10.3 Extreme value tests

mean value theorem $\Longrightarrow$

① $f' = 0 \Longrightarrow f$ is const.
(on some interval)

② $f' = g' \longrightarrow f = g + \text{const.}$

example: antiderivatives

$$f'(x) = 2x$$
$$\Longrightarrow f(x) = x^2 + C$$

③ $f' > 0 \longrightarrow f$ increasing
$f' < 0 \Longrightarrow f$ decreasing

<u>example:</u> projectile motion

assumption: gravity $\downarrow$ acceleration $-32\ \text{ft}/s^2$ (straight down)

$$x''(t) = 0$$
$$y''(t) = -32$$

$\Longrightarrow$ $x'(t) = c_1$

$y'(t) = -32t + c_2$

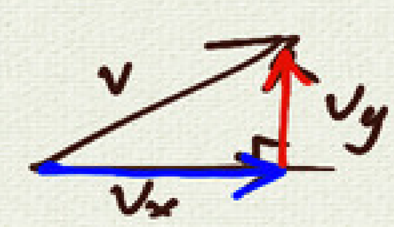
$x'(t) = v_x$

$y'(t) = -32t + v_y$

<u>$t=0$</u> $x'(0) = c_1 = v_x$

speed in x direction at $t=0$

$y'(0) = c_2 = v_y$



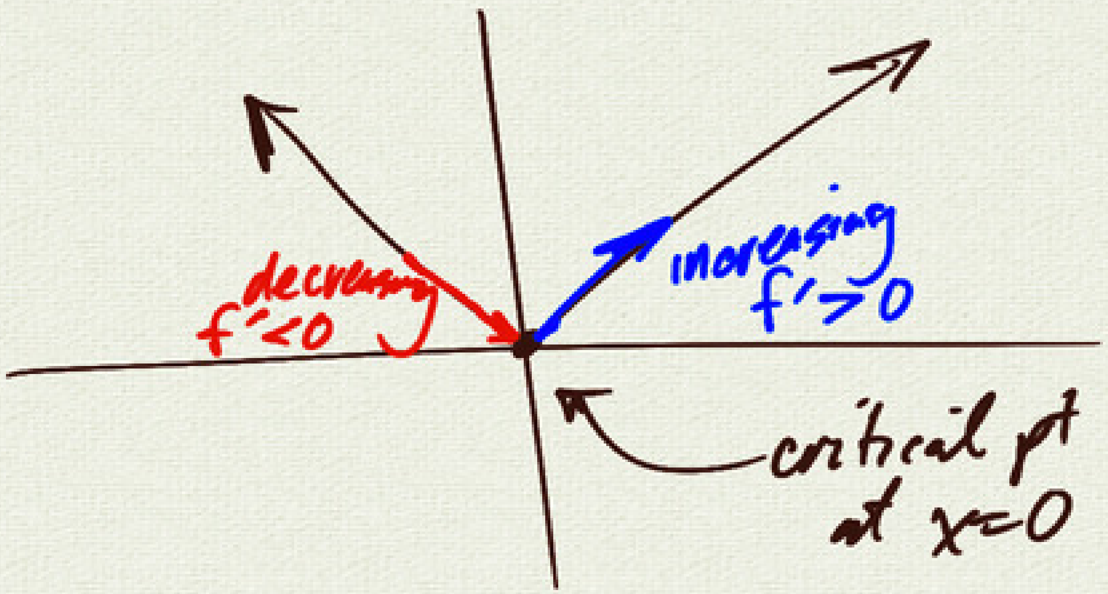
$x(t) = v_x t + c_3$

$y(t) = -16t^2 + v_y t + c_4$

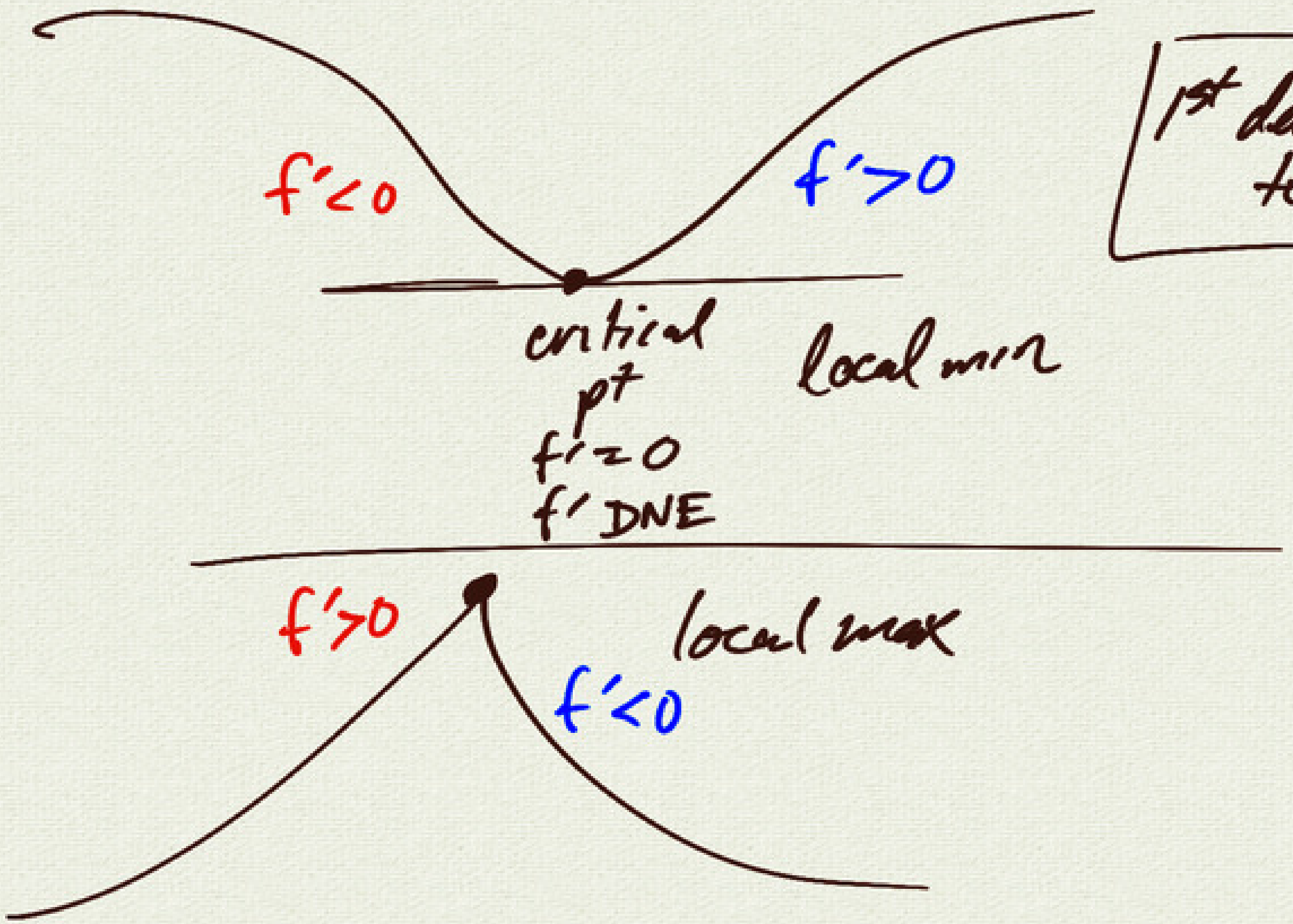
(c_3, c_4) initial position $t=0$ (x_0, y_0)

$$\Longrightarrow \boxed{\begin{aligned} x(t) &= v_x t + x_0 \\ y(t) &= -16t^2 + v_y t + y_0 \end{aligned}}$$

$f(x) = |x|$



1st derivative test



example:

