Unit 3 Group Work
PCHA 2022-23 / Dr. Kessner

Name / Pledge:

Partner(s):

You can use your notes and/or textbook. No calculator. Have fun!

1. Suppose you have the following vectors:

$$\vec{u} = \left\langle 2, 2\sqrt{3} \right\rangle$$
$$\vec{v} = \left\langle 3\sqrt{3}, -3 \right\rangle$$
$$\vec{w} = \langle 3, 0 \rangle$$

Calculate the following:

a) $|\vec{u}|$

b) $|\vec{v}|$

c) Unit vector in the direction of $\vec{v}$.

d) Angle between $\vec{u}$ and $\vec{v}$.

e) Angle between $\vec{u}$ and $\vec{w}$.

2. a) Parametrize the line segment from $(1, 2)$ to $(3, 6)$.

b) Parametrize the line segment from $(3, 6)$ to $(1, 2)$ (same points, opposite direction).

c) Parametrize the circle with center $(3, 4)$ and radius 5.

d) Parametrize the same circle, but make the period $= 6$.

3. Find all polar coordinates of the following (rectangular) points:

a) (1, 1)

b) $(-3\sqrt{3}, 3)$

Convert the following equations from rectangular to polar coordinates:

c) $3x + 4y = 5$

d) $x^2 + y^2 = 25$

Convert from polar to rectangular:

e) $r = -5 \sin \theta$

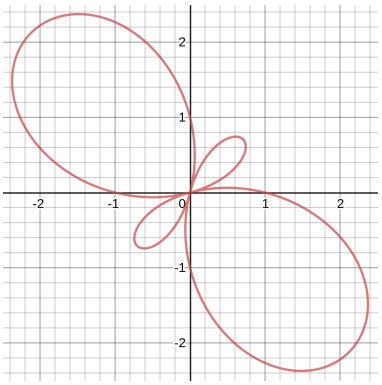
f) $r = 5 \csc \theta$

4. Analyze the graph of the polar function $r = 1 - 2\sin 2\theta$:

1) Find the max $|r|$ values and θ values where they occur.

2) State and prove any symmetry relations.

3) **Challenge:** What is going on at $\frac{\pi}{4}$ and $\frac{5\pi}{4}$?



5. For each of the following 2x2 matrices, determine whether it is invertible, and if so, find the inverse matrix and the determinant of the inverse.

$A = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

$B = \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$

$C = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$

$D = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$

Let $E = \begin{pmatrix} 6 & 5 \\ 5 & 4 \end{pmatrix}$. Find E^{-1}. Verify that $EE^{-1} = I$.

Use the inverse matrix you found to solve the following linear systems:

$6x + 5y = 1$
$5x + 4y = 0$

$6x + 5y = 0$
$5x + 4y = 1$

$6x + 5y = 1$
$5x + 4y = 2$

6. Consider the following system of linear equations:

$$\begin{aligned} x + 3z &= 4 \\ -x - 2z &= -3 \\ y - 2z &= -1 \end{aligned}$$

a. Write the linear system as a matrix equation.

b. Calculate the determinant of the matrix to verify that the matrix is invertible.

c. Find the inverse matrix and use it to solve the system.