quotient rule.
- Write $k(x)$ as $(\tan x)^{-1}$ and use the power and chain rules.

5. Let $l(x) = \sin^2 x + \cos^2 x$. Find $l'(x)$ in two ways:

- Use the power and chain rules.
- Simplify using the Pythagorean identity first.

6. Let $m(x) = 2 \sin x \cos x$. Find $m'(x)$ in two ways:

- Use the product rule.
- Write $m(x)$ as $\sin 2x$ and use the chain rule.

7. Let $n(x) = \cos^2 x - \sin^2 x$. Find $n'(x)$ in two ways:

- Use the power and chain rules.
- Write $n(x)$ as $\cos 2x$ and use the chain rule.

Challenge. Write $\frac{f(x)}{g(x)}$ as $f(x)[g(x)]^{-1}$ and prove the quotient rule using the product, power, and chain rules.