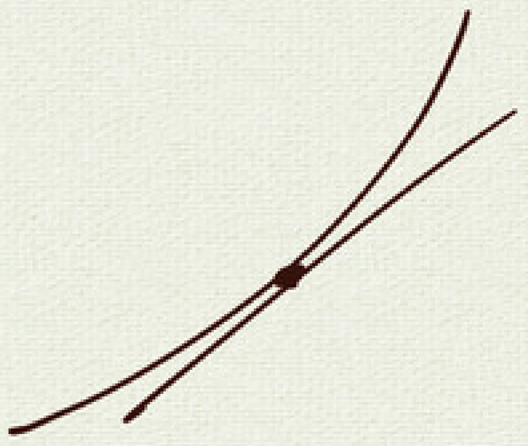
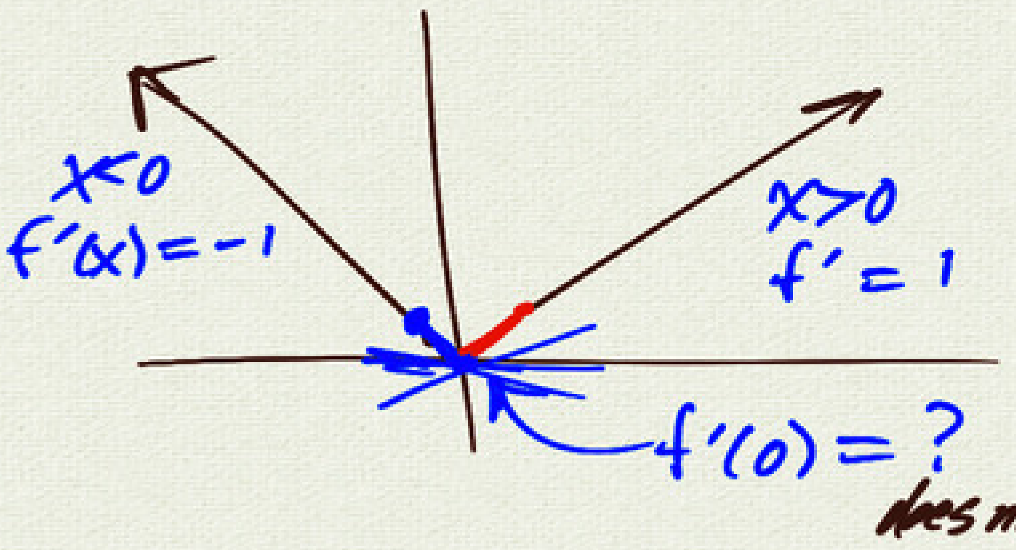
10.1 Extreme Values

$f(x) = |x|$





$|x|$ is continuous
but not differentiable
(at $x=0$)

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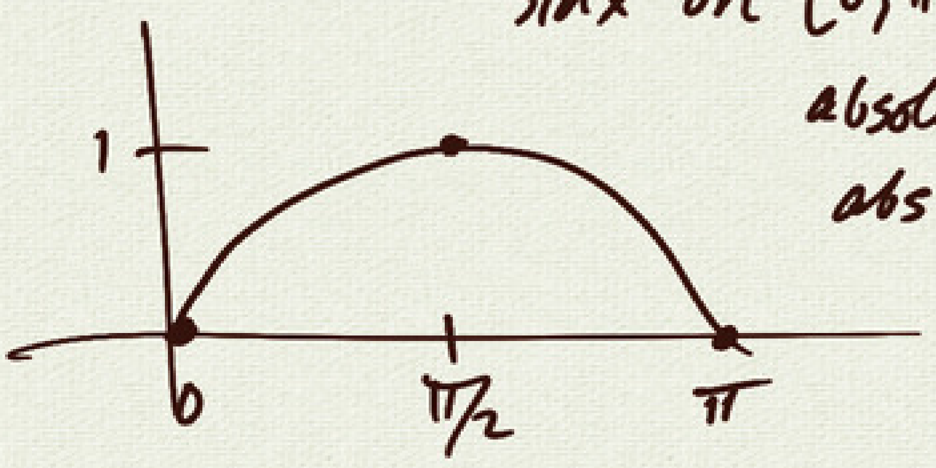
differentiable $\Rightarrow$ continuous

