KEY

No calculator, no notes – just your brain! Have fun!
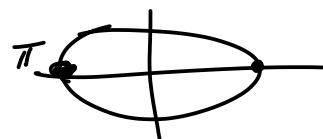
1. Evaluate the following:
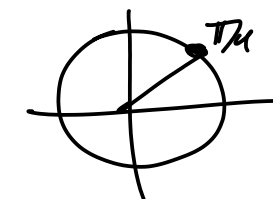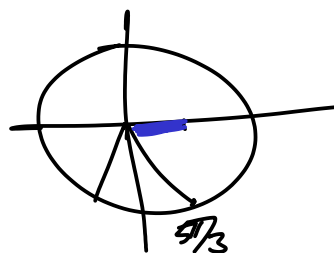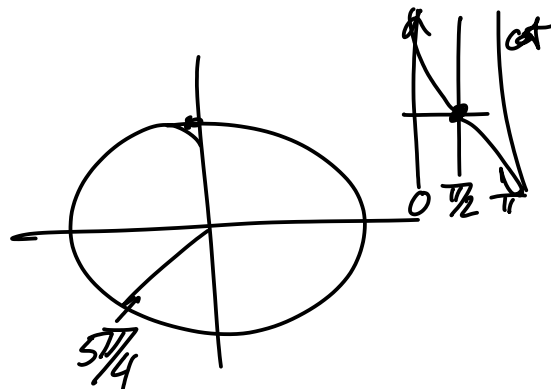
a) $\cot \frac{\pi}{2}$ $= 0$

b) $\tan \frac{5\pi}{4}$ $= 1$

c) $\sec \frac{5\pi}{3}$ $= 2$

d) $\cos \frac{25\pi}{2}$ $= \cos\left(\frac{24\pi}{2} + \frac{\pi}{2}\right) = 0$

e) $\tan^{-1}\left(\sin \frac{25\pi}{2}\right)$

$\underbrace{\sin\left(\frac{24\pi}{2} + \frac{\pi}{2}\right)}_{1} = \frac{\pi}{4}$

f) $\sin^{-1}\left(\tan(\cos^{-1}(-1))\right)$ $= 0$

$\underbrace{\underbrace{\tan\underbrace{}_{\pi}}_{0}}_{0}$

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2. A spider jumps onto a hamster wheel at the right-most (3 o'clock) position. This scares the hamster, which tries to run away from the spider quickly, rotating the hamster wheel at a rate of 1 revolution every 6 seconds. The hamster wheel has a radius of 8 inches and the bottom of the wheel is 2 inches above the ground.

a) Graph the x and y position of the spider, $x(t)$ and $y(t)$. Find equations for both $x(t)$ and $y(t)$.

period $= 6$ sec.

$$x(t) = 8\cos\left(\frac{2\pi}{6}t\right)$$

$$y(t) = 10 + 8\sin\left(\frac{2\pi}{6}t\right)$$
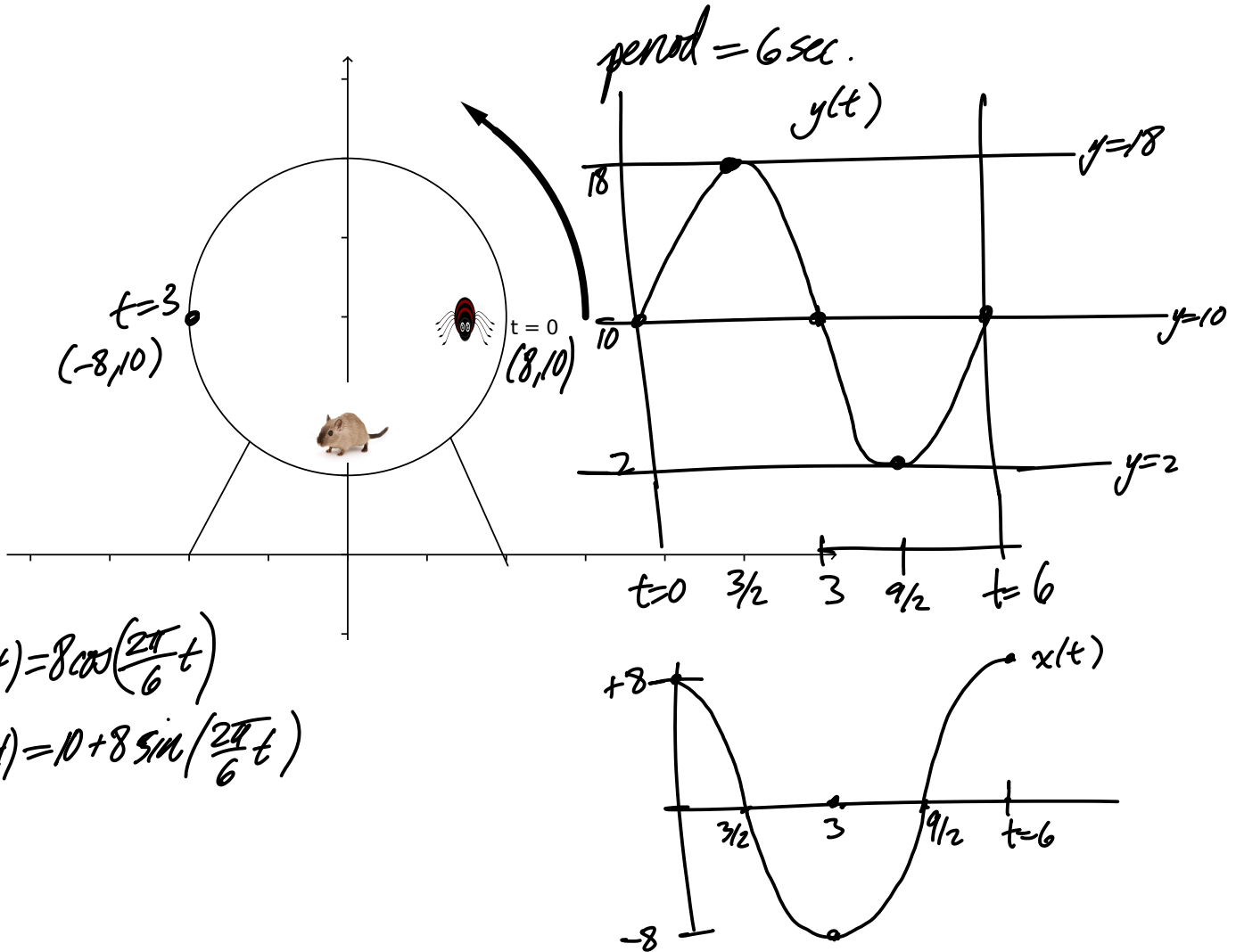
b) Calculate the position $(x(t), y(t))$ of the spider at $t = 3$ and $t = 6$. Make sure your answers make sense. When does the spider reach the hamster (assuming the hamster stays at the bottom of the wheel)?

