10.2 Mean Value Theorem

Unit 9:

1. limits
2. limit definition of derivative
3. derivative rules
4. implicit differentiation
5. application: (projectile, ferris wheel, growth/decay)

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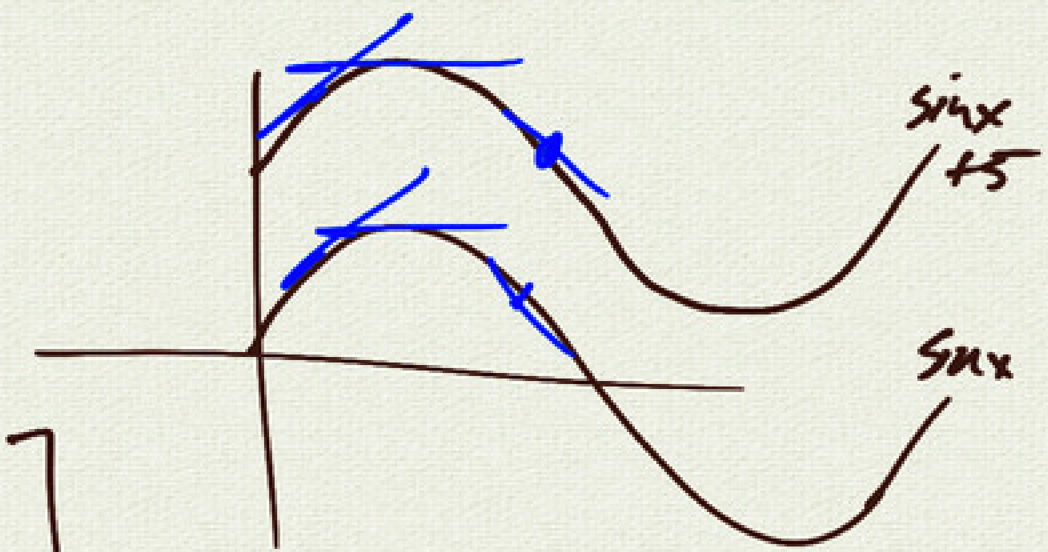
suppose f function

$f'(x) = \cos x$

$\Longrightarrow$ what's f?



$\sin x$

$5 + \sin x$

$\pi + \sin x$

$\sin x + C$

anything else?

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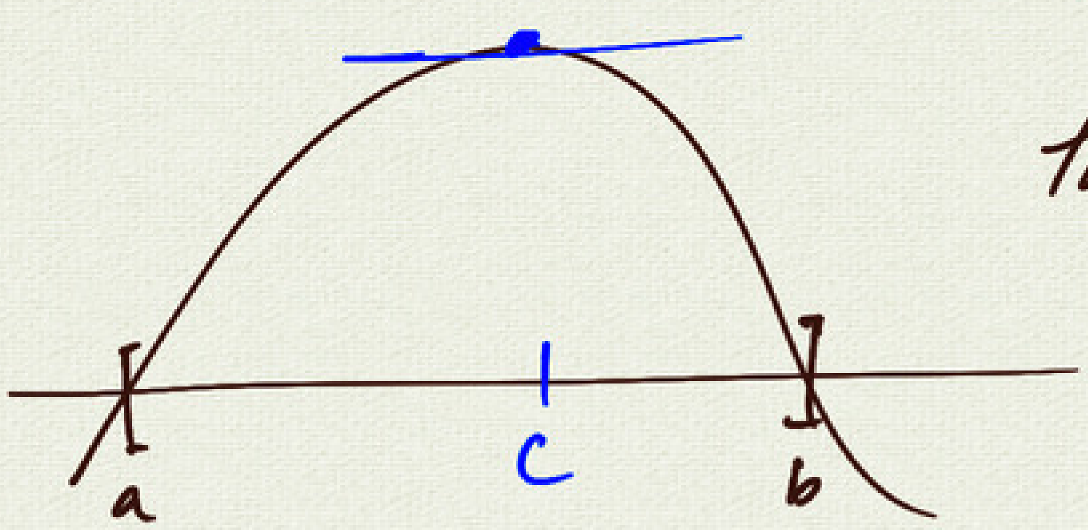
an easier question: $g(x)$ function

suppose $g'(x) = 0$ (on some interval)

$\Longrightarrow$ g is constant?