"smooth"

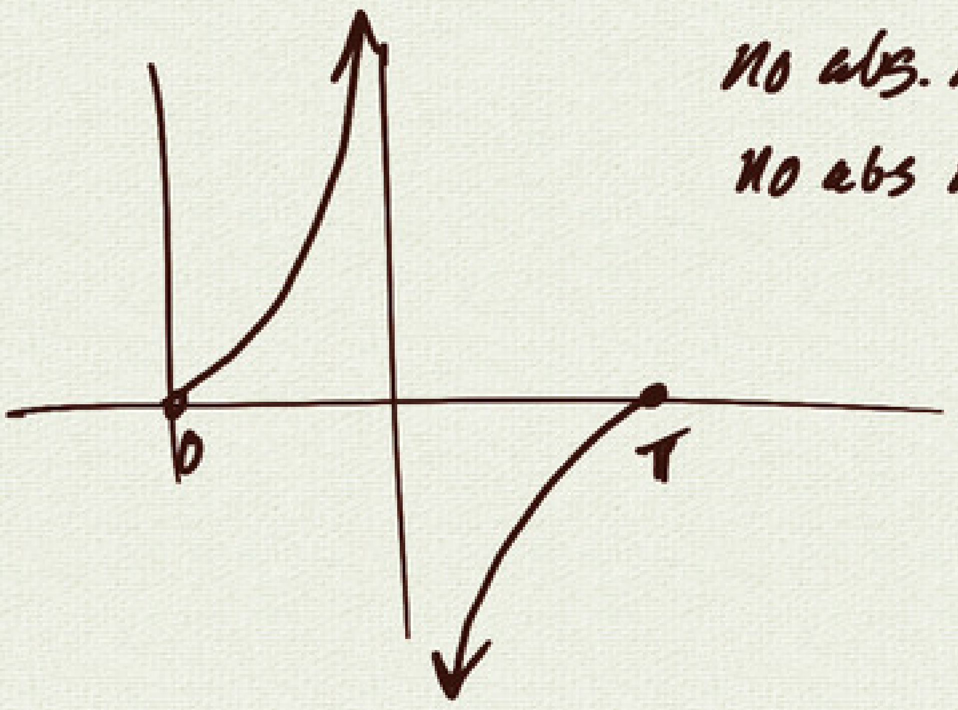
$\sin x$ on $[0, \pi]$

absolute max at $x = \pi/2$
abs min at $x = 0, \pi$



$\tan x$ on $[0, \pi]$

no abs. max
no abs min.



Extreme Value Theorem:

suppose: ① f is continuous
on
② closed interval $[a, b]$

then f has an absolute min + max on the interval

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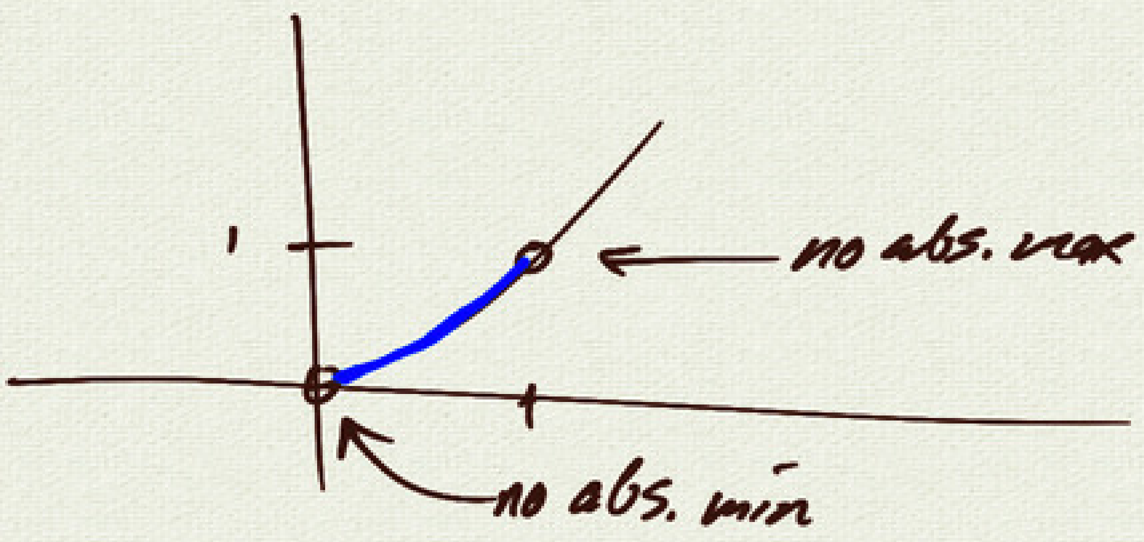
example: $g(x) = x^2$ on $(0, 1)$



