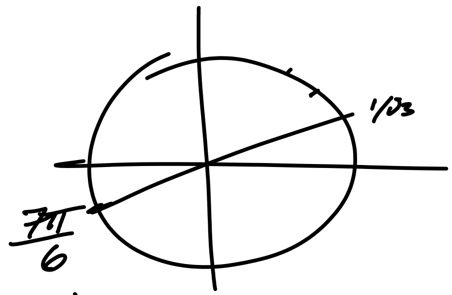
Unit 2 Group Work
PCHA 2022-23 / Dr. Kessner

Name & Pledge:

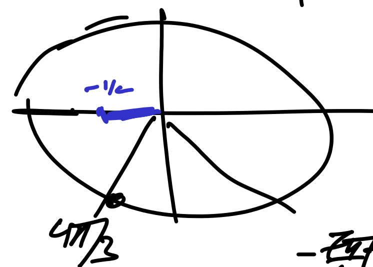
KEY

No calculator! Have fun!

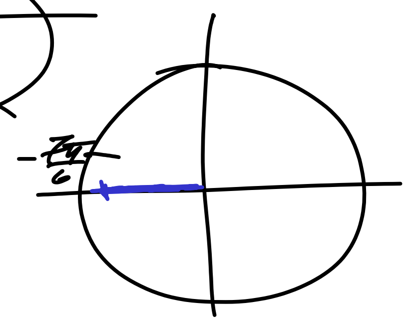
1. Evaluate the following:



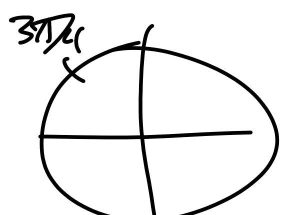
a) $\tan \frac{7\pi}{6} = \frac{1}{\sqrt{3}}$



b) $\sec \frac{4\pi}{3} = -2$

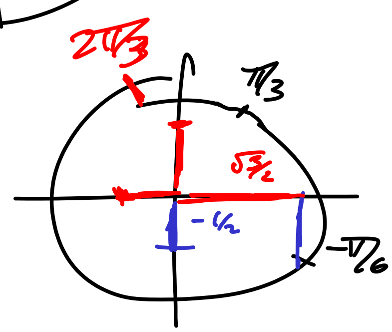


c) $\cos(-\frac{7\pi}{6}) = -\frac{\sqrt{3}}{2}$



d) $\cot \frac{99\pi}{4} = \cot\left(\frac{96\pi}{4} + \frac{3\pi}{4}\right)$

$= -1$



e) $\cos^{-1} \underbrace{\sin(-\frac{\pi}{6})}_{-1/2} = \frac{2\pi}{3}$

f) $\sin^{-1} \underbrace{\cos(-\frac{\pi}{6})}_{\sqrt{3}/2} = \frac{\pi}{3}$

2. Write down all the relevant properties (period, amplitude, shifts/scales, asymptotes) of the following trig functions, and then graph by hand.

$f(x) = 3\sec 2\pi x$

vertical scale 3

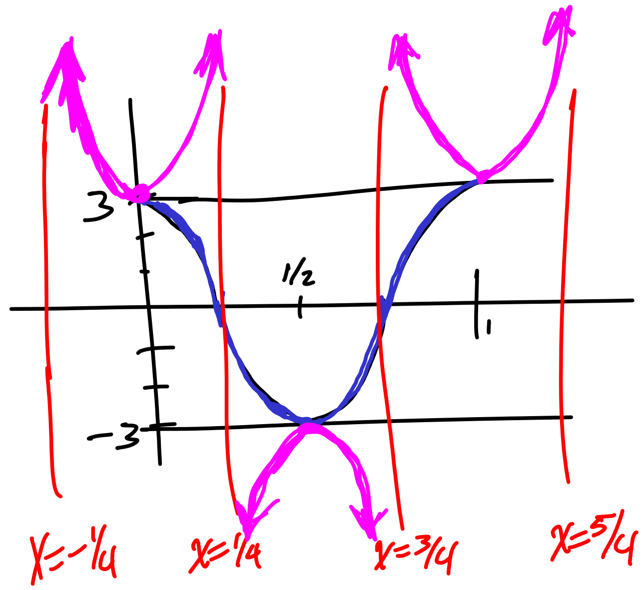
period 1

asymptotes $x = \frac{1}{4} + \frac{n}{2}$ $(n \in \mathbb{Z})$

forgot to add:

domain: $x \neq \frac{1}{4} + \frac{n}{2}$ $(n \in \mathbb{Z})$

range $= (-\infty, -3] \cup [3, \infty)$



$g(x) = -3\tan \pi x$

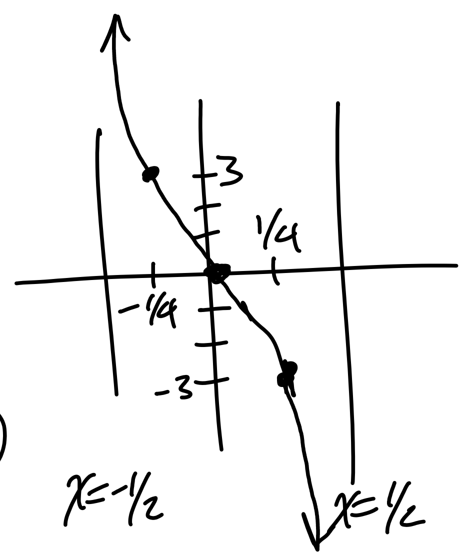
vertical scale 3 (flipped)