$t = 3$ $x(3) = 8\underset{-1}{\cos\pi} = -8$ $(-8, 10)$

$y(3) = 10 + 8\underset{0}{\sin\pi} = 10$

$t = 6$ $x(6) = 8\underset{1}{\cos 2\pi} = 8$ $(8, 10)$

$y(6) = 10 + 8\underset{0}{\sin 2\pi} = 10$

$\overline{\text{bottom at } t = \frac{9}{2}}$: $x(\frac{9}{2}) = 8\underset{0}{\cos\frac{3\pi}{2}} = 0$ $(0, 2)$ ✓

$y(\frac{9}{2}) = 10 + 8\underset{-1}{\sin\frac{3\pi}{2}} = 2$

3. Write down all the relevant properties (period, amplitude, shifts/scales, asymptotes) of the following trig functions, and then graph by hand. Write the domain and range of the function.
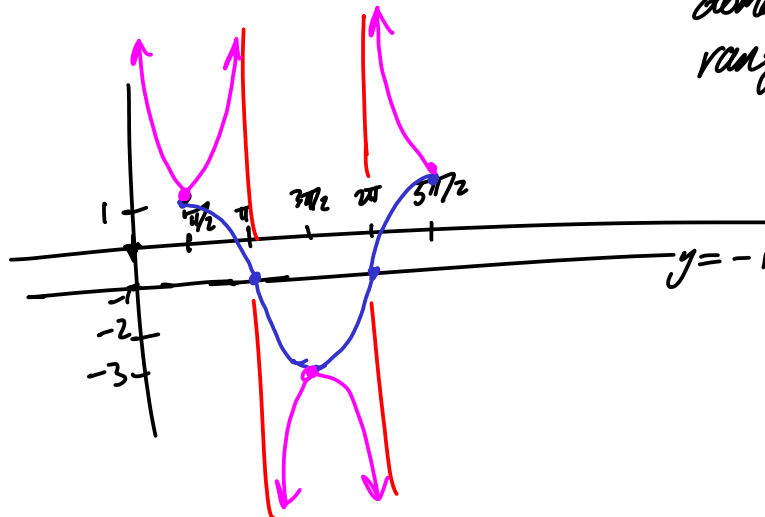
a. $f(x) = 2\sec(x - \frac{\pi}{2}) - 1$ ← *vertical shift −1*

vertical scale ×2

horizontal shift + π/2

period 2π

domain $x \neq n\pi$ $(n \in \mathbb{Z})$
range $(-\infty, -3] \cup [1, \infty)$

y = −1

b. $g(x) = \frac{1}{2}\tan(\frac{x}{3}) + 1$

vertical scale ×½

vertical shift +1

period $\frac{\pi}{(1/3)} = 3\pi$

domain:
$x \neq \frac{3\pi}{2} + n \cdot 3\pi$
$(n \in \mathbb{Z})$

range: $\mathbb{R}$

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