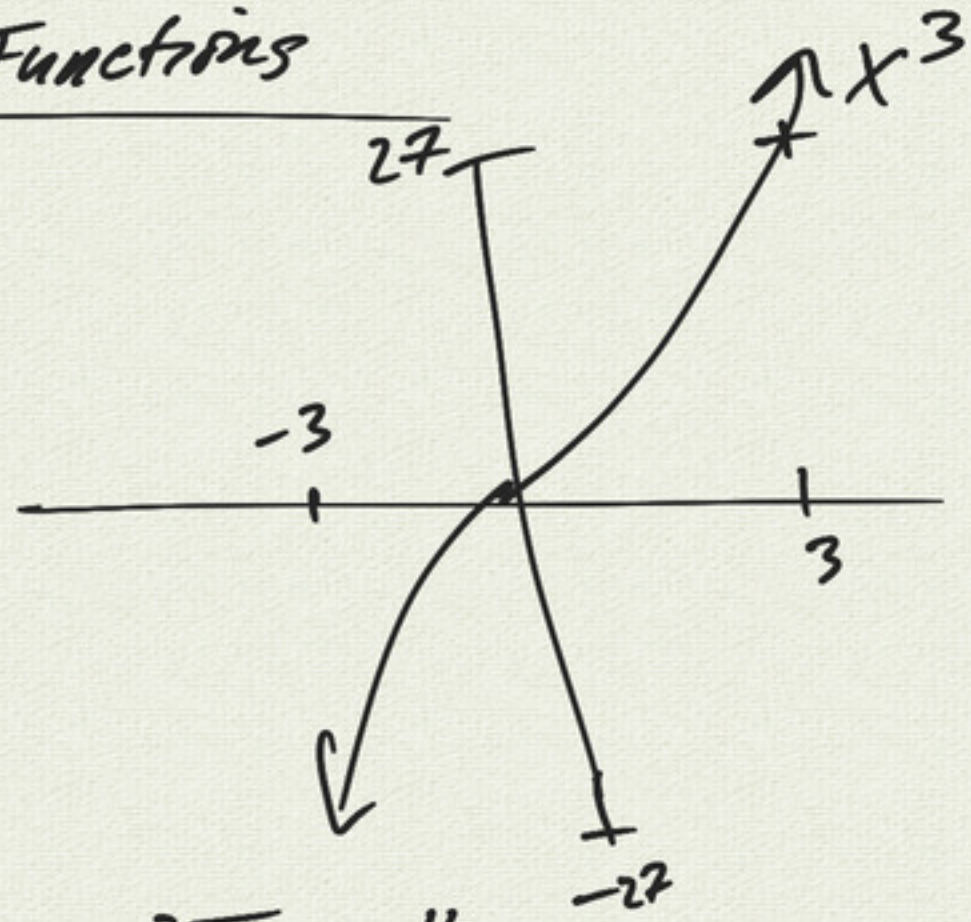
# 1.3 Inverse Trig Functions

$f(x) = x^3$

| $x$ | $x^3$ |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 8 |
| 3 | 27 |
| -1 | -1 |
| -2 | -8 |
| -3 | -27 |