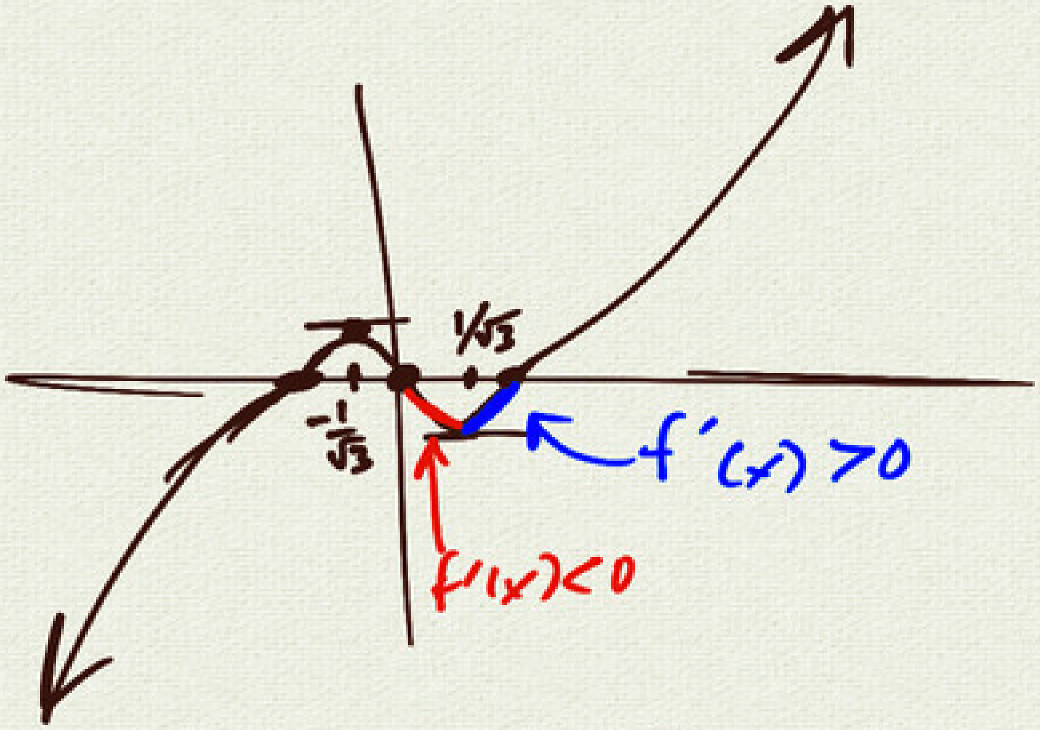
$f(x) = x^3 - x$

find all local min/max.

