10.3 Extreme value tests

mean value theorem $\Rightarrow$

① $f' = 0 \implies f$ constant
(on some interval)

② $f' = g' \implies f = g + C$

example:

$f'(x) = 2x$

$\implies f(x) = x^2 + C$ (antiderivative)

③ $f' > 0 \implies f$ increasing

$f' < 0 \implies f$ decreasing

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example:
projectile motion

assumption: gravity $\downarrow$ acceleration $-32\ \text{ft}/\text{s}^2$ down

$$x''(t) = 0$$
$$y''(t) = -32$$

$$x'(t) = c_1 \leftarrow v_x$$
$$y'(t) = -\underline{32t} + c_2 \leftarrow v_y$$

$t=0$
$x'(0) = v_x$
$y'(0) = v_y$
] initial velocity

$$x'(t) = v_x$$
$$y'(t) = -32t + v_y$$

$$x(t) = v_x t + c_3 \leftarrow x_0$$
$$y(t) = -16t^2 + v_y t + c_4 \leftarrow y_0$$

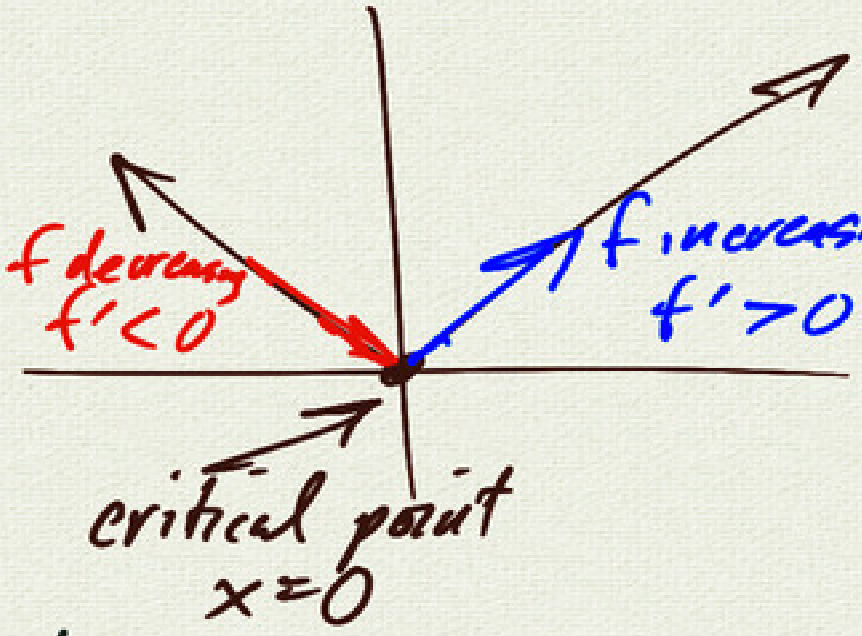
| initial position (x_0, y_0)

$$\boxed{\begin{aligned} x(t) &= v_x t + x_0 \\ y(t) &= -16t^2 + v_y t + y_0 \end{aligned}}$$

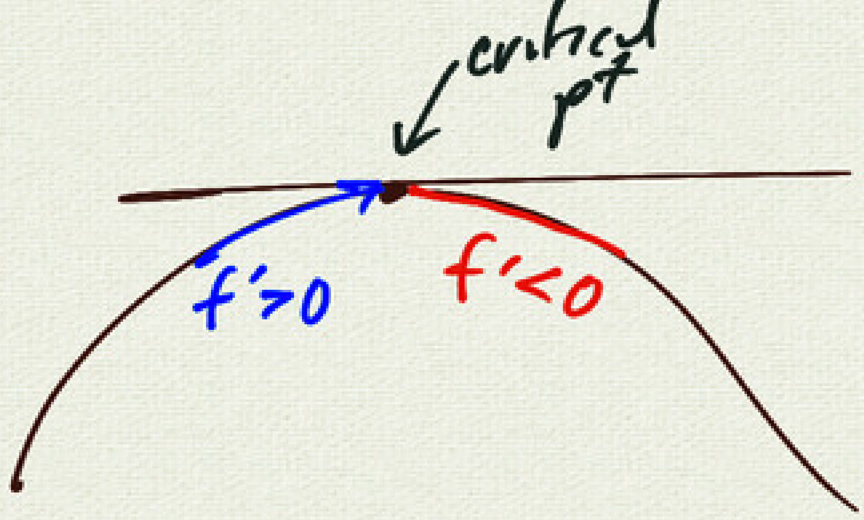
parametric equations for projectile motion