$g(x) = \sqrt[3]{x} = x^{1/3}$

$f(g(x)) = x$
$g(f(x)) = x$
] inverse functions

notation $f^{-1}(x) = g(x)$

$f^{-1}(x) = \sqrt[3]{x}$

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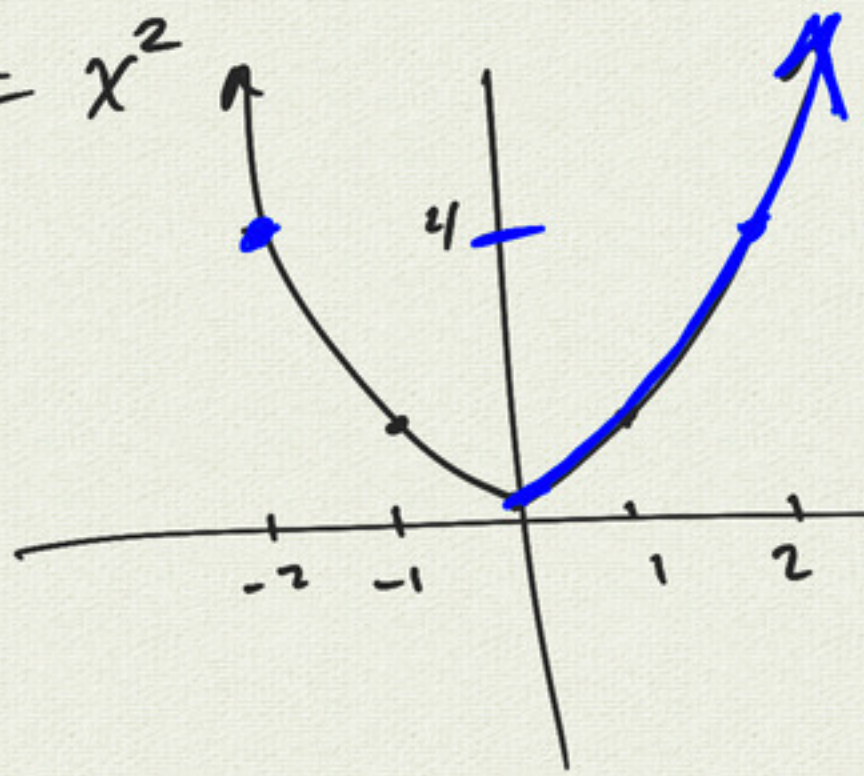
$f(x) = x^2$




