Unit 9 Group Work B
PCHA 2022-23 / Dr. Kessner

No calculator! Have fun!

1. Evaluate the following limits, evaluating left and right side limits where applicable.

a. $\lim\limits_{x\to 0} x\csc\dfrac{x}{3}$

b. $\lim\limits_{x\to 0} x\sin\dfrac{x}{3}$

c. $\lim\limits_{x\to \infty} 10^{-x}\sin\dfrac{x}{3}$

d. $\lim\limits_{x\to 0} \cot\dfrac{x}{3}$

e. $\lim\limits_{x\to 0} \dfrac{\sin(\pi+x)-\sin(\pi)}{x}$.

2. a. Find the derivative of $f(x) = \cos 2x$ using a limit definition. Recall that $\lim_{x \to 0} \frac{\sin x}{x} = 1$ and $\lim_{x \to 0} \frac{\cos x - 1}{x} = 0$.
Hint: Use the sum angle formula $\cos(u + v) = \cos u \cos v - \sin u \sin v$, but don't use the double angle formula.

b. Find the derivative of $g(x) = \frac{1}{x}$ using the limit definition:

$$g'(a) = \lim_{x \to a} \frac{g(x) - g(a)}{x - a}$$

3. Using the various rules for differentiation, calculate the derivatives of the following functions.

a. $p(x) = e^{\sin x}$

b. $q(x) = \sin^2 x + \cos^2 x$ (Practice using power and chain rules!)

c. $r(x) = \sin^4 x - \cos^4 x$

d. $s(x) = -\cos 2x$ (Notice that $s'(x) = r'(x)$. Challenge: verify that $r(x) = s(x)$.)

e. $t(x) = 2^{\sin x^2}$

4. Consider the curve $x = 10^y$.

a. Sketch the graph of this curve.

b. Find $\frac{dy}{dx}$ (in terms of x and y) by implicit differentiation.

c. Solve for y in terms of x.

d. Find $\frac{dy}{dx}$ using the expression for y you found above.

e. Verify that these two formulas for $\frac{dy}{dx}$ are the same.

5. Suppose you have 128 kg of ^{14}C, which has a half-life of 5730 years.

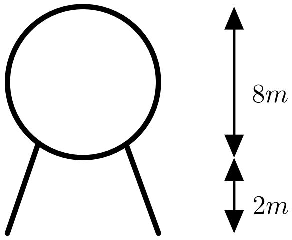
a. Write an equation to model the amount $A(t)$ of ^{14}C as a function of time.

b. Find the average rate of change in the amount over the first 5 half-lives ($5 \cdot 5730$ years). Use a calculator to get approximate values.

c. Find $A'(t)$.

d. Calculate the rate of change (exact) at $t = 0$, $t = 2 \cdot 5730$, and $t = 5 \cdot 5730$ years. Use a calculator to get approximate values.

6.



Model the motion of a Ferris wheel with diamter 8m, sitting 2m off the ground. Suppose you start ($t = 0$) at the 9 o'clock position (furthest left on diagram), traveling counter-clockwise, and that the period is 8 minutes.

a. Write parametric equations $x(t)$ and $y(t)$ to model the position as a function of time.

b. Find $x'(t)$ and $y'(t)$.

c. Evaluate $x'(t)$ and $y'(t)$ at the bottom position.

d. Find $x''(t)$ and $y''(t)$.

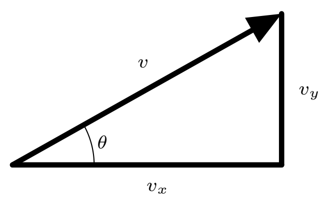
e. Evaluate $x''(t)$ and $y''(t)$ at the bottom position.

7. Recall that you can model projectile motion with parametric equations:

$$x(t) = x_0 + v_x t$$

$$y(t) = y_0 + v_y t - 16t^2$$

where (x_0, y_0) is the initial position of the object, and v_x and v_y are the components of the initial velocity vector v:



Suppose that you launch a rocket straight up from the ground (i.e. at an angle of $\frac{\pi}{2}$), with an initial speed of 96 ft/sec.

a. Write equations for $x(t)$ and $y(t)$.

b. Find $x'(t)$ and $y'(t)$.

c. Find $x''(t)$ and $y''(t)$.

d. Using the derivatives you found above, find the maximum height of the rocket.

e. When does the rocket hit the ground, and how far has it traveled in the x-direction?