# 10.2 Mean Value Theorem

1. limits
2. limit def of derivative
3. derivative rules
4. implicit (e.g. $\sin^{-1} x$)
5. application

---

suppose $f$ function

$f'(x) = \cos x$

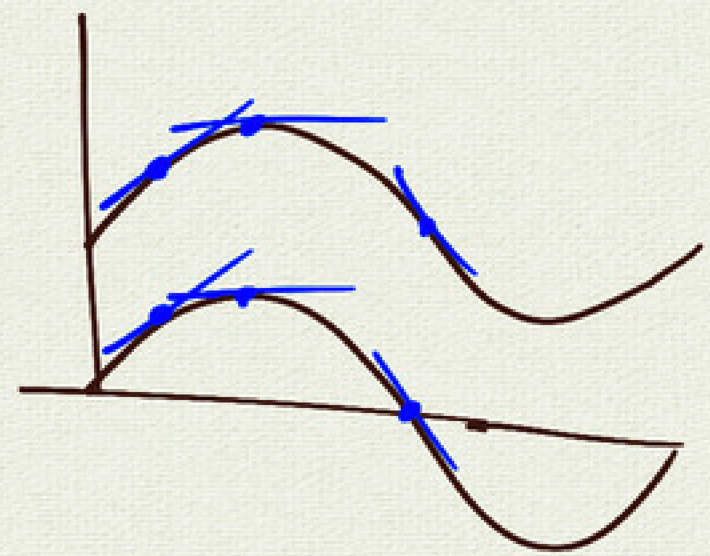
What is $f$?

$f(x) = \sin x$

$\sin x + 5$

$\sin x + C$

anything else?



---

suppose $g$ function

$g'(x) = 0 \longrightarrow g(x)$ constant?

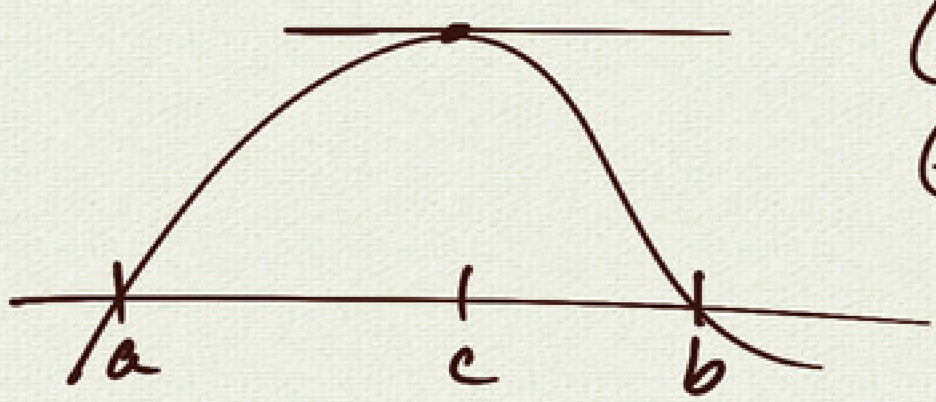
# Rolle's Theorem:

Suppose:

① $f(a) = 0 = f(b)$

② $f$ is continuous on $[a,b]$

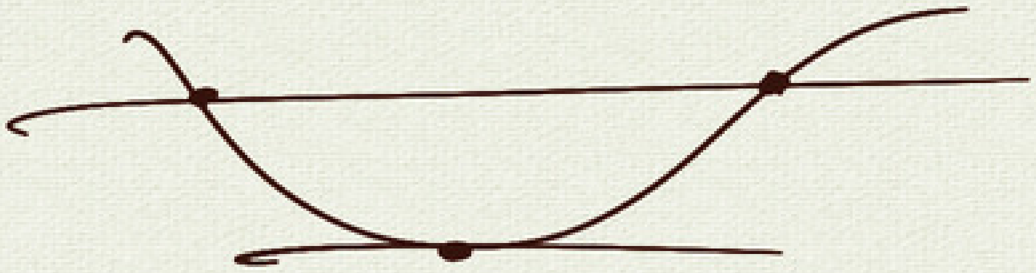
③ $f$ is differentiable on $(a,b)$

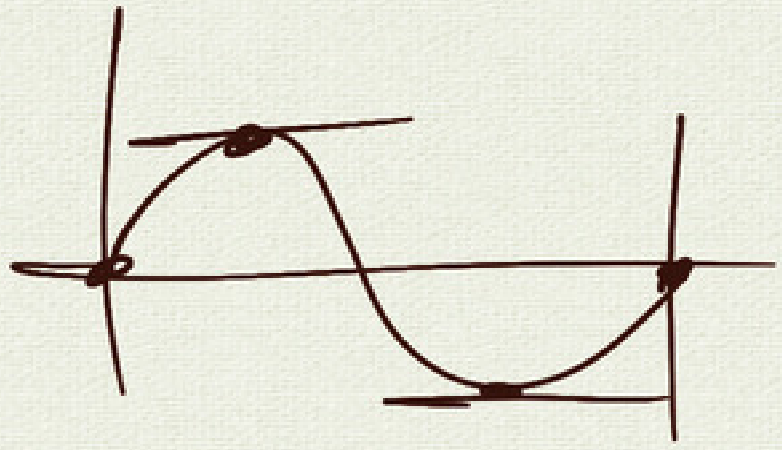



Then $\exists c$ in $(a,b)$ such that

$$f'(c) = 0$$

| $\exists$ there exists
| $\forall$ for all





# Mean Value Theorem

Suppose

① $f$ continuous on $[a,b]$

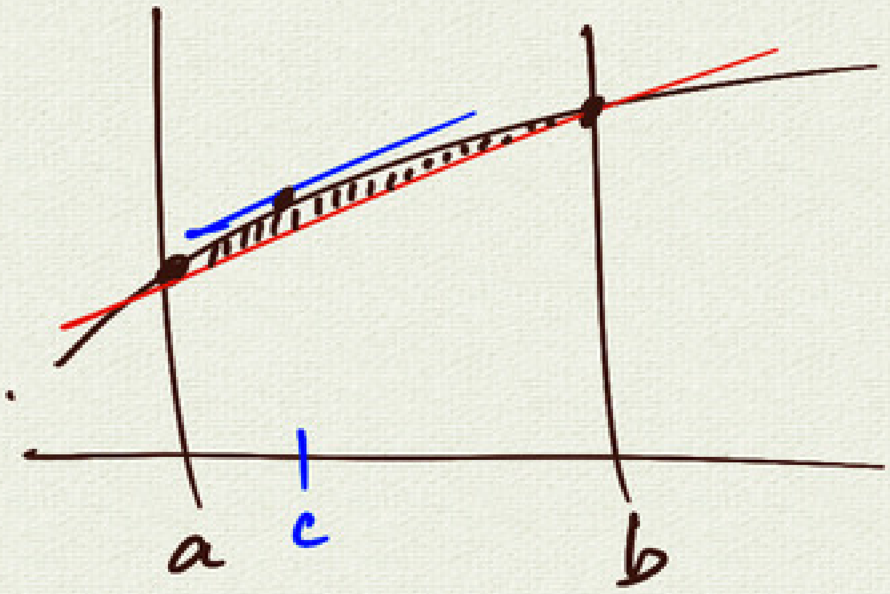
② $f$ differentiable on $(a,b)$

Let $m = \dfrac{f(b) - f(a)}{b-a}$

Then

$\exists c$ in $(a,b)$ such that

$$f'(c) = m$$

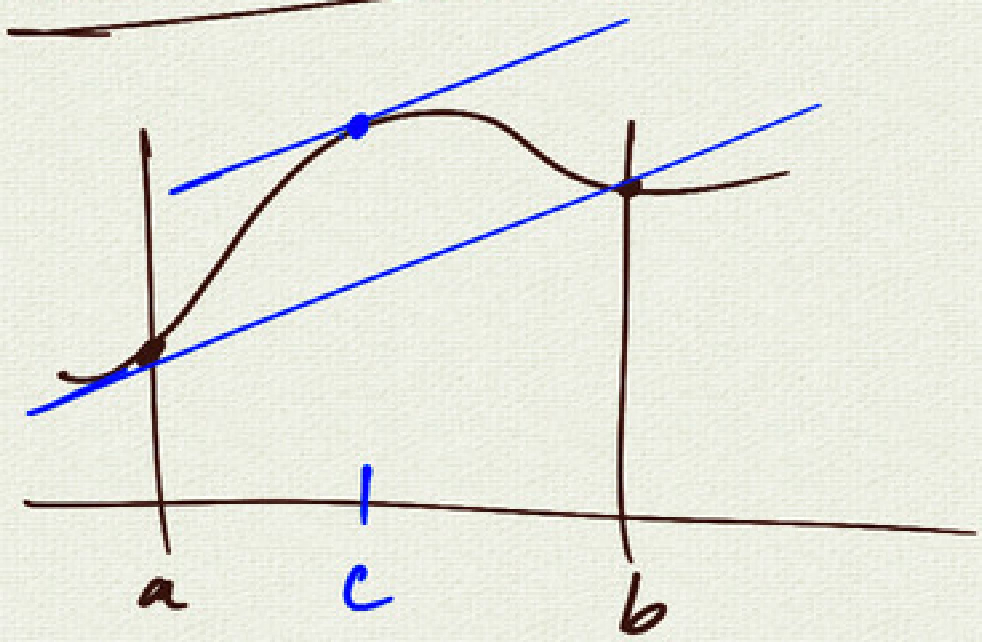



---

idea of proof: let $\ell(x) =$ secant line

consider $g(x) = f(x) - \ell(x)$

# Mean Value Theorem




$\exists c$ in $(a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$f(b)-f(a) = f'(c)(b-a)$$

## Corollary 1

Suppose $f(x)$ function with $f'(x)=0$ on some interval.

Then $f(x) = \text{const.}$

**Proof:** take any $a, b$ (in interval)

then $\exists c$ such that $f(b)-f(a) = \underbrace{f'(c)}_{0}(b-a)$

$$f(b)-f(a)=0$$

$$f(b)=f(a)$$

$f$ constant.