$2 \xrightarrow{x^2} 4 \xrightarrow{\sqrt{x}} 2$

$-2 \rightarrow 4 \rightarrow 2$

"partial" inverse

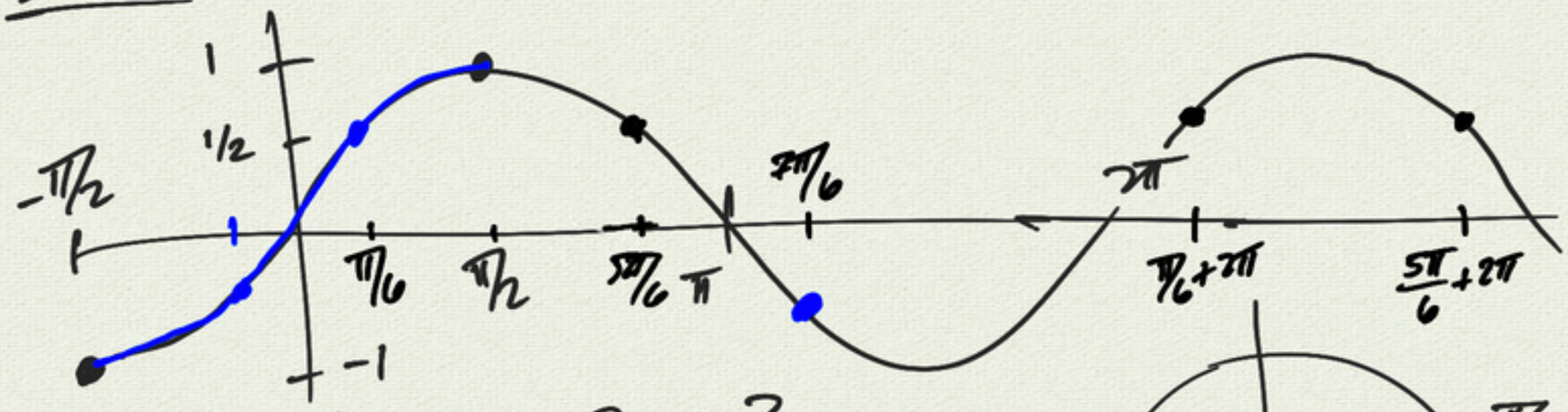
$g(x) = \sqrt{x}$

$f(x)$ is not 1-1

1-1: for a given y value, there is a unique x-value

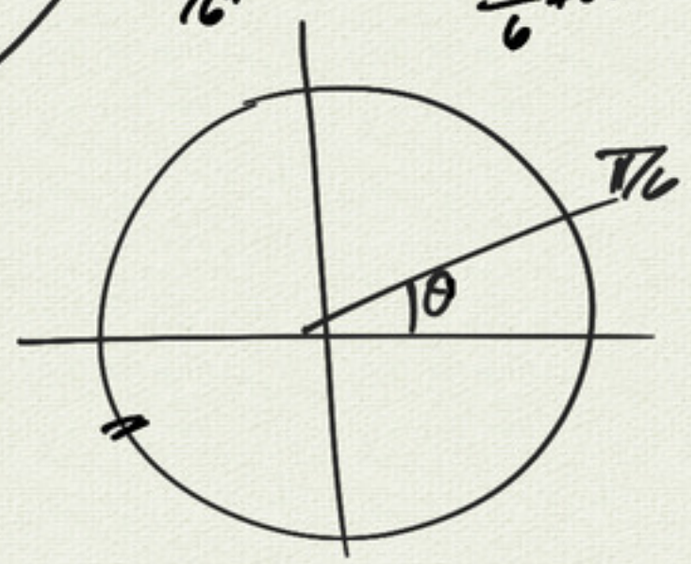
horizontal line test

## $\sin x$




$\sin\theta = \frac{1}{2} \Rightarrow \theta = ?$

$\theta = \pi/6, \frac{5\pi}{6}$




**define:**

$\sin^{-1}$ : pick value in $[-\pi/2, \pi/2]$

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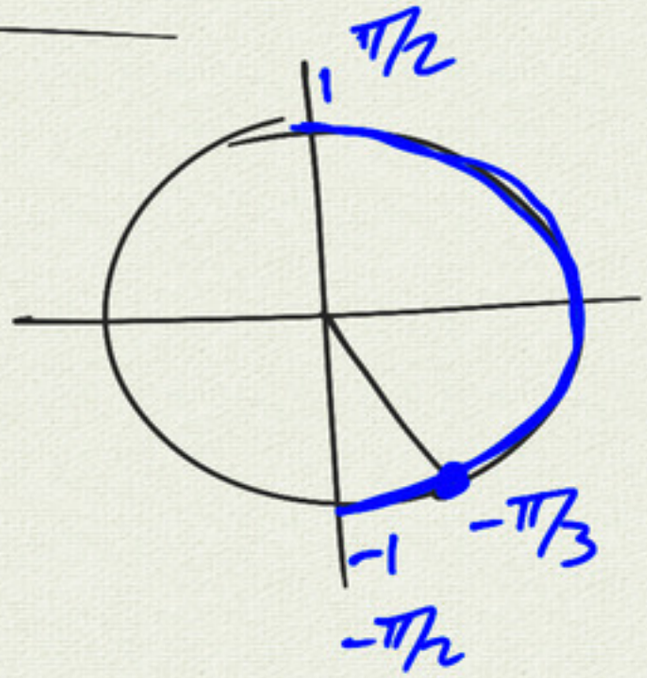
$\sin^{-1}(1) = \frac{\pi}{2}$

