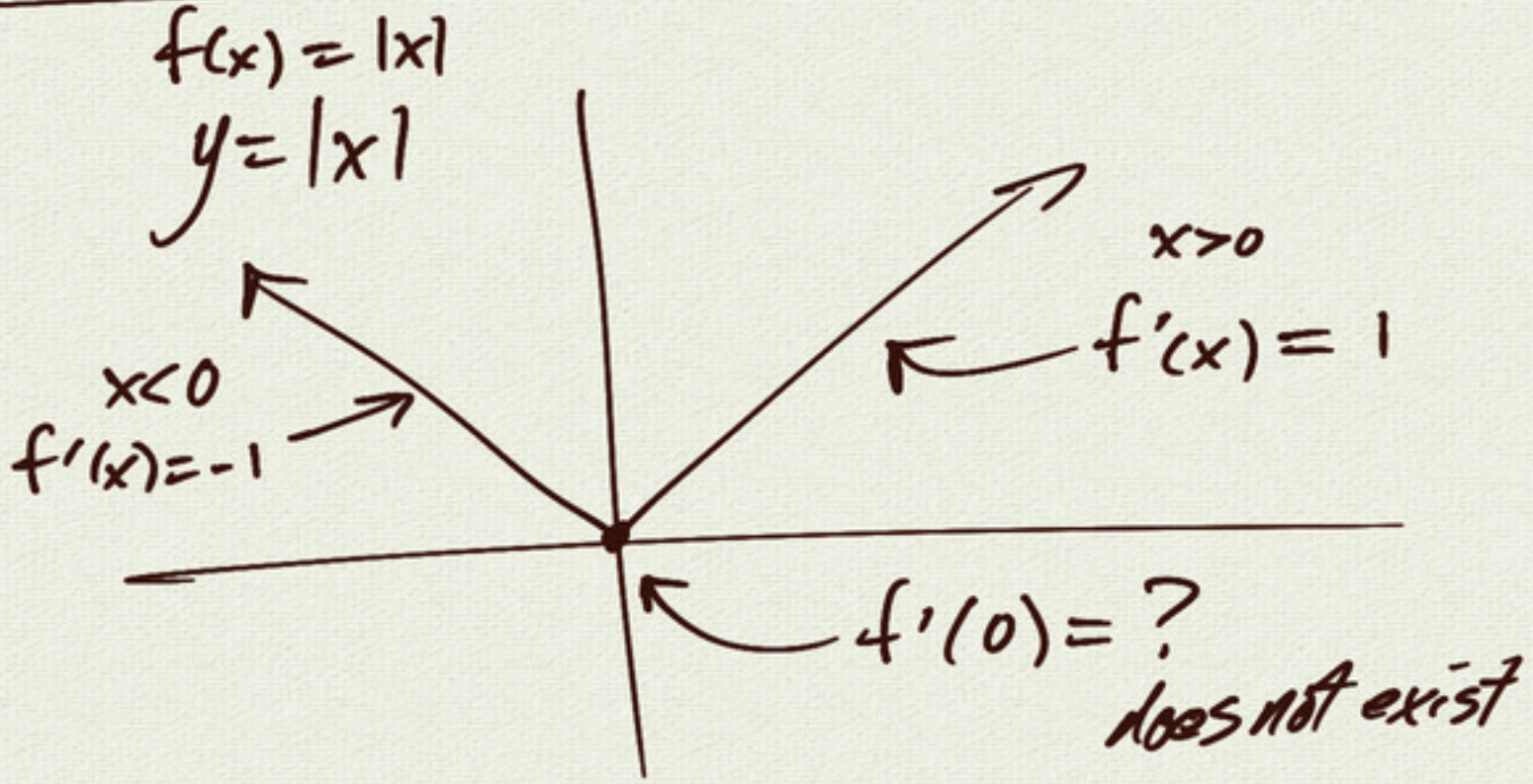
10.1 Extreme Values



f is continuous at $x=0$

f is not differentiable at $x=0$

smoother