## Corollary 2

Suppose $f' = g'$ on some interval.

Then $f = g + \text{const.}$

**Proof:** let $h(x) = (f-g)(x)$

then $h'(x) = \underbrace{f'(x) - g'(x)}_{0}$

$\Rightarrow h$ constant

$f-g$ constant

$\Rightarrow f = g + \text{constant}$

example:

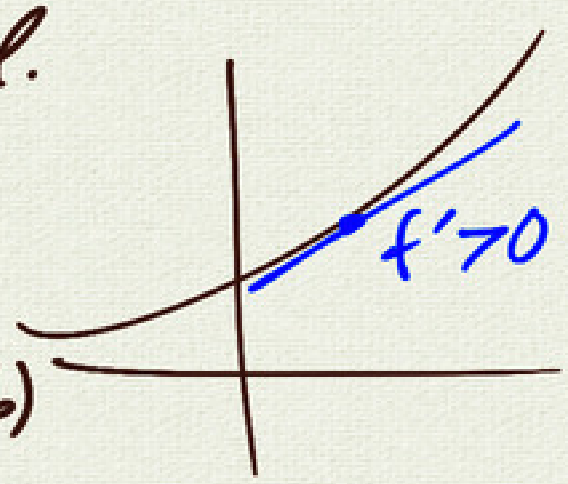
$f'(x) = \cos x$

then:

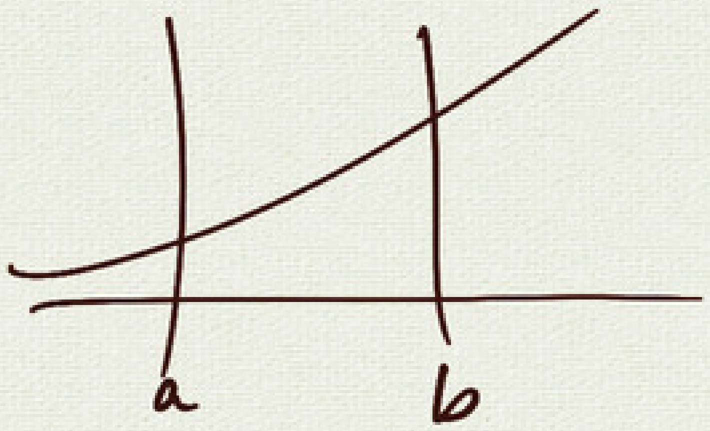
$f(x) = \sin x + C$

only possibilities

**Cor 3** Suppose $f' > 0$ on some interval.
Then $f$ is increasing




**Proof:** take any $a, b$ in interval $(a<b)$
$\Rightarrow \exists c$ such that
$$f(b) - f(a) = \underbrace{f'(c)}_{+}\underbrace{(b-a)}_{+}$$




$\Rightarrow f(b) > f(a)$
$f$ increasing

---

notation: $\frac{d}{dx}(\sin x + C) = \cos x$

anti-derivative of $\cos x = \sin x + C$

$$\int \cos x \, dx = \sin x + C$$

$\int$ ← antiderivative (integral)

---

examples:

$f'(x) = 2x \Rightarrow f(x) = x^2 + C$

$g'(x) = 5x^4 \rightarrow g(x) = x^5 + C$

$h'(x) = x^4 \Rightarrow h(x) = \frac{1}{5}x^5 + C$

$k'(x) = e^x \Rightarrow k(x) = e^x + C$