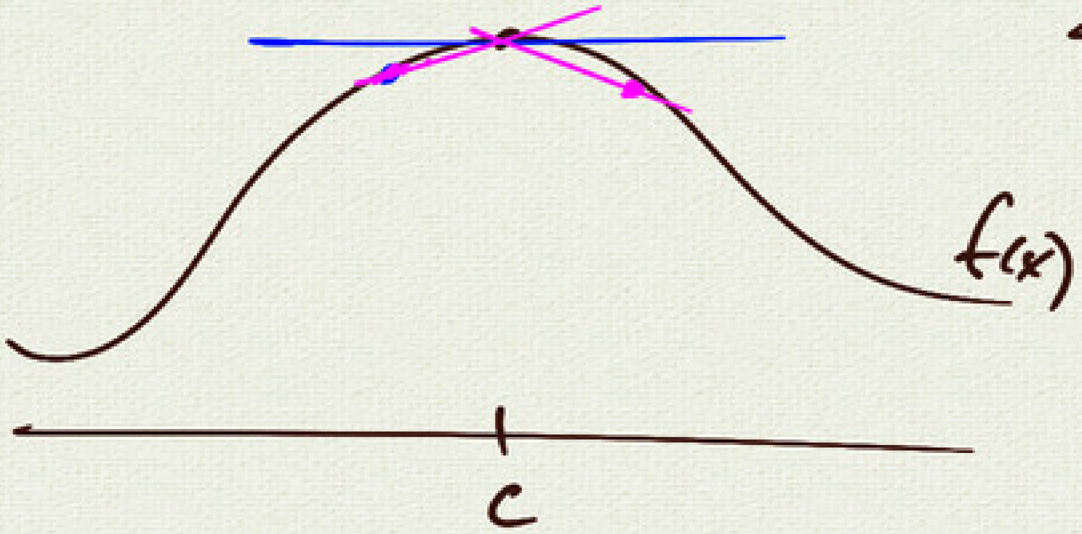
observation:

suppose:

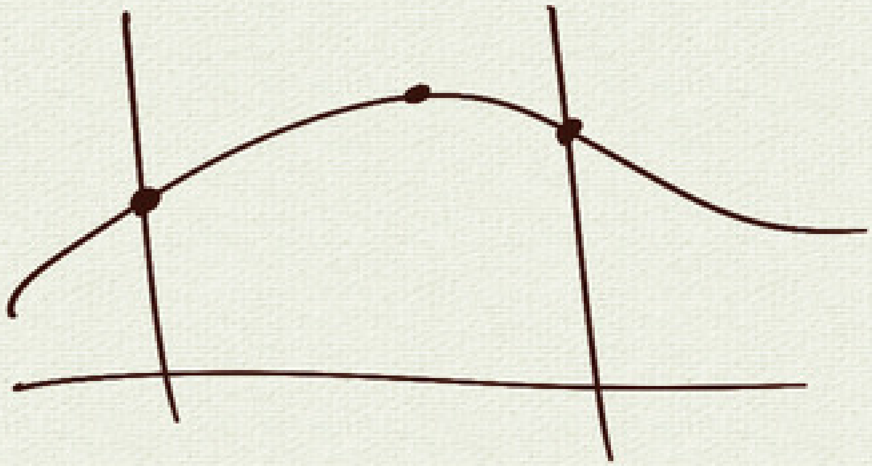
f has local max at $x=c$

$f'(c)$ exists

then: $f'(c)=0$



local min, $f'(0)$ does not exist



f continuous on $[a,b]$

closed interval

find abs. min/max:

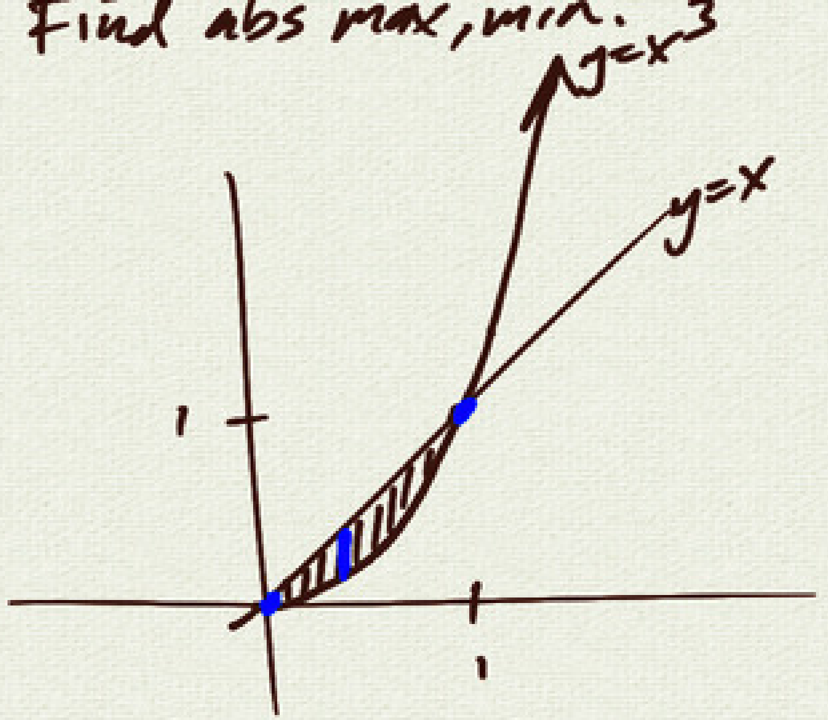
1. end point
2. local min/max:
   - f' does not exist
   - $f'=0$

critical point

example:

$h(x) = x - x^3$ on $[0,1]$

Find abs max, min.