$f(x) = |x|$

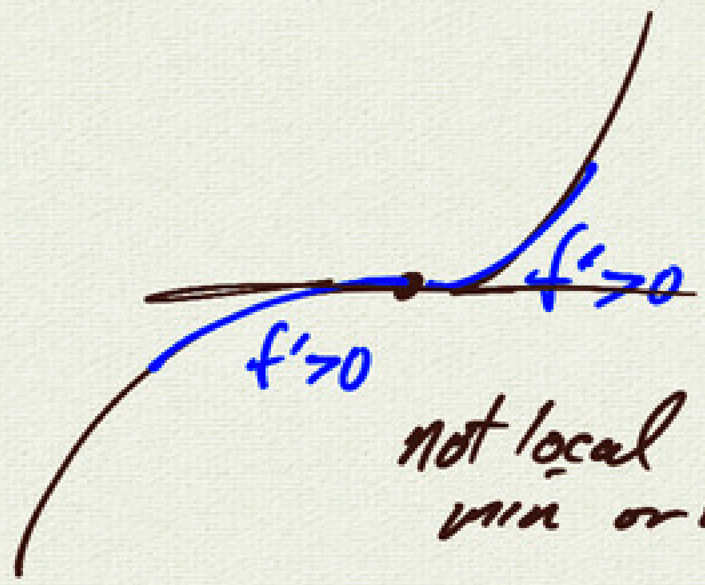
find local min/max



1st derivative test



local max



not local min or max

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example: $g(x) = x^3 - x$

$= x(x^2 - 1)$

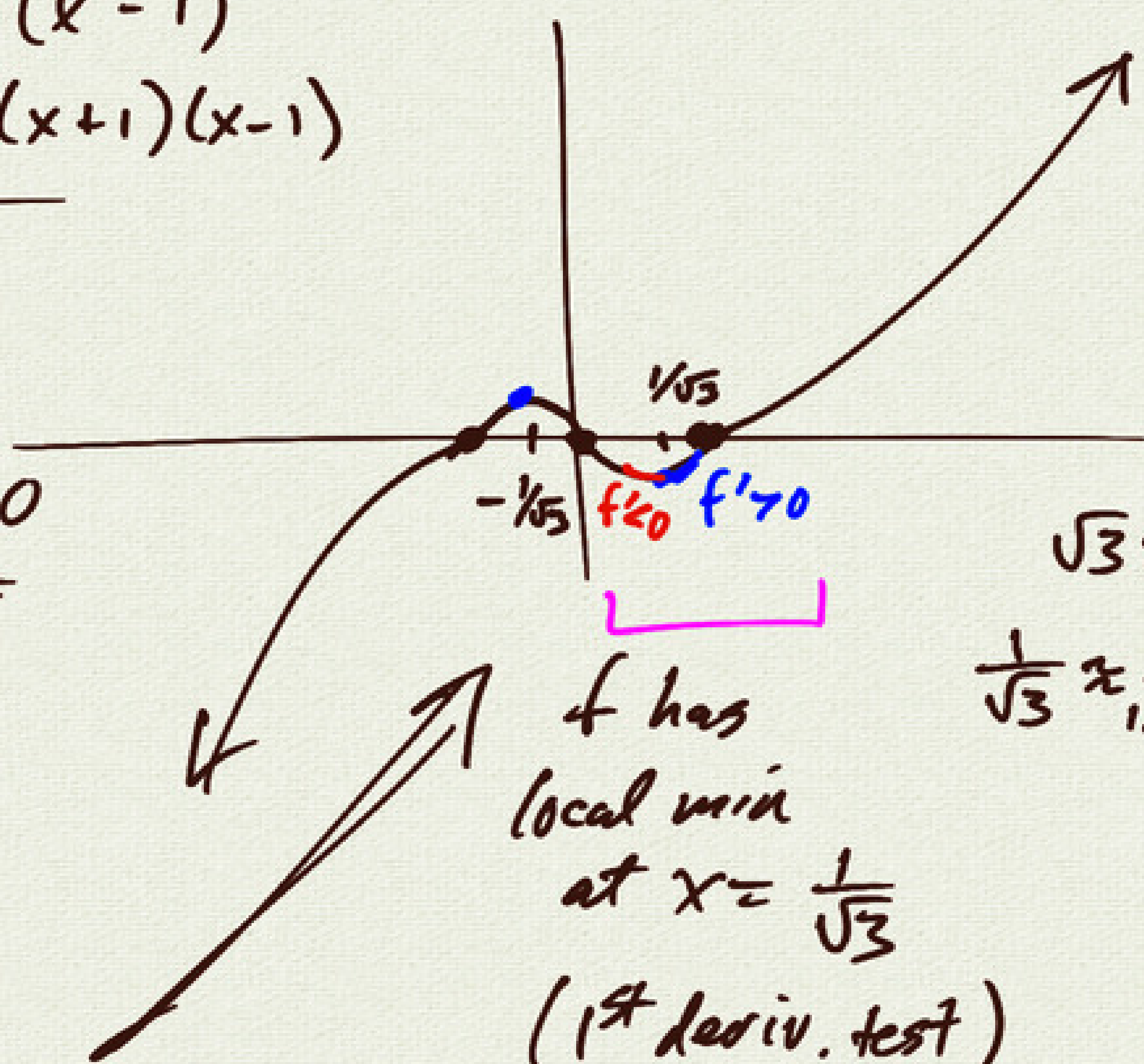
$= x(x+1)(x-1)$

$g'(x) = 3x^2 - 1$