$\sin^{-1}(\frac{1}{2}) = \pi/6$

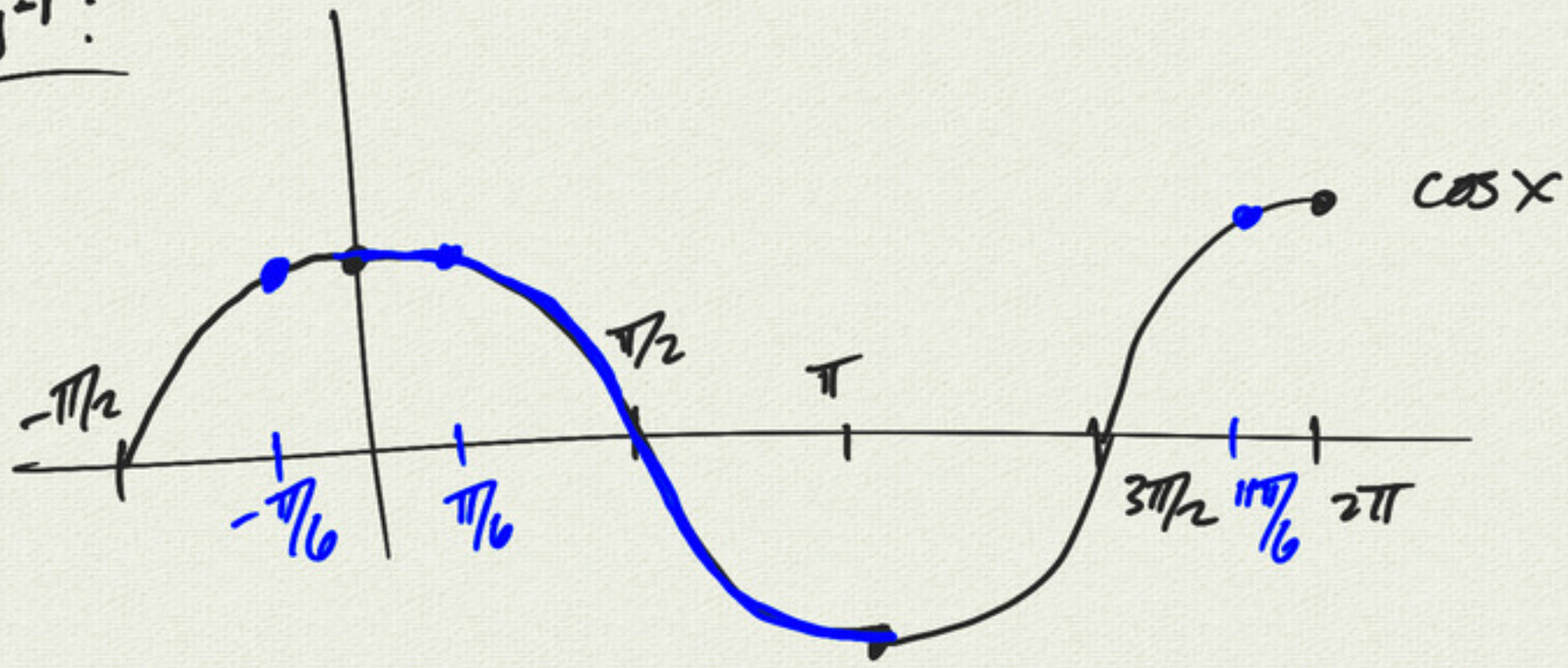
$\sin^{-1}(0) = 0$

$\sin^{-1}(-\frac{1}{2}) = -\pi/6$

$\sin^{-1}(-\frac{\sqrt{3}}{2}) = -\pi/3$



$\cos^{-1}$:




$\cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \pi/6$

define $\cos^{-1}$: pick value in $[0, \pi]$

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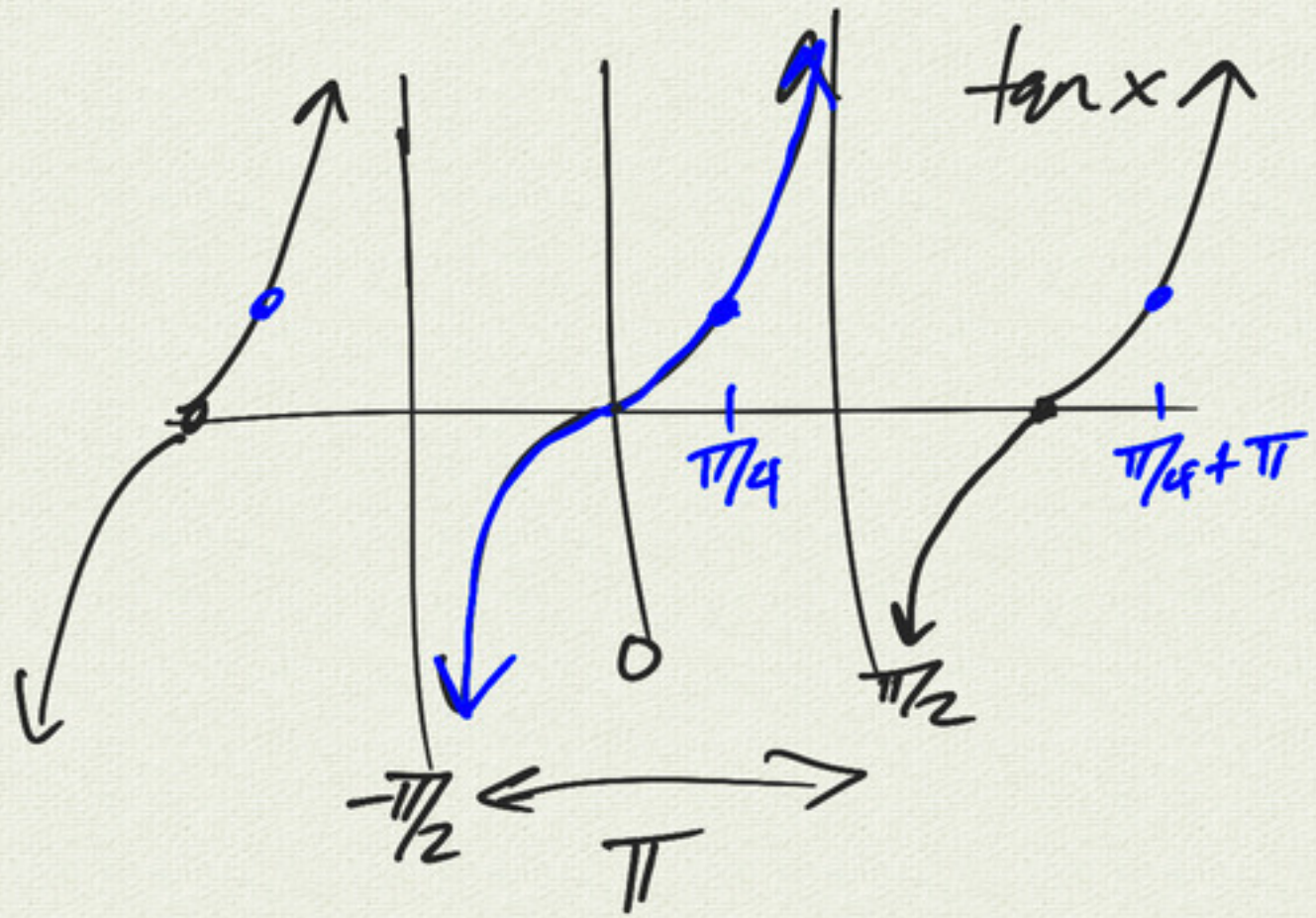
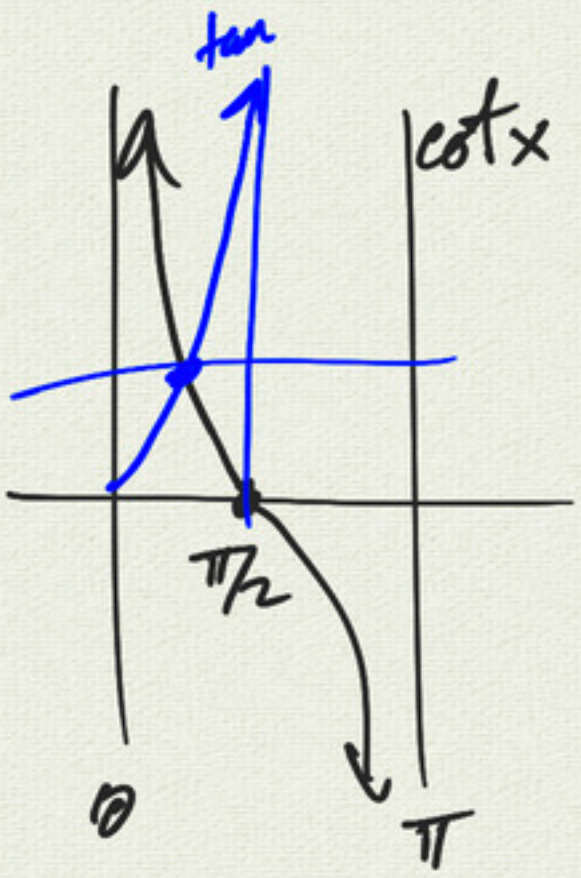
"partial":

$$\cos^{-1}\left(\underbrace{\cos\frac{\pi}{6}}_{\sqrt{3}/2}\right) = \pi/6$$

$$\cos^{-1}\left(\underbrace{\cos\frac{11\pi}{6}}_{\frac{\sqrt{3}}{2}}\right) = \pi/6$$

