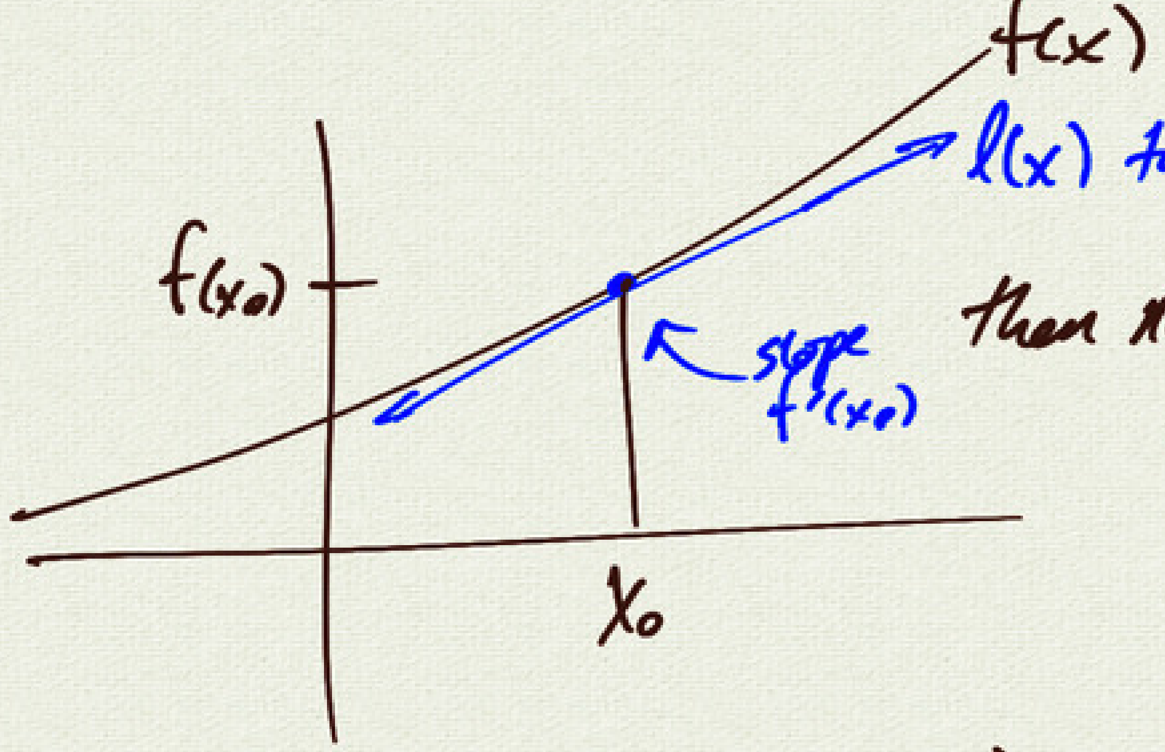
10.5 Linearization



then near x_0:

$$f(x) \approx l(x)$$

line $l(x)$: point $(x_0, f(x_0))$

slope $f'(x_0)$

$$l(x) = \underbrace{f(x_0) + f'(x_0)(x - x_0)}$$

slope-point:

$$y - y_1 = m(x - x_1)$$

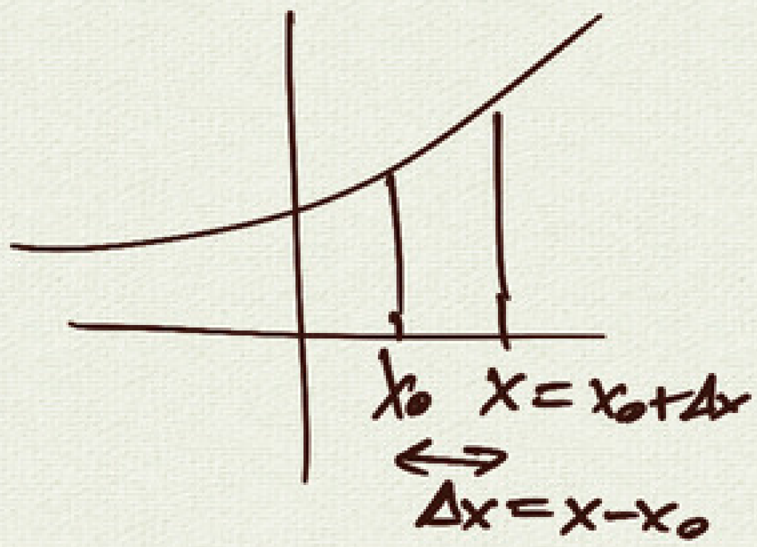
$$y - y_0 = m(x - x_0)$$

$$y = y_0 + m(x - x_0)$$

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

when Δx is small:

$$f'(x_0) \approx \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$



$$f(x_0 + \Delta x) - f(x_0) \approx f'(x_0)\Delta x$$

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x$$

$$f(x) \approx \underbrace{f(x_0) + f'(x_0)(x - x_0)}_{l(x)}$$

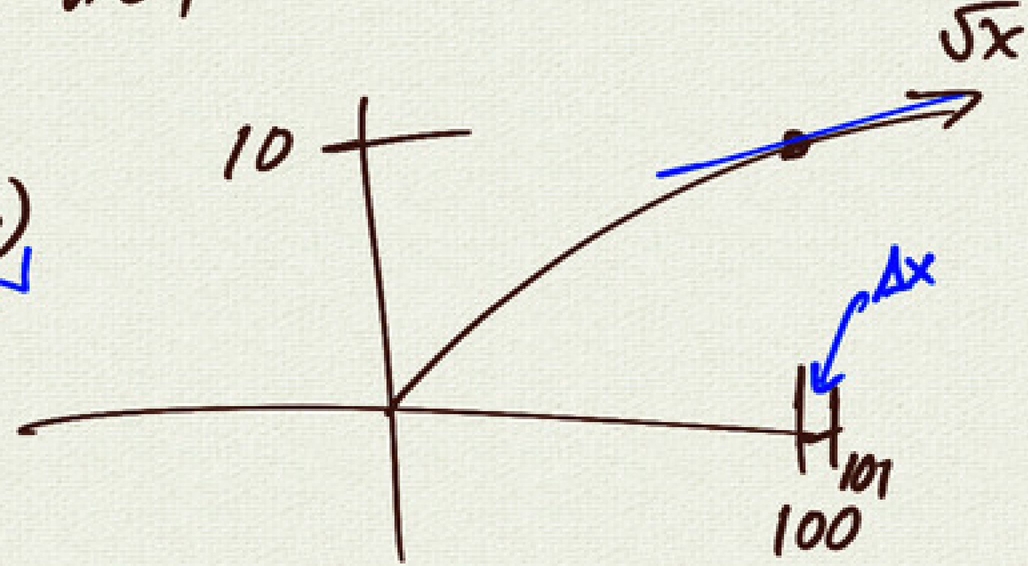
$f(x) \approx l(x)$ (when Δx small)

near x_0

example: approximate $\sqrt{101}$

$$f(x) \approx f(x_0) + f'(x_0)\underbrace{(x-x_0)}_{\Delta x = 1}$$

100



$$f(x) = \sqrt{x}$$

$$f'(x) = \frac{1}{2}x^{-1/2}$$

$$f'(100) = \frac{1}{2\sqrt{100}} = \frac{1}{20}$$

$$f(101) \approx f(100) + f'(100)(1)$$

$$= 10 + \frac{1}{20}(1)$$

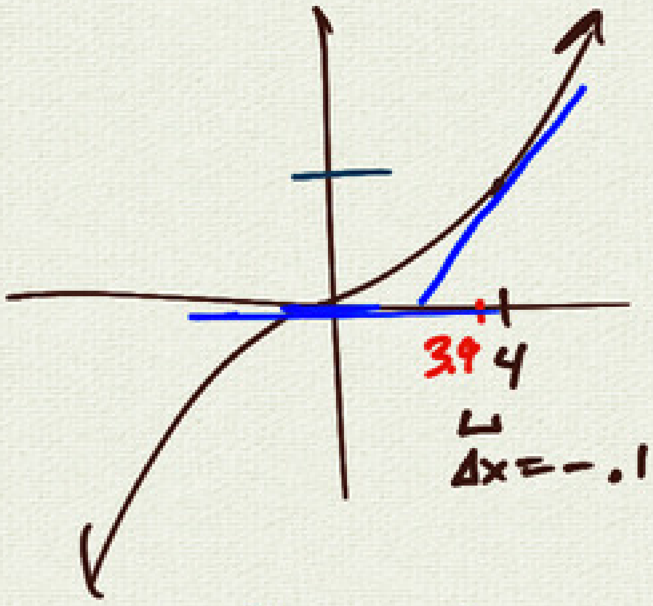
$$= 10.05$$

calculator: $\sqrt{101} \approx 10.04987$

example: approximate $(3.9)^3$

$f(x) = x^3$ near $x_0 = 4$

$\Delta x = -.1$



$$f(x) \approx f(\underbrace{x_0}_{4}) + f'(\underbrace{x_0}_{4})(\underbrace{x - x_0}_{\Delta x})$$