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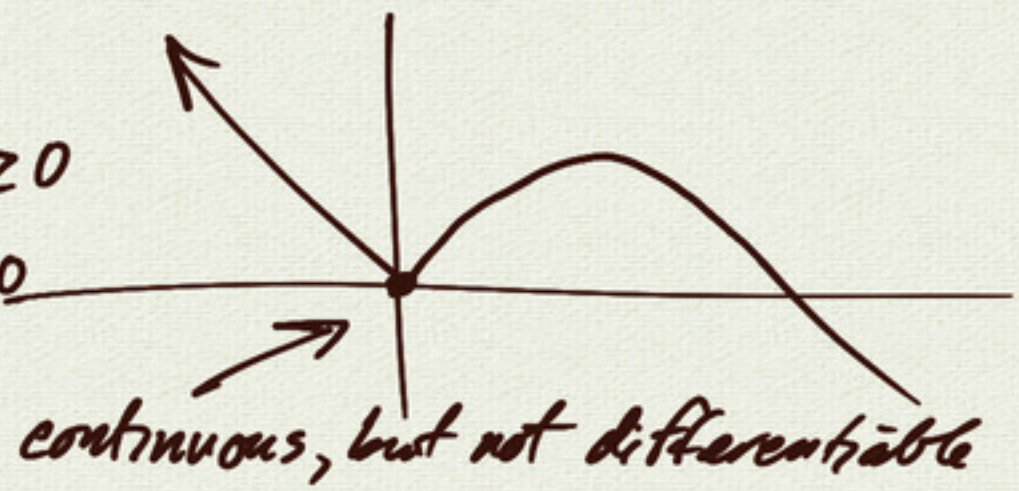
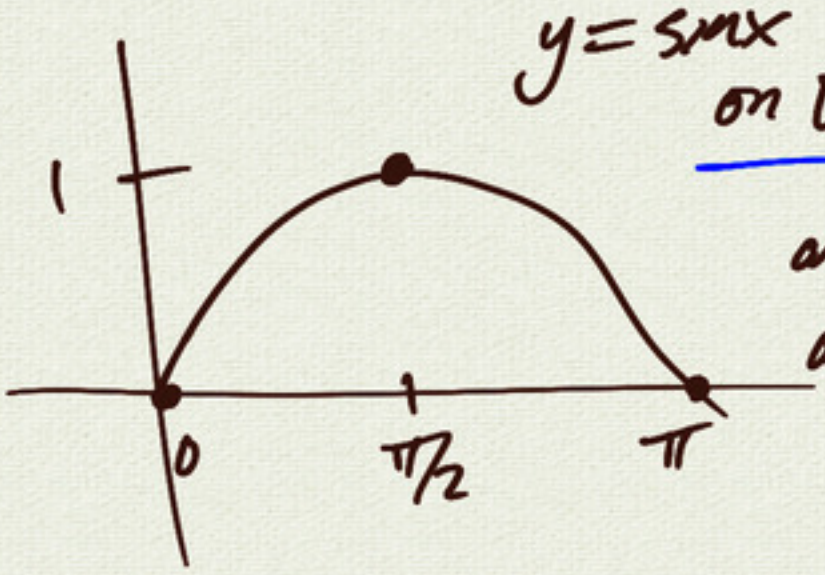
piecewise:

$$g(x)=\begin{cases}\sin x & x\geq 0\\ -x & x<0\end{cases}$$



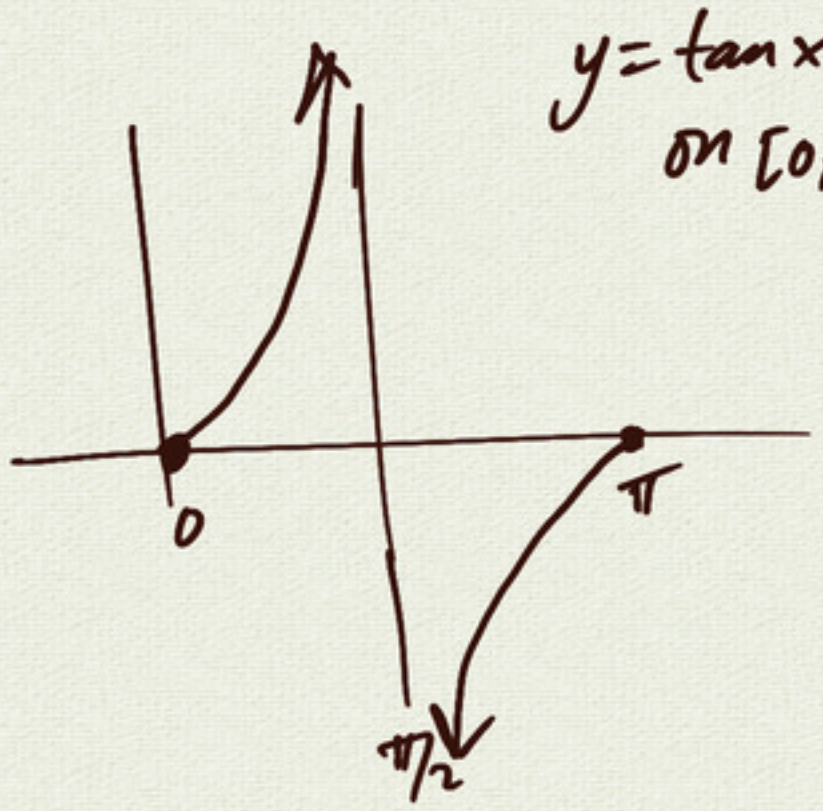
$y = \sin x$ on $[0, \pi]$



absolute max at $x = \pi/2$

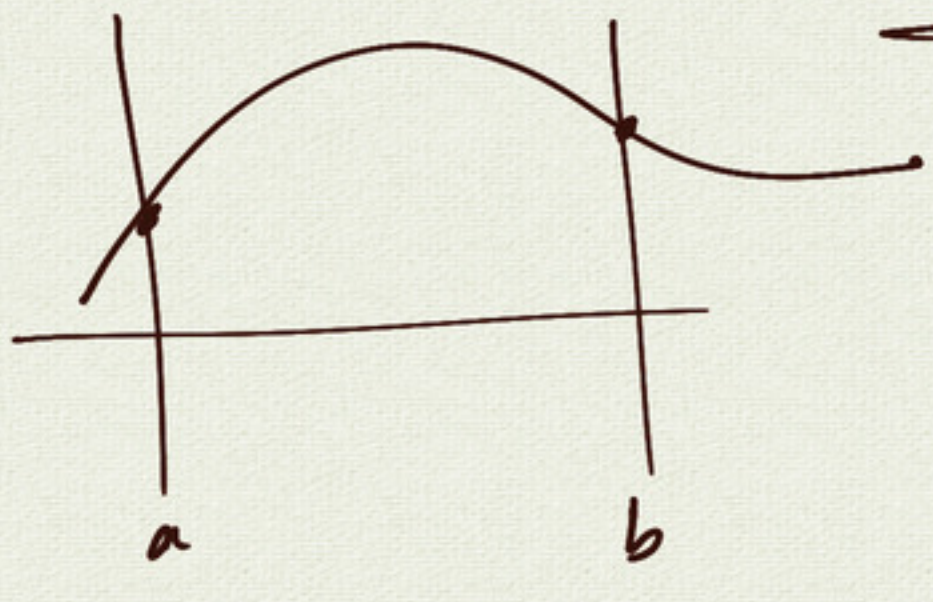
absolute min at $x = 0, \pi$

$y = \tan x$ on $[0, \pi]$



No absolute max (on $[0, \pi]$)