period 1

asymptotes $x = \frac{1}{2} + n$ $(n \in \mathbb{Z})$

domain $x \neq \frac{1}{2} + n$ $(n \in \mathbb{Z})$

range $\mathbb{R}$



3. Prove the identities:

$$(\sec\theta - \cos\theta)^2 + \sin^2\theta = \tan^2\theta$$

$$(\sec\theta - \cos\theta)^2 + \sin^2\theta = \sec^2\theta \underbrace{-2\sec\theta\cos\theta}_{-2} + \underbrace{\cos^2\theta + \sin^2\theta}_{1}$$

$$= \sec^2\theta - 1$$

$$= \tan^2\theta \quad \checkmark$$

$$\frac{\sin\theta}{\sec\theta - \cos\theta} = \cot\theta$$

$$\frac{\sin\theta}{\sec\theta - \cos\theta} = \frac{\sin\theta}{\frac{1}{\cancel{\cos\theta}} - \cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{\cancel{\cos\theta}}$$

$$= \frac{\sin\theta\cos\theta}{1 - \cos^2\theta}$$

$$= \frac{\cancel{\sin\theta}\cos\theta}{\sin^2\theta}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta \quad \checkmark$$

4. Use a sum formula to find cos(195°).

$$\cos(195^\circ) = \cos(\underset{3\pi/4}{135^\circ} + \underset{\pi/3}{60^\circ})$$

$$= \cos\tfrac{3\pi}{4}\cos\tfrac{\pi}{3} - \sin\tfrac{3\pi}{4}\sin\tfrac{\pi}{3}$$

$$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}$$

$$= \frac{-\sqrt{2}-\sqrt{6}}{4}$$

Derive the following half angle formula from the relevant double angle formula:

$$\cos u = \pm\sqrt{\frac{1+\cos 2u}{2}}$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$\cos 2u = 2\cos^2 u - 1$$

$$2\cos^2 u = 1 + \cos 2u$$

$$\cos^2 u = \frac{1+\cos 2u}{2}$$

$$\cos u = \pm\sqrt{\frac{1+\cos 2u}{2}}$$

Use the half angle formula above to find cos(195°).

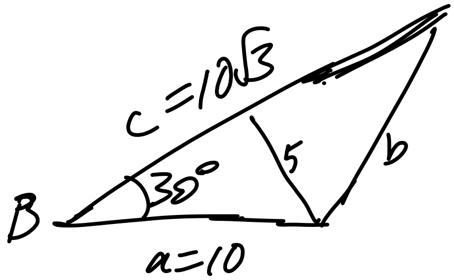
$$\cos 195^\circ = \pm\sqrt{\frac{1+\cos 390^\circ}{2}}$$

$$= \pm\sqrt{\frac{1+\sqrt{3}/2}{2}}$$

$$= -\frac{\sqrt{2+\sqrt{3}}}{2}$$

from quadrant

5. Solve the following triangle: $a = 10$, $c = 10\sqrt{3}$, $B = 30°$.



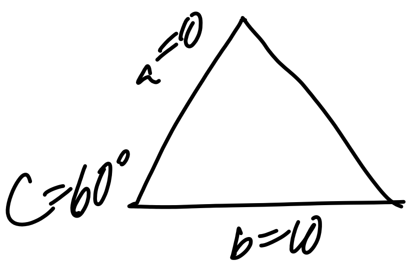
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$= 100 + 300 - 2\cdot 10 \cdot 10\sqrt{3}\cdot \frac{\sqrt{3}}{2}$$
$$= 100$$
$$\boxed{b = 10}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} \Longrightarrow \sin A = \frac{a \sin B}{b} = \frac{10 \cdot \frac{1}{2}}{10} = \frac{1}{2}$$

$$\boxed{A = \frac{\pi}{6}} \text{ or } \frac{5\pi}{6} \quad \text{not possible: } B + A = \frac{\pi}{6} + \frac{5\pi}{6} = \pi$$

$$\boxed{C = \pi - A - B = 2\pi/3}$$

Solve the following triangle: $a = 10$, $b = 10$, $C = 60°$.



$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$= 100 + 100 - 200 \cdot \frac{1}{2}$$
$$= 100$$
$$\boxed{c = 10}$$

$$\frac{\sin A}{a} = \frac{\sin C}{c} \Longrightarrow \sin A = \frac{a \sin C}{c} = \frac{10 \cdot \sqrt{3}/2}{10} = \frac{\sqrt{3}}{2}$$

$$\boxed{A = \frac{\pi}{3}} \text{ or } \frac{2\pi}{3} \quad \text{can't happen}$$

$$\boxed{B = \pi - A - C = \frac{\pi}{3}}$$