Rolle's Theorem



Suppose

(1) $f(a) = 0 = f(b)$

(2) $f(x)$ differentiable on (a,b)

(3) $f(x)$ continuous on $[a,b]$

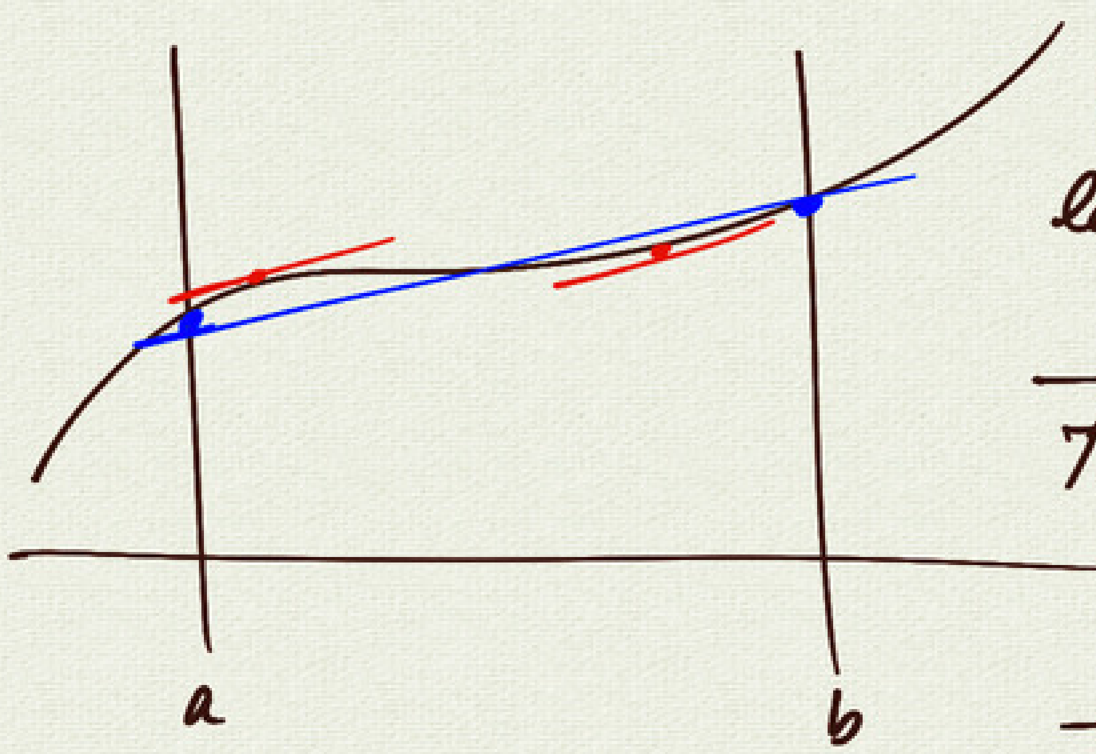
Then $\exists c$ in $[a,b]$ such that

$$f'(c) = 0$$

$\exists$ "there exists"

$\forall$ "for all"