$f(4) = 64$

$f'(x) = 3x^2$

$f'(4) = 48$

$= 64 + 48(-.1)$

$= 64 - 4.8$

$(3.9)^3 \approx 59.2$

calculator:
$(3.9)^3 \approx 59.3$

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example: approximate $\sin(.01)$

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

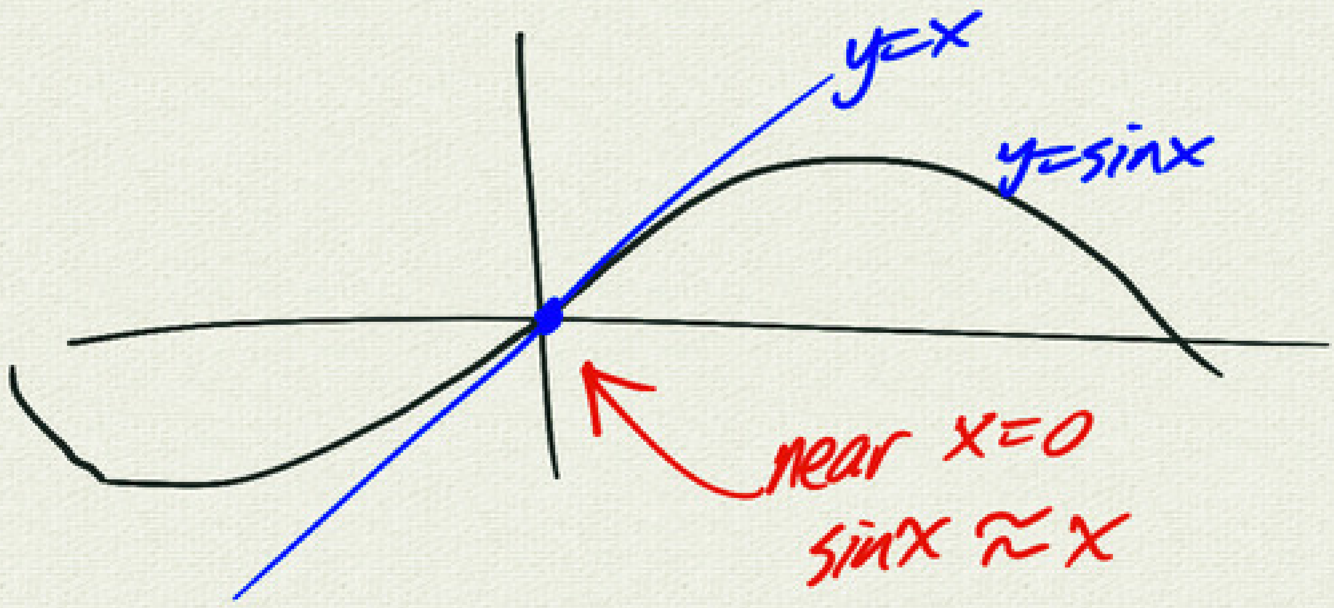
$$\sin(x) \approx \underbrace{\sin(0)}_{0} + \underbrace{\cos(0)}_{1}(x - 0)$$