$f(x) = x(x^2 - 1)$

$\quad = x(x+1)(x-1)$

$f'(x) = 3x^2 - 1$



$f'(x) = 0 \Rightarrow 3x^2 - 1 = 0$

$x^2 = \pm \frac{1}{\sqrt3}$

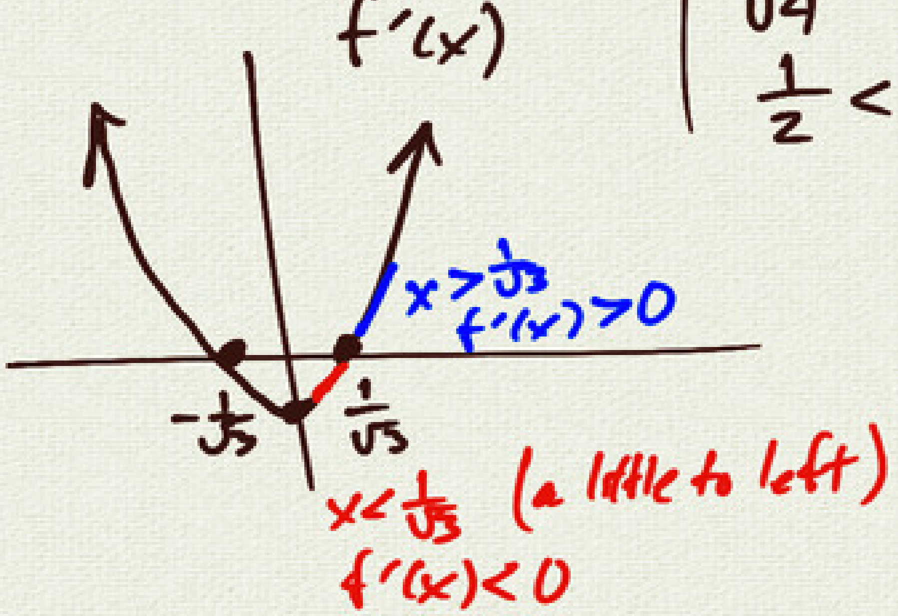
$\sqrt3 \approx 1.732$

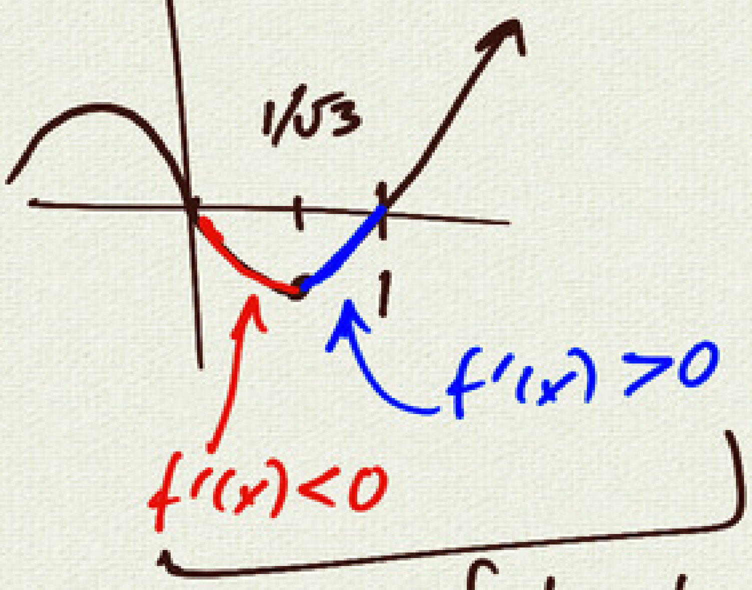
$\frac{1}{4} < \frac{1}{3}$

$\frac{1}{\sqrt4} < \frac{1}{\sqrt3}$

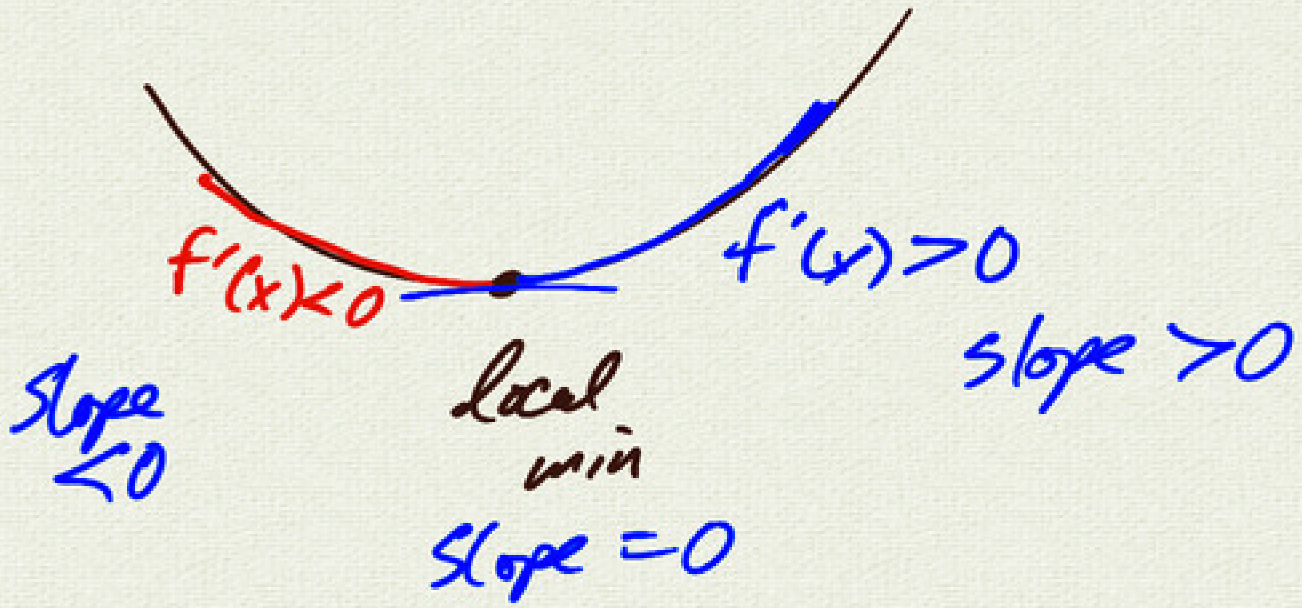
