

KEY

Unit 4 Group Work 2  
PCHA 2022-23 / Dr. Kessner

No calculator! Have fun!

1. Factor the following polynomial completely, both over  $\mathbb{R}$  (as a product of real linear and irreducible quadratic factors) and over  $\mathbb{C}$  (as a product of complex linear factors). Sketch the graph of the function.

$$p(x) = x^4 - 2x^3 + x^2 + 2x - 2$$

potential rational zeros:  $\pm 1, 2$

$$p(1) = 1 - 2 + 1 + 2 - 2 = 0$$

$$\begin{array}{r|rrrrr} 1 & 1 & -2 & 1 & 2 & -2 \\ & & 1 & -1 & 0 & 2 \\ \hline & 1 & -1 & 0 & 2 & \boxed{0} \end{array}$$

$$p_2(x) = x^3 - x^2 + 2$$

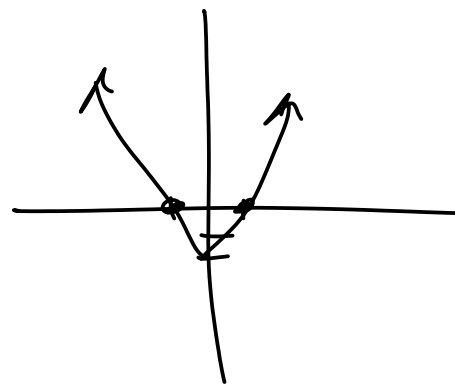
$$p_2(1) = 1 - 1 + 2 \neq 0$$

$$p_2(-1) = -1 - 1 + 2 = 0$$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & 0 & 2 \\ & & -1 & 2 & -2 \\ \hline & 1 & -2 & 2 & \boxed{0} \end{array}$$

$$x^2 - 2x + 2 \Rightarrow \text{roots } x = \frac{2 \pm \sqrt{4-8}}{2} = 1 \pm i$$

$$\begin{aligned} p(x) &= (x-1)(x+1)(x^2-2x+2) \\ &= (x-1)(x+1)(x-(1+i))(x-(1-i)) \end{aligned}$$



2. Factor the following polynomial completely, both over  $\mathbb{R}$  (as a product of real linear and irreducible quadratic factors) and over  $\mathbb{C}$  (as a product of complex linear factors). Sketch the graph of the function. A little bird tells you that  $2 - 2i$  is a zero.

$$q(x) = x^5 - 4x^4 + 5x^3 + 14x^2 - 32x + 16$$

$$\begin{array}{r|rrrrrr} 2-2i & 1 & -4 & 5 & 14 & -32 & 16 \\ & & 2-2i & -8 & -6+6i & 28-4i & -16 \\ \hline & 1 & -2-2i & -3 & 8+6i & -4-4i & 0 \\ & & 2+2i & 0 & -6-6i & 4+4i & \\ \hline & 1 & 0 & -3 & 2 & 0 & \end{array}$$

$$p_2(x) = x^3 - 3x + 2$$

potential rational zeros:  $\pm 1, 2$

$$p_2(1) = 1 - 3 + 2 = 0$$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$$x^2 + x - 2 = (x+2)(x-1)$$

$$\Rightarrow p(x) = (x-1)^2(x+2) \underbrace{(x-(2+2i))(x-(2-2i))}_{x^2-4x+8} \text{ over } \mathbb{C}$$

$$(x-1)^2(x+2) (x^2-4x+8) \text{ over } \mathbb{R}$$

$$\begin{aligned} (2-2i)(-2-2i) &= -(2-2i)(2+2i) \\ &= -8 \end{aligned}$$

$$\begin{aligned} (2-2i)(8+6i) &= 2 \cdot 2 (1-i)(4+3i) \\ &= 4(4-i+3) \\ &= 28-4i \end{aligned}$$

$$(-4-4i)(2-2i) = -8-8 = -16$$