critical pts:

$g'(x) = 0 \Longrightarrow 3x^2 - 1 = 0$

$x = \pm \frac{1}{\sqrt{3}}$

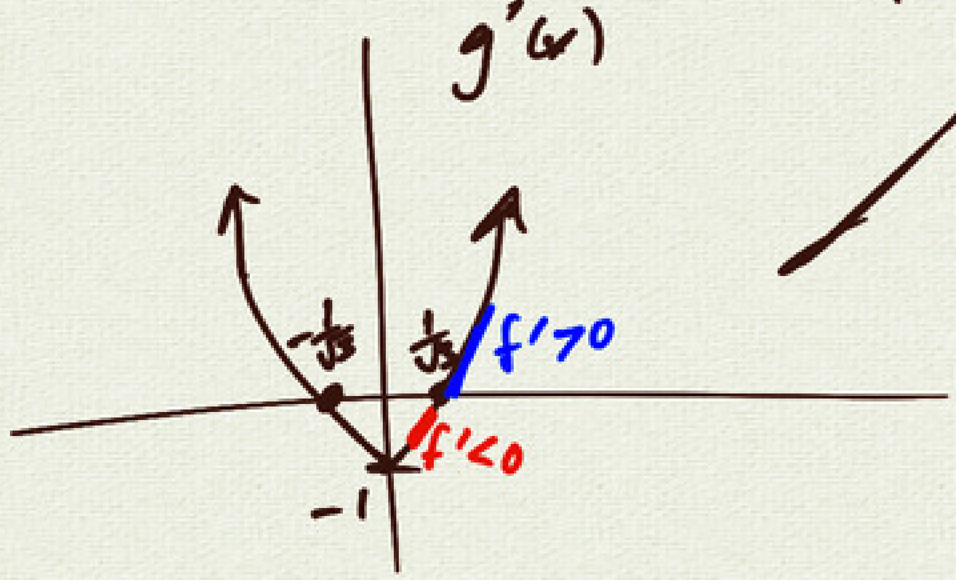


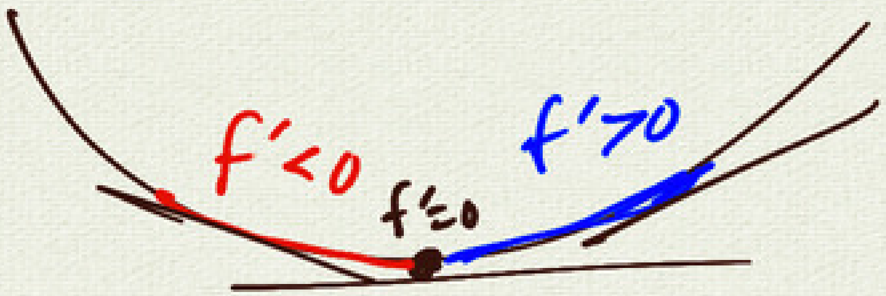
$\sqrt{3} \approx 1.732$

$\frac{1}{\sqrt{3}} \approx \frac{1}{1.7} > \frac{1}{2}$

f has local min at $x = \frac{1}{\sqrt{3}}$

(1st deriv. test)