Mean Value Theorem



(1) f continuous on $[a,b]$

(2) f differentiable on (a,b)

let $m = \dfrac{f(b) - f(a)}{b - a}$

Then $\exists c$ in (a,b) such that

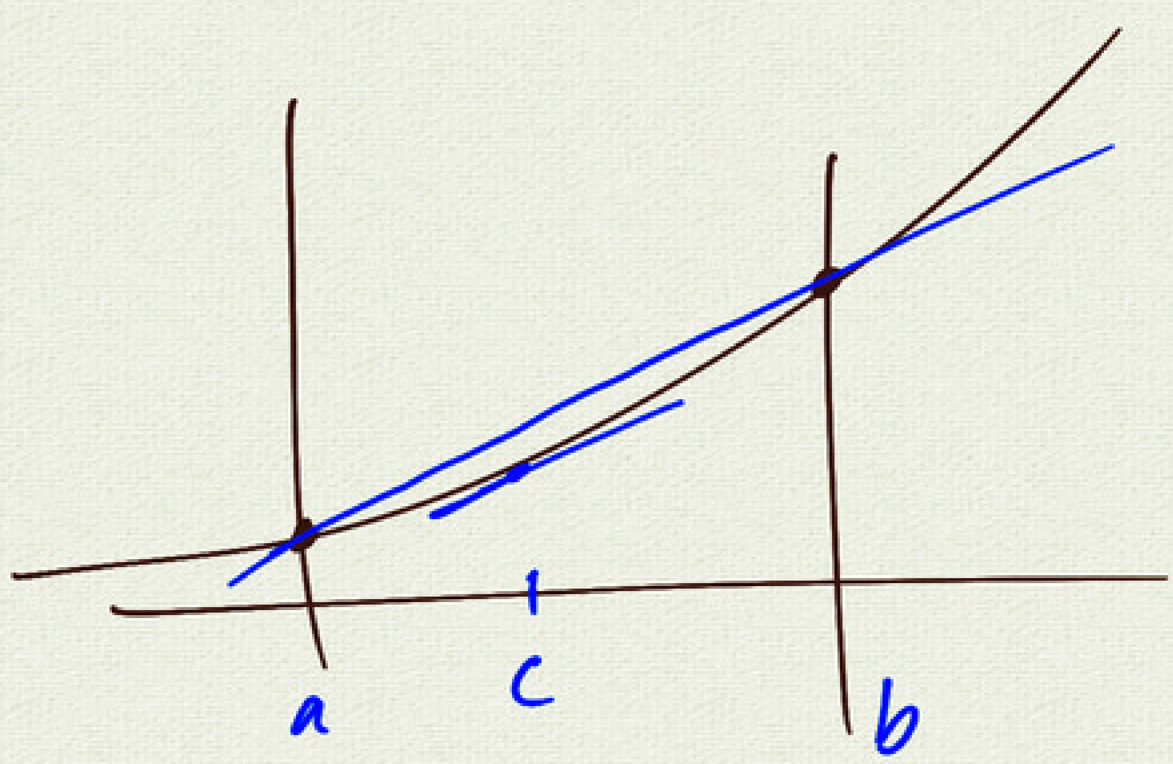
$$f'(c) = m$$

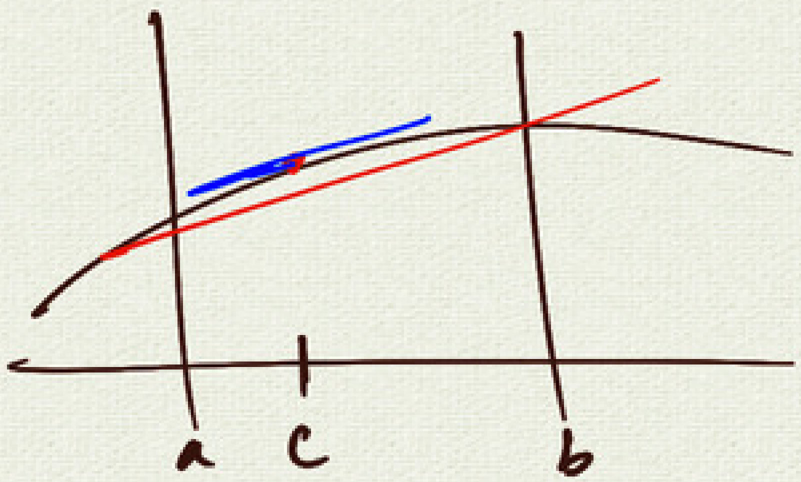
To prove: "tilt it"

