Unit 4 Group Work 3
PCHA 2022-23 / Dr. Kessner

KEY

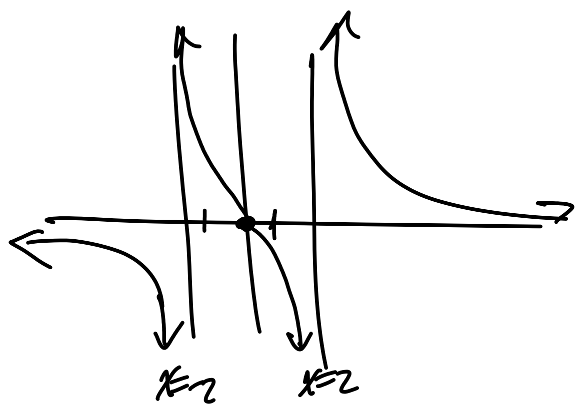
No calculator! Have fun!

1. Sketch the graph of the following rational function. Use limit notation to describe the end behavior of the function, as well as the behavior at any asymptote.

$$f(x) = \frac{x}{x^2 - 4} = \frac{x}{(x-2)(x+2)}$$

end behavior: $\lim_{x \to \pm\infty} f(x) = 0$

		-2		0		2	
$x+2$	$-$	0	$+$	$+$	$+$	$+$	$+$
x	$-$	$-$	$-$	0	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	$-$	$-$	0	$+$
	$-$	X	$+$	0	$-$	X	$+$



$\lim_{x \to -2^-} f(x) = -\infty$

$\lim_{x \to -2^+} f(x) = \infty$

$\lim_{x \to 2^-} f(x) = -\infty$

$\lim_{x \to 2^+} f(x) = \infty$

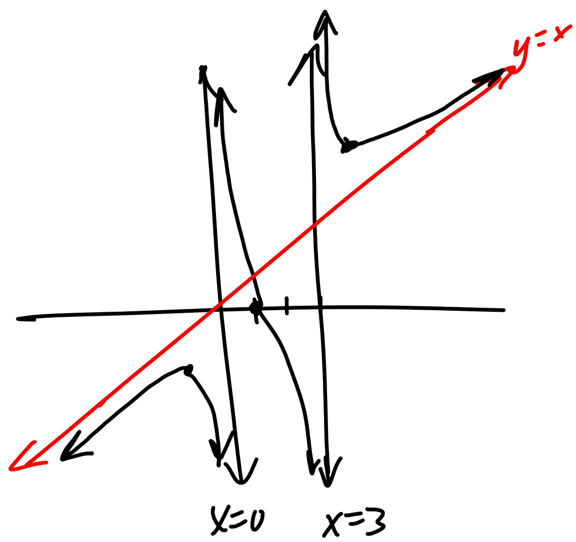
2. Sketch the graph of the following rational function. Use limit notation to describe the end behavior of the function, as well as the behavior at any asymptote. Challenge: describe the asymptotic behavior of the function.

$$g(x) = \frac{(x-1)^3}{x(x-3)}$$

$g(x) = \dfrac{x^3 + \cdots}{x^2 + \cdots} \Rightarrow$ end behavior $\lim_{x\to\infty} g(x) = \infty$

$\lim_{x\to -\infty} g(x) = -\infty$

		0		1		3	
x	$-$	0	$+$	$+$	$+$	$+$	$+$
$(x-1)^3$	$-$	$-$	$-$	0	$+$	$+$	$+$
$x-3$	$-$	$-$	$-$	$-$	$-$	0	$+$
$g(x)$	$-$	X	$+$	0	$-$	X	$+$



asymptotic behavior:

$g(x) = \dfrac{x^3 - 3x^2 + 3x - 1}{x^2 - 3x}$

$g(4) = \frac{27}{4} = 6.75$

$g(-1) = \frac{-8}{4} = -2$

$$\Rightarrow \quad x^2 - 3x \overset{\displaystyle x}{\overline{\big)\, x^3 - 3x^2 + 3x - 1}}$$
$$\underline{x^3 - 3x^2}$$
$$0 + 3x - 1$$

$\Rightarrow g(x) = x + \dfrac{3x-1}{x^2-3x} \rightarrow 0$ as $x \to \infty$

$g(x) \approx x$ as $x \to \infty$

slant asymptote $y = x$

near asymptotes:

$\lim_{x\to 0^-} g(x) = -\infty$

$\lim_{x\to 0^+} g(x) = +\infty$

$\lim_{x\to 3^-} g(x) = -\infty$

$\lim_{x\to 3^+} g(x) = \infty$