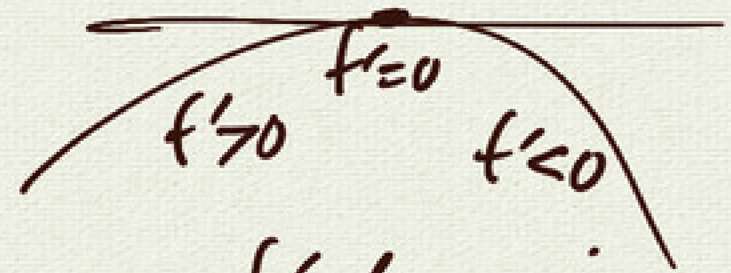




local min

f' increasing (slope increasing)

$f'' > 0$



f' decreasing

$f'' < 0$

2nd derivative test: $f'(c) = 0$ (slope = 0 at $x = c$)

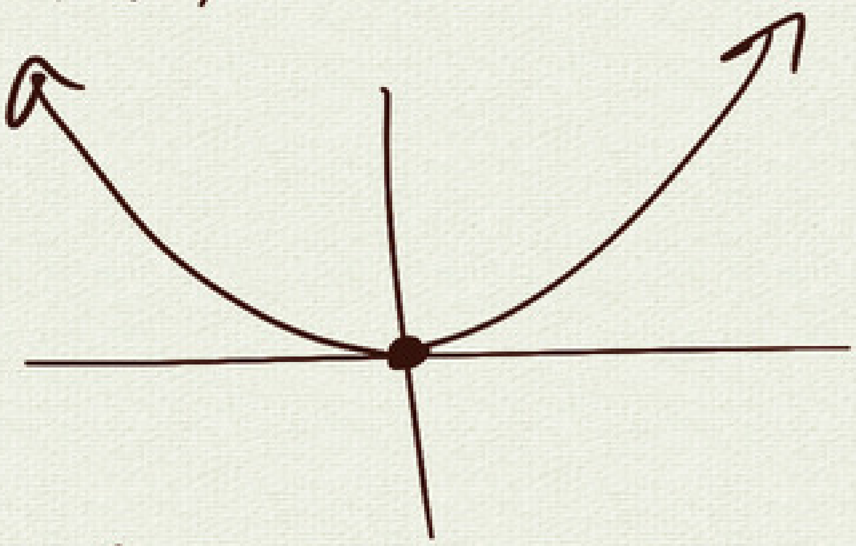
suppose $f''(c)$ exists

then ① $f''(c) > 0 \implies$ local min at $x = c$

② $f''(c) < 0 \implies$ local max at $x = c$

③ $f''(c) = 0 \implies$ inconclusive (I don't know) examples:

$f(x) = x^2$



$f'(x) = 2x$

$\Rightarrow f'(0) = 0$

$f''(x) = 2$

$f''(0) = 2 > 0$ local min

$g(x) = -x^2$

$g'(x) = -2x$

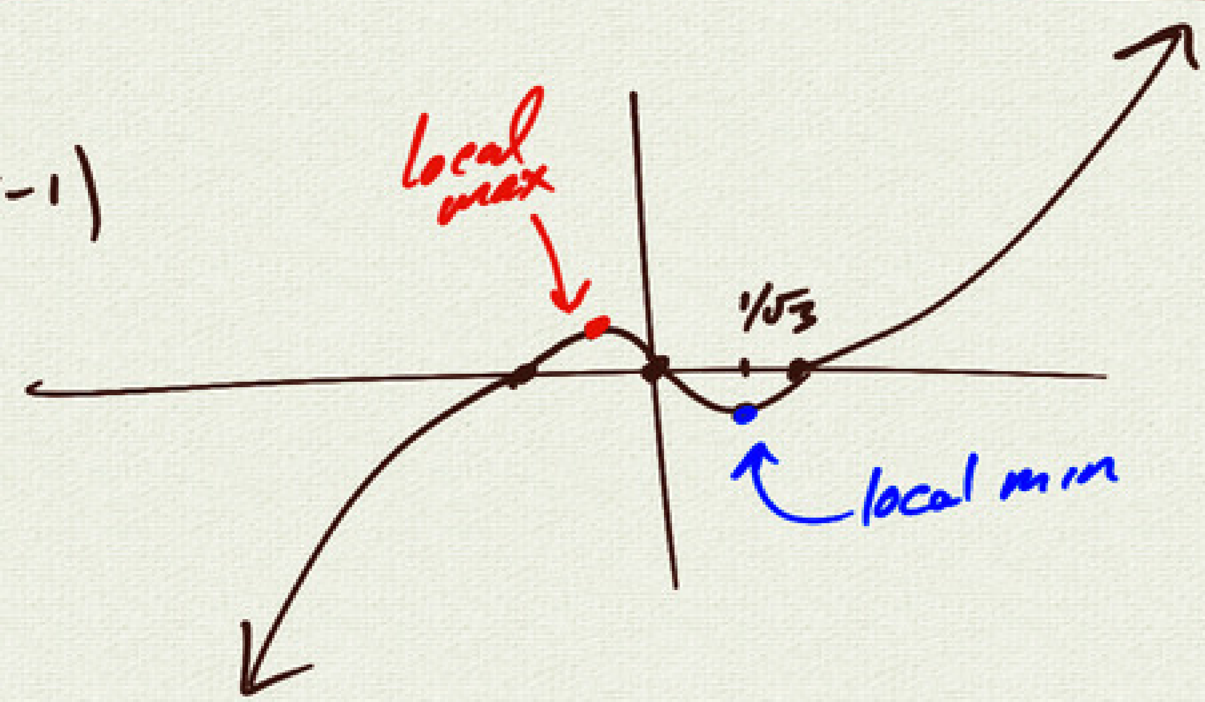
$g'(0) = 0$ critical pt

$g''(x) = -2$

$g''(0) = -2 < 0$ local max

$h(x) = x^3 - x$

$= x(x+1)(x-1)$



$h'(x) = 3x^2 - 1$

critical pts

$3x^2 - 1 = 0$

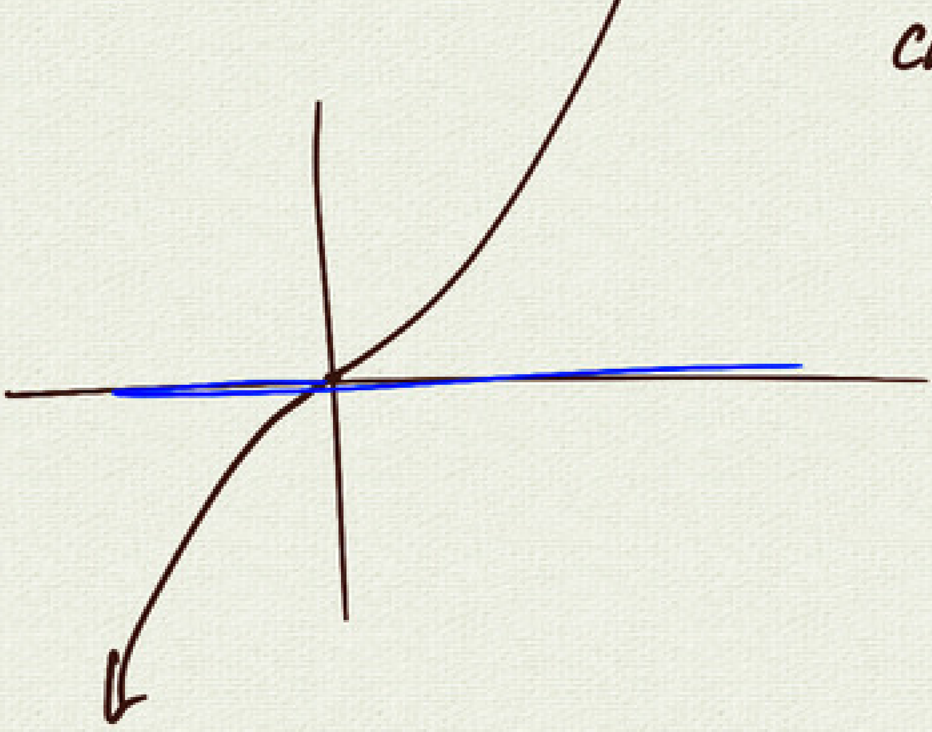
$x = \pm \frac{1}{\sqrt{3}}$

$h''(x) = 6x$

$h''\left(\frac{1}{\sqrt{3}}\right) = 6/\sqrt{3} > 0$ local min