① end points: $h(0) = 0$, $h(1) = 0$ abs min $x=0$, $x=1$

② critical pts: $h'(x) = 1 - 3x^2$

$h'(x) = 0 \Rightarrow 1 - 3x^2 = 0$

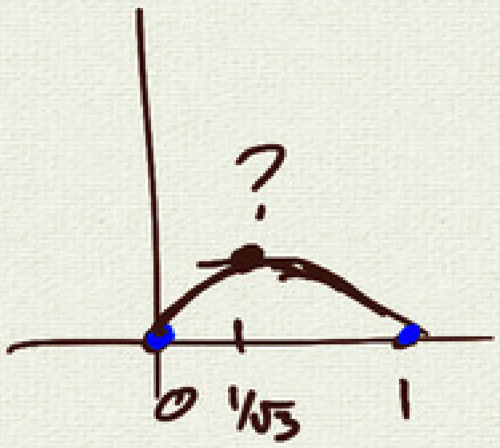
$x = \pm \frac{1}{\sqrt{3}}$

$h\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}}\right)^3$

$= \frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{3}}$

$= \frac{2}{3\sqrt{3}} > 0$

abs max at $x = \frac{1}{\sqrt{3}}$



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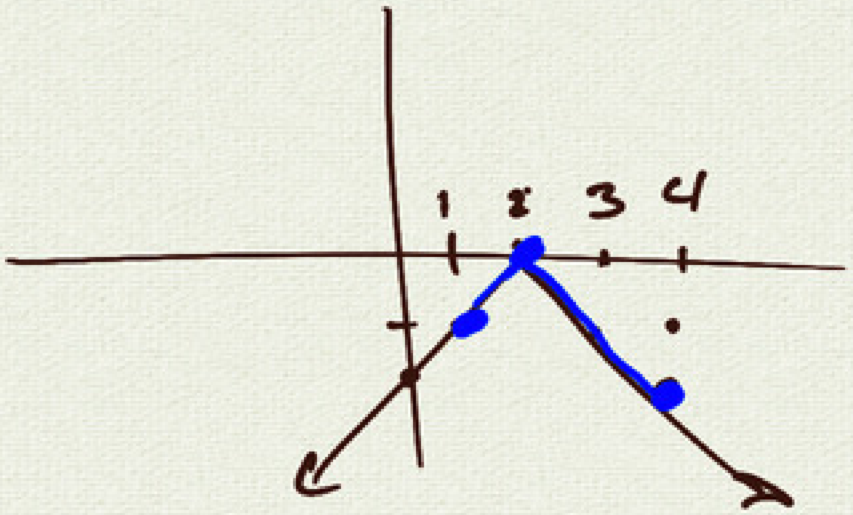
example:

$f(x) = -|x-2|$

on $[1, 4]$

Find abs min and abs max



① end pts: $f(1) = -1$

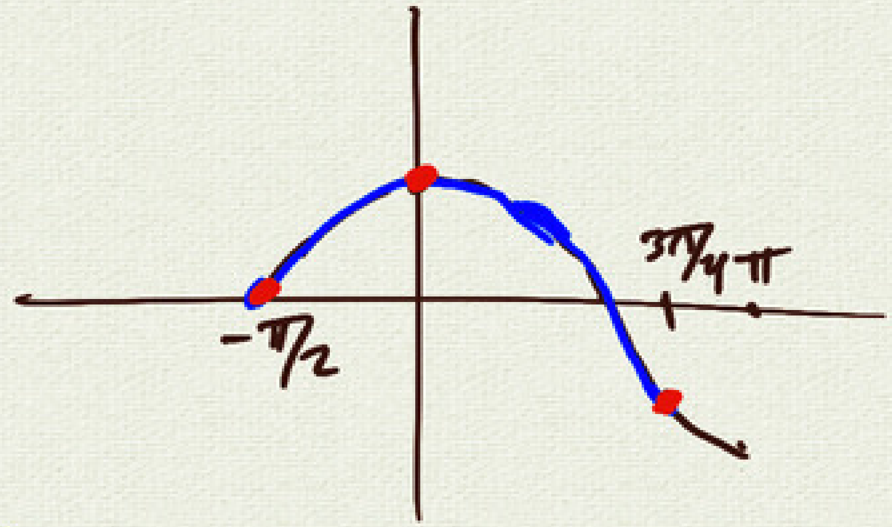
$f(4) = -2$ abs min

② critical pt $x=2$ ($f'(2)$ does not exist)

$f(2) = 0$ abs max

$g(x) = \cos x$ on $\left[-\frac{\pi}{2}, \frac{3\pi}{4}\right]$

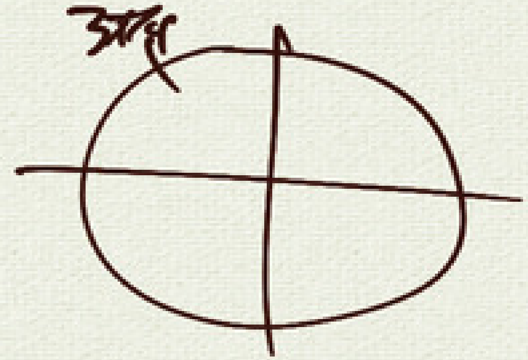
find abs min + max



① end pts:

$$g\left(-\frac{\pi}{2}\right) = 0$$

$$g\left(3\pi/4\right) = -\frac{\sqrt{2}}{2}$$ abs min.



② critical pts:

$$g'(x) = -\sin x$$

$$g'(x) = 0 \Rightarrow \sin x = 0$$

$$x = 0, \pi, 2\pi, \ldots$$

in $[-\pi/2, 3\pi/4]$

$$g(0) = 1$$ abs max