$\frac{1}{2} < \frac{1}{1.7}$

$x = +\frac{1}{\sqrt3}$:



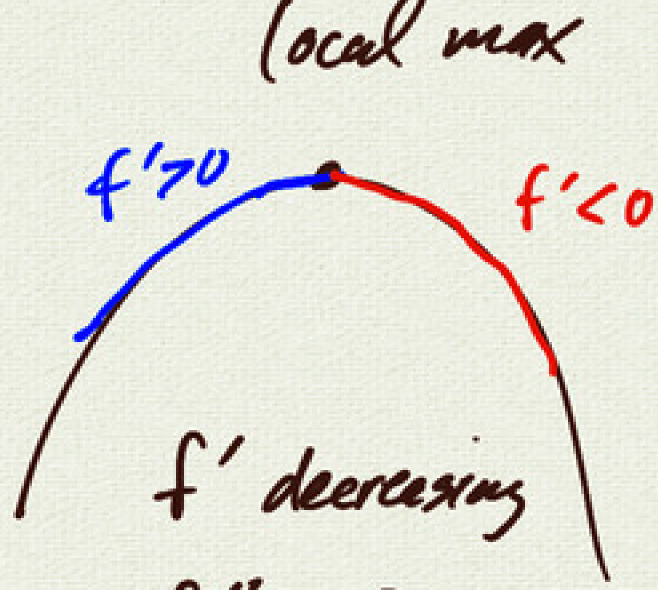


f has local min at $x = \frac{1}{\sqrt3}$ (1st deriv test)



$\Rightarrow$ f' increasing

$f'' > 0$



f' decreasing

$f'' < 0$

2nd deriv test:

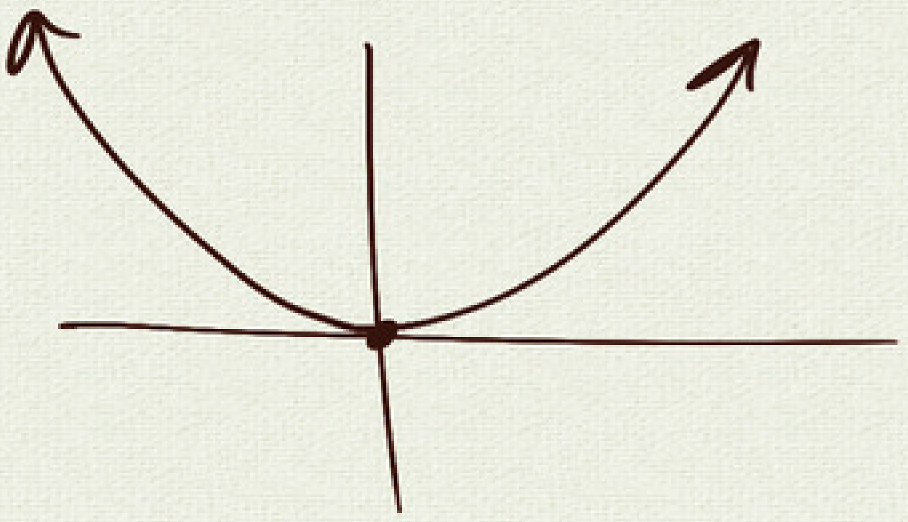
suppose f has $f'(c) = 0$, $f''(c)$ exists