No absolute min

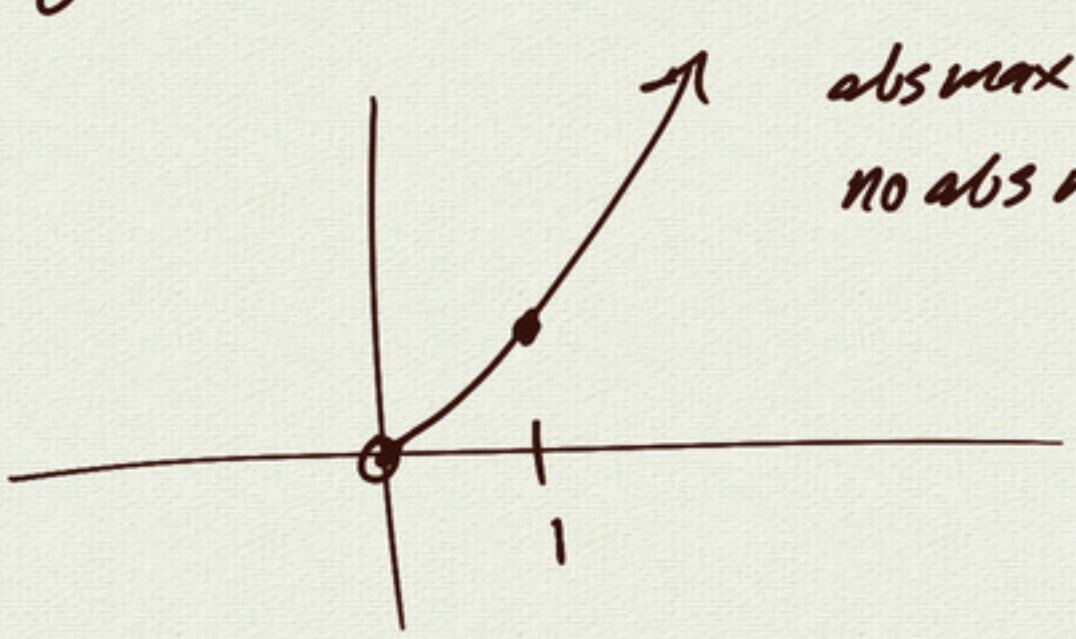


Extreme Value Theorem:

Suppose ① f is continuous on closed interval $[a, b]$

② Then f has an absolute min and max on the interval

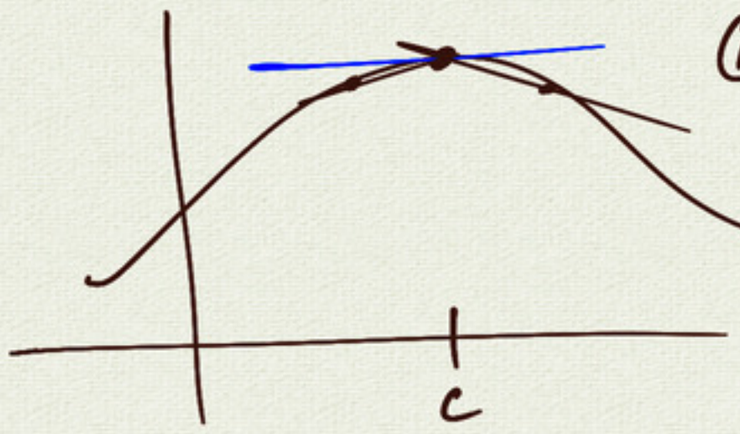
$g(x) = x^2$ on $(0, 1]$



abs max at $x = 1$

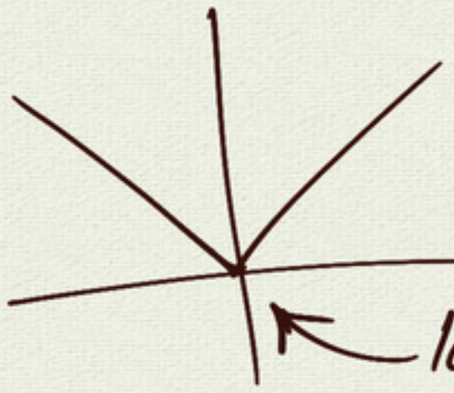
no abs min on $(0, 1]$

observation:

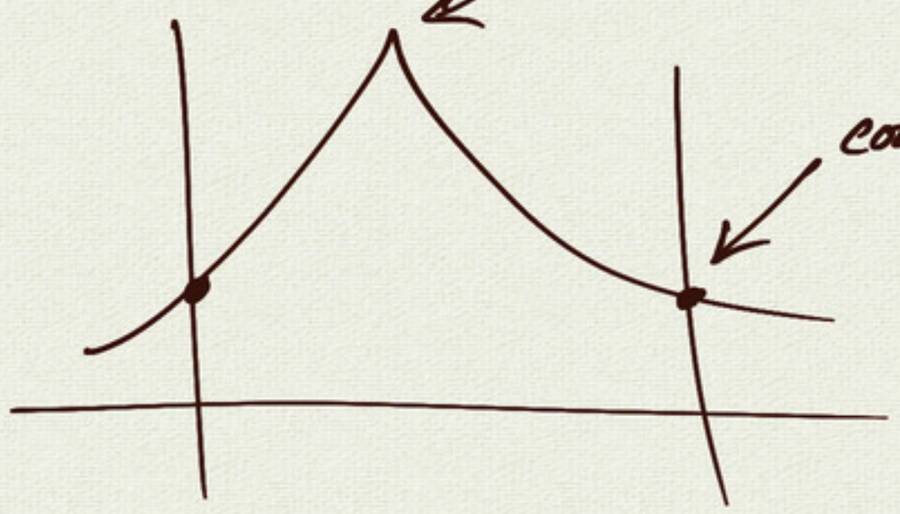


suppose
① $f(x)$ has local max (or local min) at $x=c$
② $f'(c)$ exists

then $f'(c)=0$



find extreme values (abs max or abs min) of f on $[a,b]$

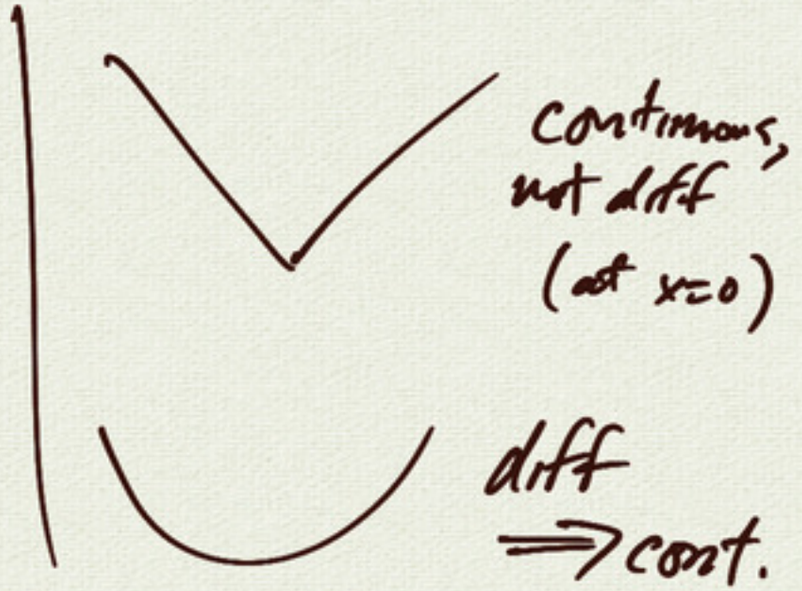


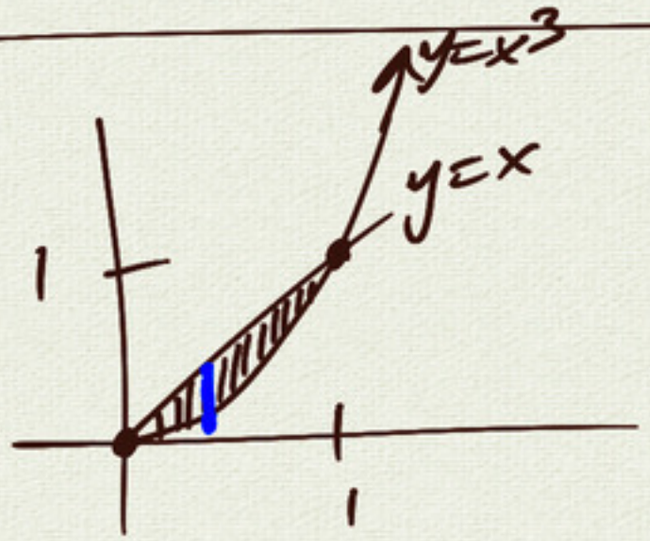
① end points
② find x where
$f'(x)$ does not exist
$f'(x)=0$
] critical points

example:

$f(x) = x - x^3$ on $[0,1]$

find abs max, min.



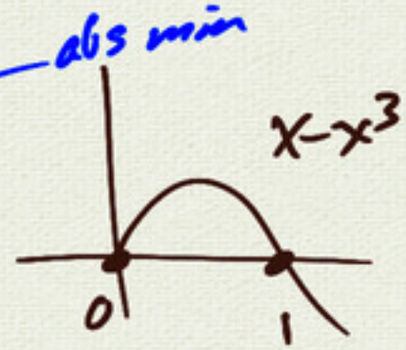


① end points:

$f(0) = 0$

$f(1) = 0$

abs min



② critical pts:

$f'(x) = 1 - 3x^2$

$f'(x) = 0 \longrightarrow \quad 1 - 3x^2 = 0$

$x = \pm \frac{1}{\sqrt{3}}$

$f\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}}\right)^3$

$= \frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{3}}$

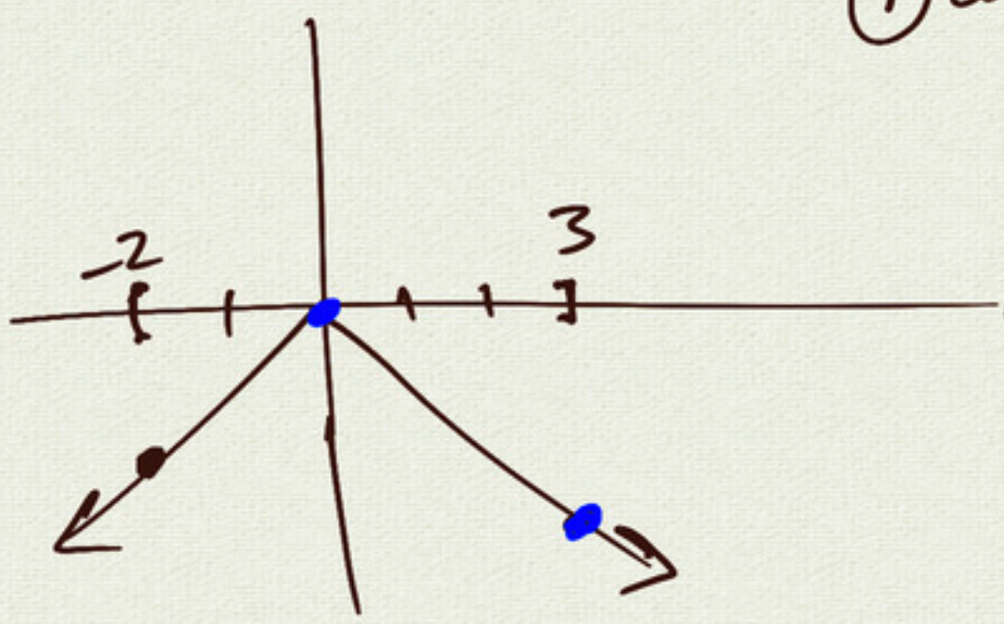
$f\left(\frac{1}{\sqrt{3}}\right) = \frac{2}{3\sqrt{3}} > 0$

abs max

example 2

$g(x) = -|x|$ on $[-2, 3]$

find abs max, min



① end pts:

$g(-2) = -2$

$g(+3) = -3$ abs min

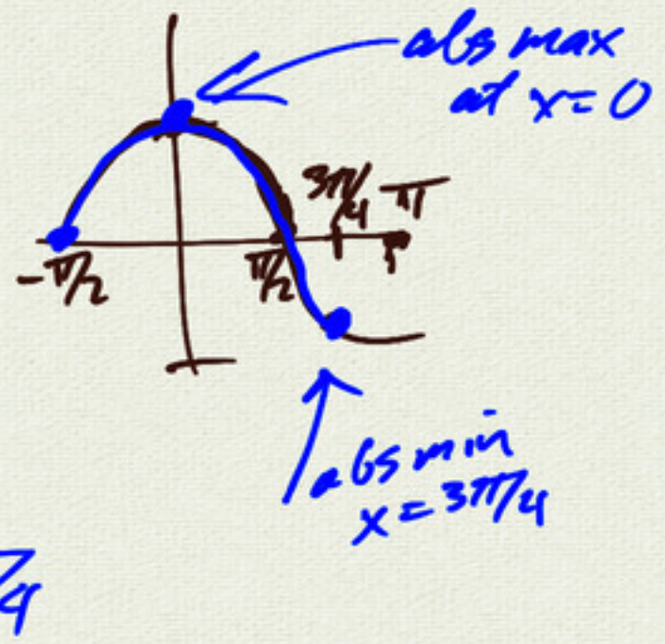
② critical pts:

$x = 0$:

$g(0) = 0$ abs max

example 3

$h(x) = \cos x$ on $[-\frac{\pi}{2}, \frac{3\pi}{4}]$



① end pts:

$\cos(-\pi/2) = 0$

$\cos(3\pi/4) = -\frac{\sqrt{2}}{2}$ abs min at $x = 3\pi/4$

② critical pts:

$h'(x) = -\sin x$

$h'(x) = 0 \Rightarrow -\sin x = 0$

$x = 0, \pi, 2\pi, \ldots$ critical pt

in $[-\pi/2, 3\pi/4]$: $x = 0$

$h(0) = \cos(0) = 1$ abs max at $x = 0$