$\tan^{-1}$




$\tan\theta = 1 \longrightarrow \theta = \pi/4, \frac{3\pi}{4}, \cdots$

define $\tan^{-1}$: pick value in $[\frac{-\pi}{2}, \pi/2]$

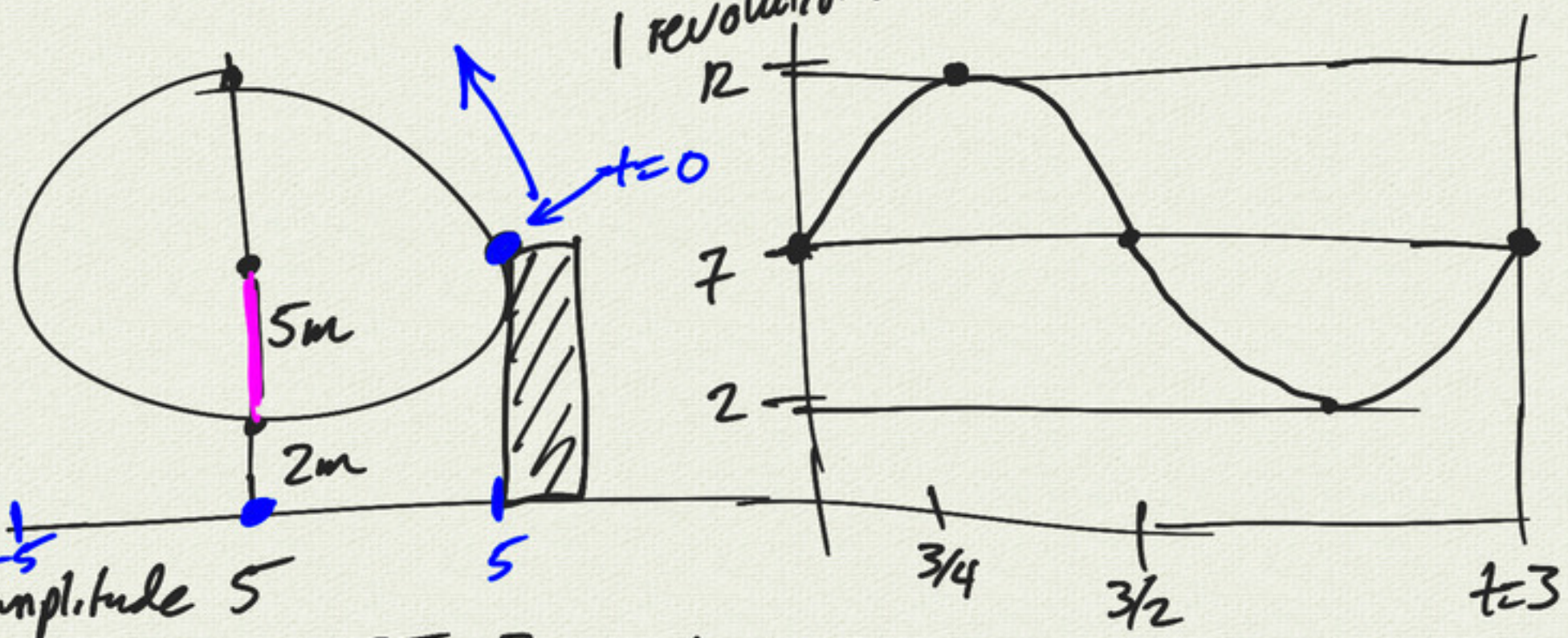
$\tan^{-1} 1 = \frac{\pi}{4} \iff 1 = \tan\frac{\pi}{4}$

$\tan^{-1} 0 = 0$

$\tan^{-1} -1 = -\frac{\pi}{4}$

# 1.4 Word Problems

**Example** Ferris wheel




amplitude 5

period 3 $\Rightarrow \frac{2\pi}{b} = 3 \Rightarrow b = \frac{2\pi}{3}$

$a\sin[b(x-h)]+k$

vertical shift +7

$$y(t) = 5\sin\left[\frac{2\pi}{3}t\right] + 7$$

**Check:**

$t=3$: $y(3) = 5\sin\left(\frac{2\pi}{3}3\right) + 7 = 7$ ($\sin(2\pi) = 0$)

$t=3/4$: $y\left(\frac{3}{4}\right) = 5\sin\left(\frac{\pi}{2}\right) + 7 = 12$ ($\sin\frac{\pi}{2} = 1$)