then ① if $f''(c) > 0$, then f has local min at $x = c$

② if $f''(c) < 0$, then f has local max at $x = c$

③ if $f''(c) = 0$, then I don't know

$f(x) = x^2$



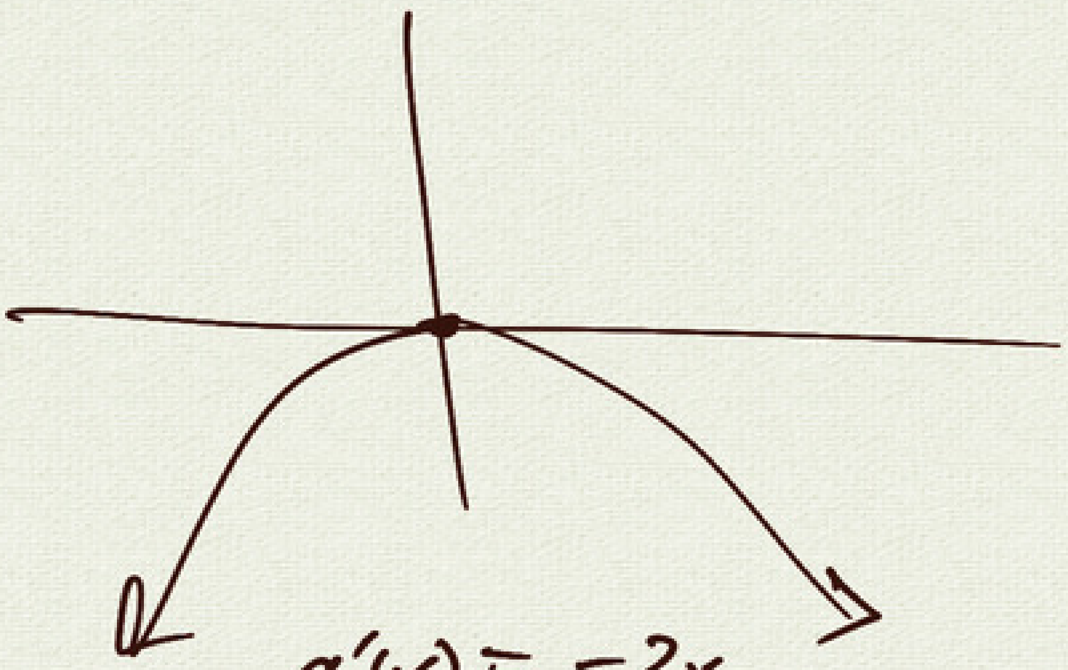
$f'(x) = 2x$

$f'(0) = 0$ $\quad x=0$ critical pt.