consider $l(x) =$ secant line

$$g(x) = f(x) - l(x)$$

use Rolle's Thm





Mean Value Thm ① f cont on $[a,b]$
② f diff on (a,b)

$$\Rightarrow \exists c \quad f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$f(b)-f(a) = f'(c)(b-a)$$

Cor 1 Suppose $f'(x) = 0$ on some interval.
Then f is constant.

Pf: take any a, b

$\Rightarrow \exists c$ where $f(b) - f(a) = \underbrace{f'(c)}_{0}(b-a)$

$f(b) = f(a)$

f constant

Cor 2 suppose $f'(x) = g'(x)$ on some interval
Then $f(x) = g(x) + \text{const}$.

Pf: consider $h(x) = (f-g)(x)$

$= f(x) - g(x)$

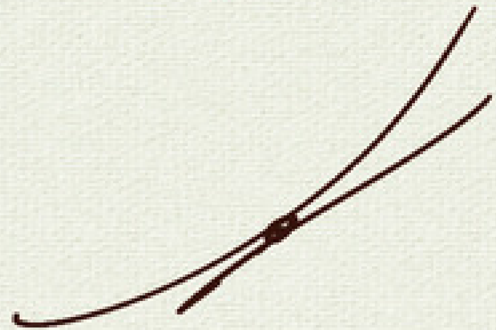
then $h'(x) = f'(x) - g'(x)$

$= 0$

$\Rightarrow h$ const.

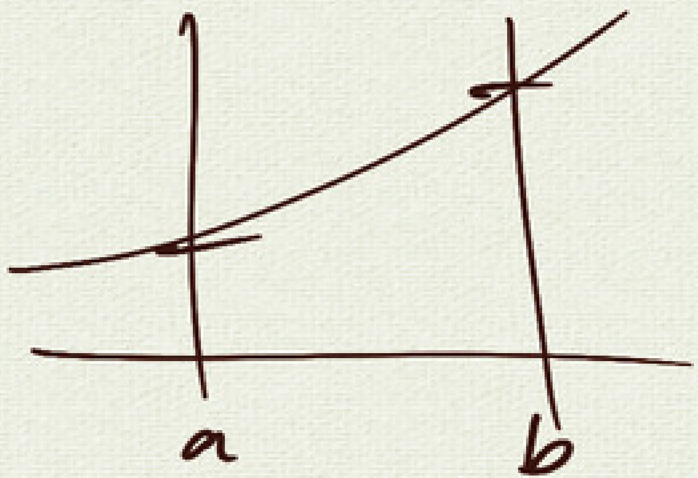
$f-g$ const.

__Cor 3__ Suppose $\underbrace{f'(x)>0}$ on some interval then f is increasing



__Pf:__ take any a, b

$\exists c$ where $f(b)-f(a) = \underbrace{f'(c)}_{+}\underbrace{(b-a)}_{+}$



$f(b)-f(a)>0$

$f(b)>f(a)$

f increasing

examples

$f'(x) = \cos x$

$\Longrightarrow f(x) = \sin x + C$

$g'(x) = 5x^4$

$\Longrightarrow g(x) = x^5 + C$

$h'(x) = x^4$

$\longrightarrow h(x) = \frac{1}{5}x^5 + C$

$k'(x) = e^x$

$\Longrightarrow k(x) = e^x + C$

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notation preview

$f'(x) = \cos x \Longrightarrow f(x) = \sin x + C$

$\frac{d}{dx}(\sin x + c) = \cos x$

$\int \cos x \, dx = \sin x + C$

antiderivative (integral)