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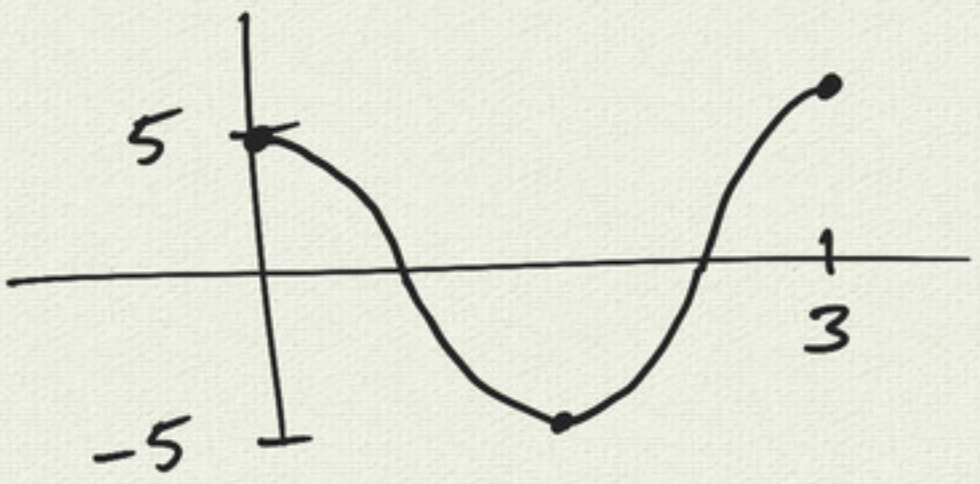
$x(t)$:

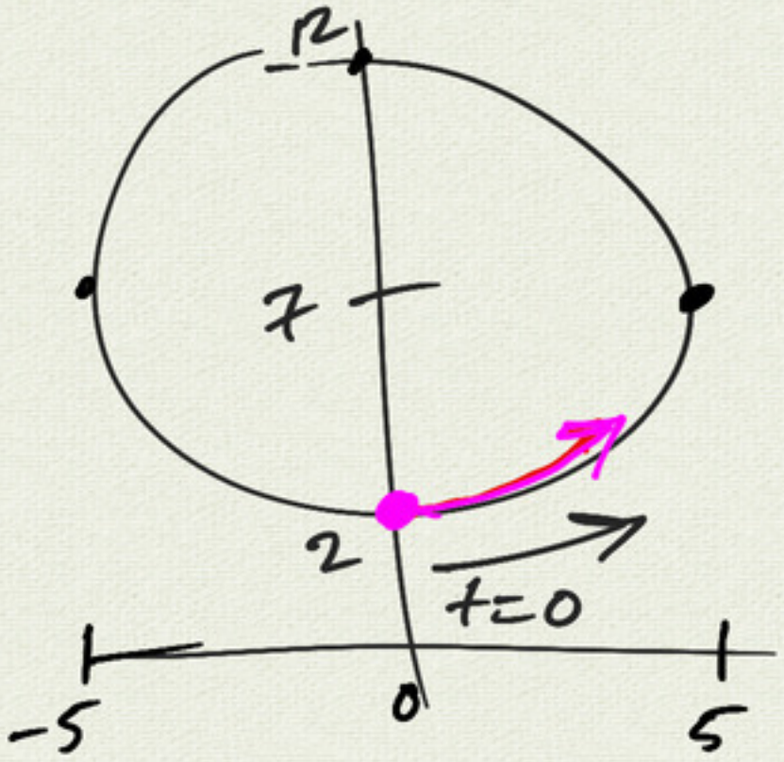


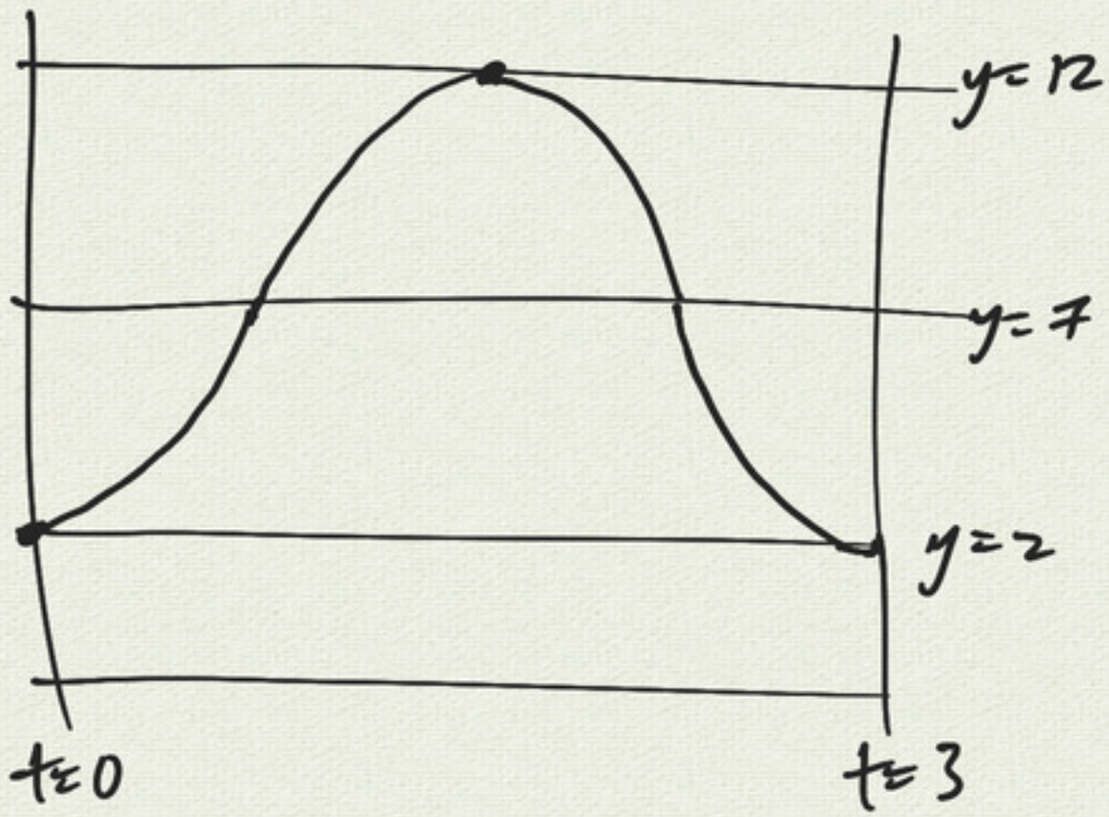

amplitude 5

no shift

$$x(t) = 5\cos\left(\frac{2\pi}{3}t\right)$$

different starting point




$$y(t) = -5\cos\left(\frac{2\pi}{3}t\right) + 7$$

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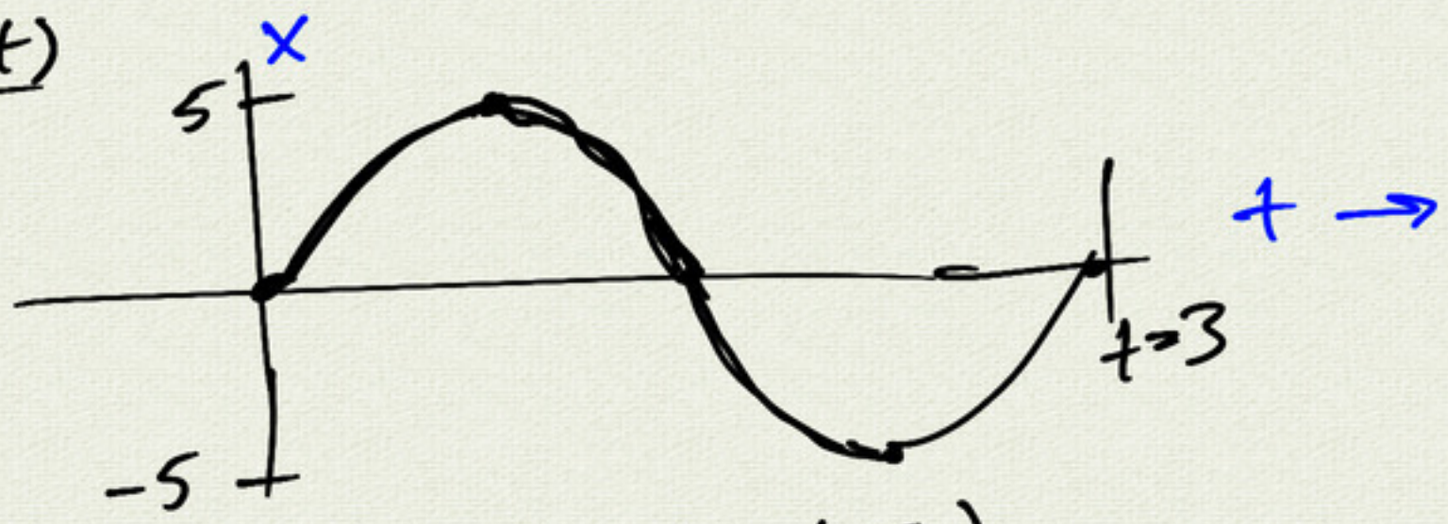
x(t)




$$x(t) = 5\sin\left(\frac{2\pi}{3}t\right)$$

x is a function of t