$f''(x) = 2 > 0$

local min at $x=0$

$g(x) = -x^2$



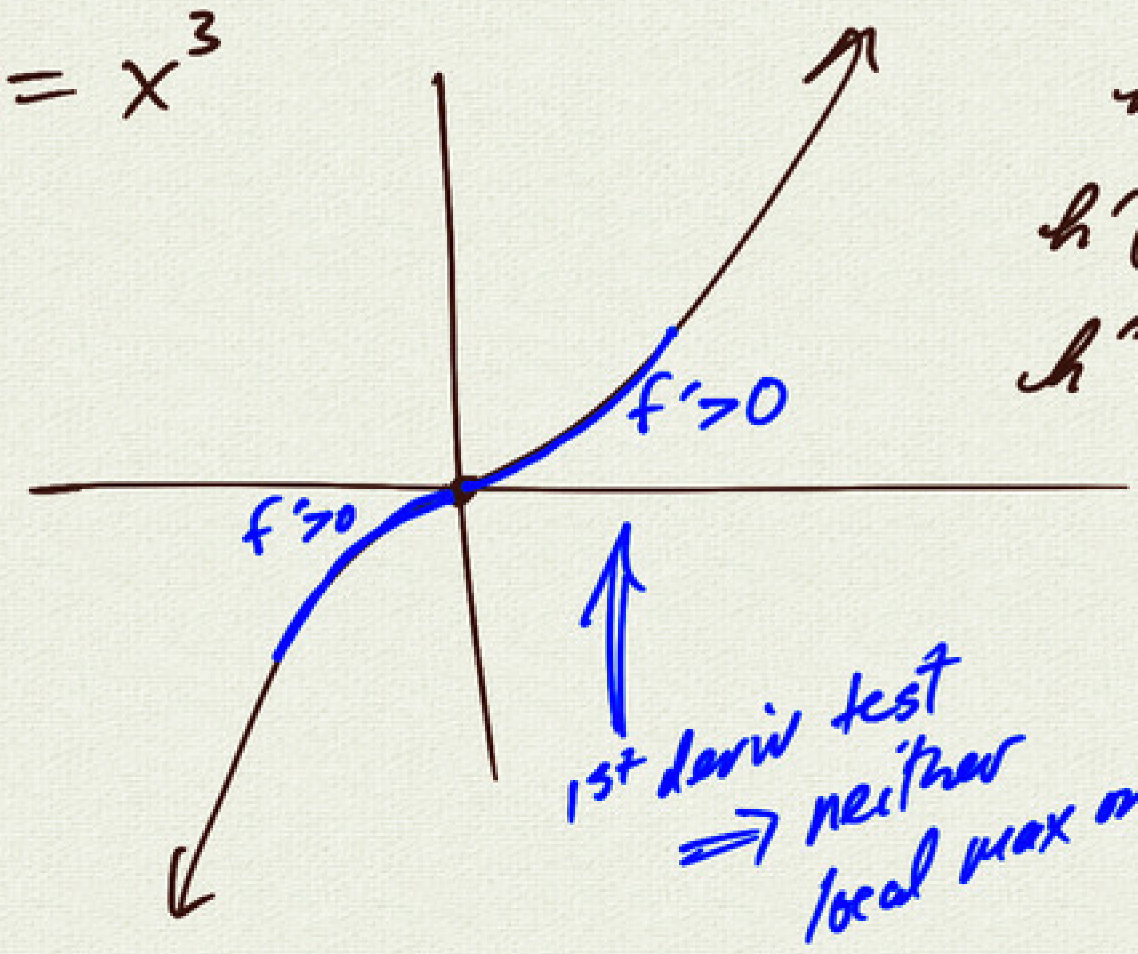
$g'(x) = -2x$

$g'(0) = 0$ $\quad x=0$ critical pt

$g''(x) = -2$

$g''(0) = -2 < 0$ local max at $x=0$

$h(x) = x^3$



1st deriv test $\Rightarrow$ neither local max or min

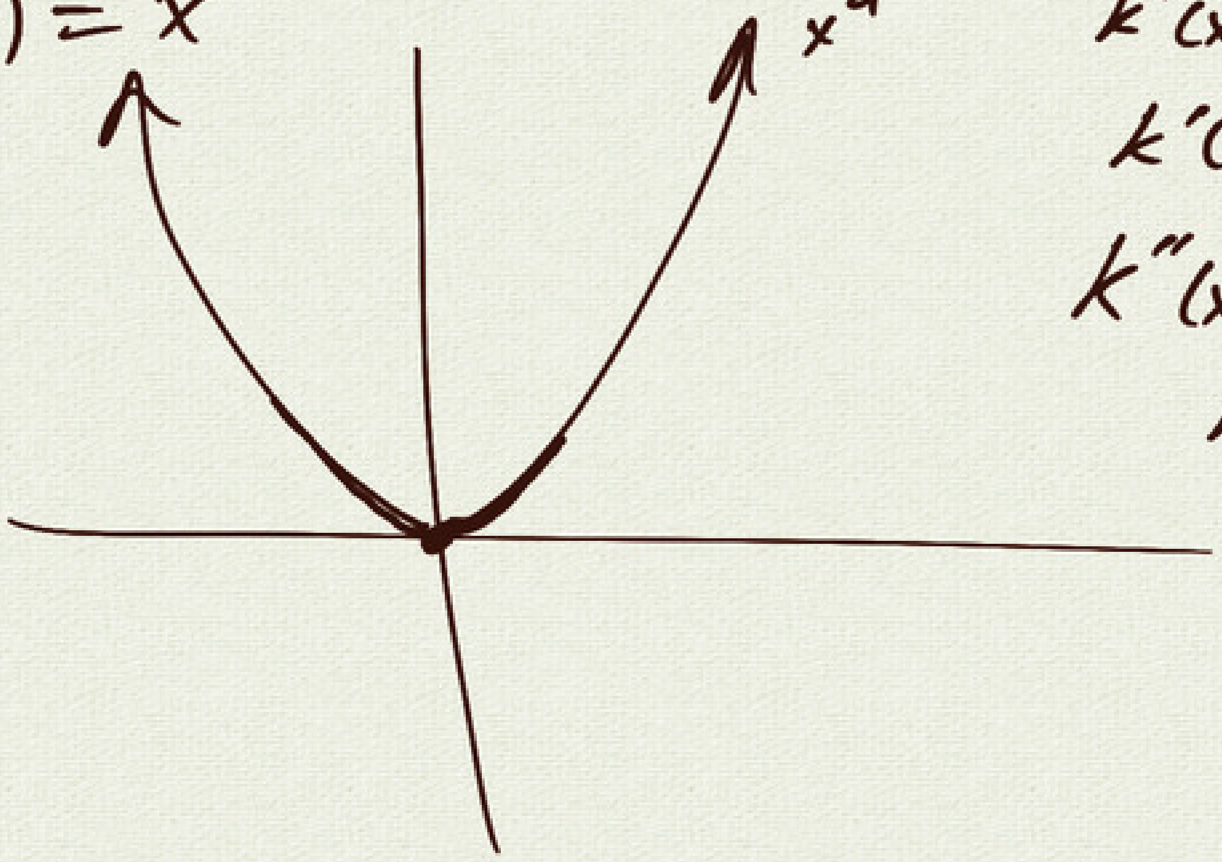
$h'(x) = 3x^2$

$h'(0) = 0$ $\quad$ critical pt $x=0$

$h''(x) = 6x$

$h''(0) = 0 \Leftarrow$ 2nd deriv test inconclusive

$k(x) = x^4$



$k'(x) = 4x^3$

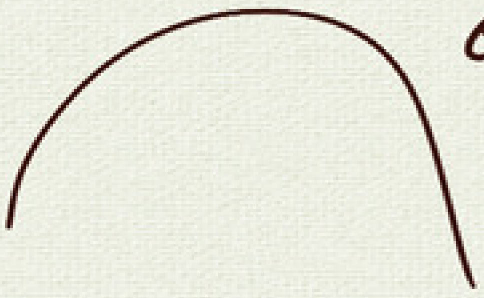
$k'(0) = 0$ $\quad$ critical pt at $x=0$

$k''(x) = 12x^2$

$k''(0) = 0 \Leftarrow$ 2nd deriv test inconclusive

concave up

$f'' > 0$



concave down

$f'' < 0$