$h''\left(-\frac{1}{\sqrt{3}}\right) = -\frac{6}{\sqrt{3}} < 0$ local max

$k(x)=x^3$



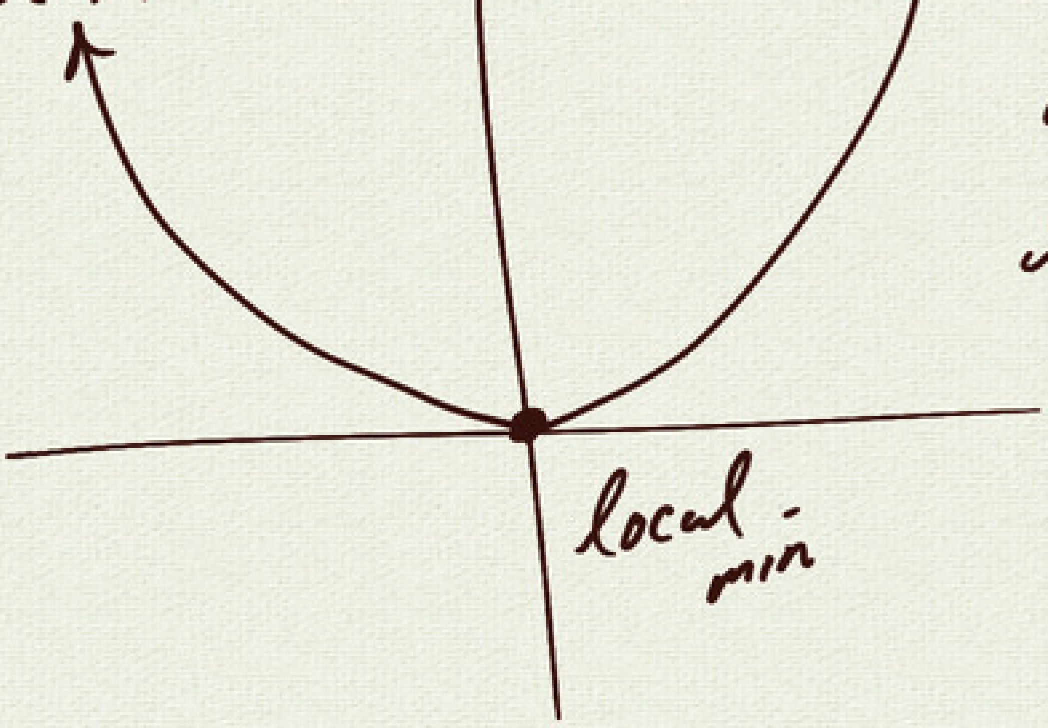
$k'(x)=3x^2$

critical pt $k'(0)=0$

$k''(x)=6x$

$k''(0)=0$

$l(x)=x^4$



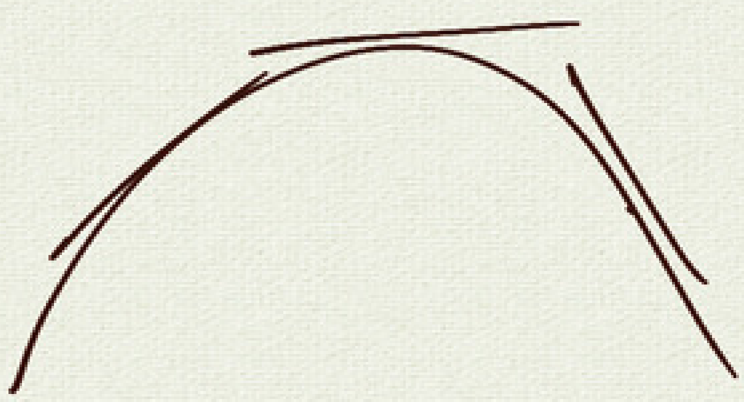
$l'(x)=4x^3$

critical pt $l'(0)=0$

$l''(x)=12x^2$

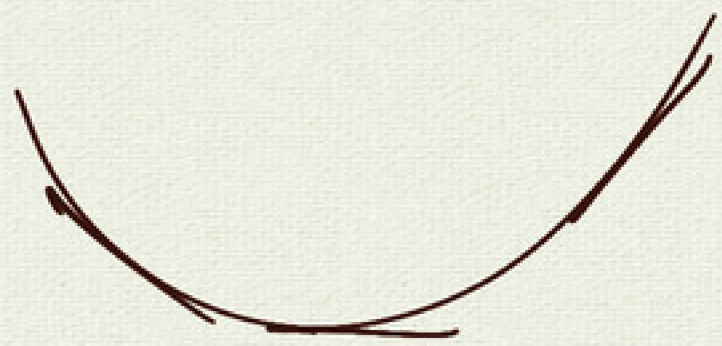
$l''(0)=0$

nomenclature:



concave down

$f''<0$



concave up

$f''>0$