$\sin x \approx x$

$\sin(.01) \approx .01$

$\boxed{\text{special limit: } \lim_{x \to 0} \frac{\sin x}{x} = 1}$

calculator:
$\sin(.01) \approx .0099998$



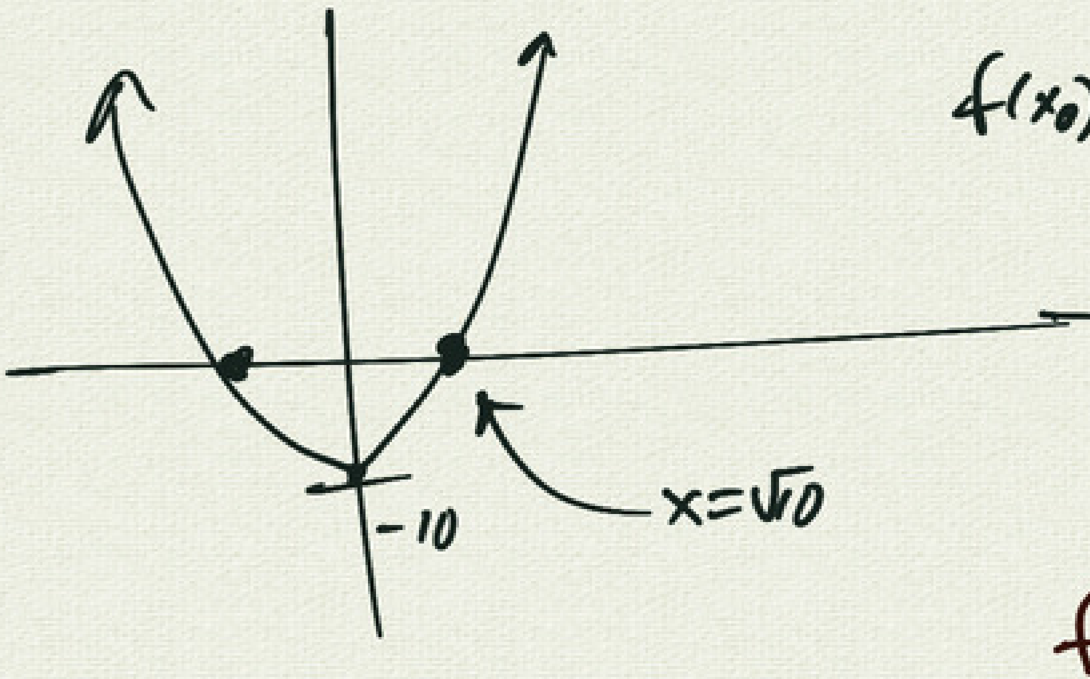
Newton's Method

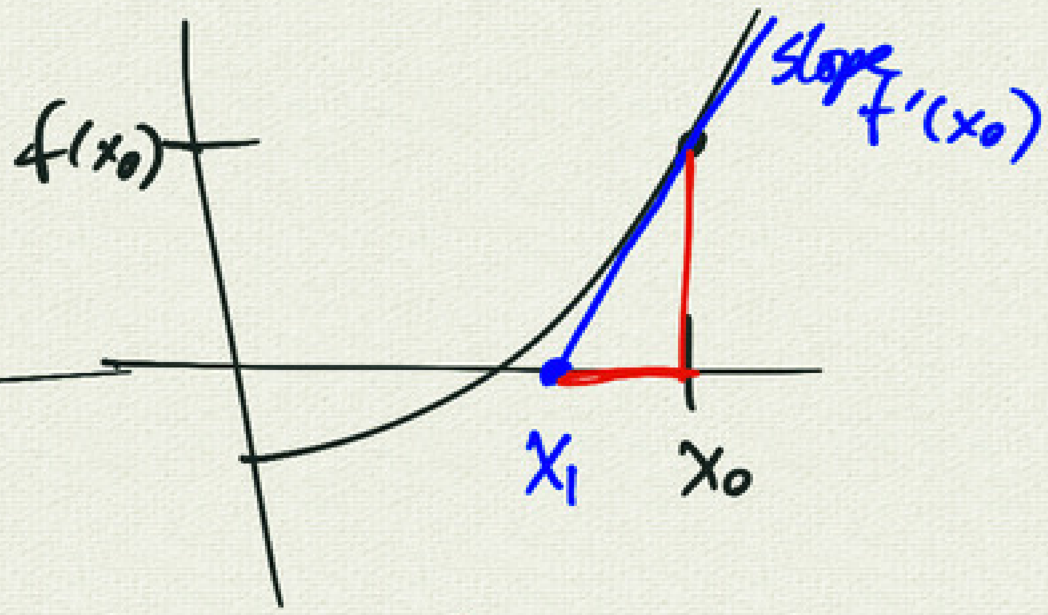
find $\sqrt{10}$

Babylonian/Greek: guess x_0 $(=3)$

$\Longrightarrow$ next guess $x_1 = \dfrac{x_0 + {}^{10}/_{x_0}}{2}$

Newton: $f(x) = x^2 - 10 \Longleftarrow$ find root





$$f'(x_0) = \frac{f(x_0)}{(x_0 - x_1)}$$

$$f'(x_0)(x_0 - x_1) = f(x_0)$$

$$x_0 - x_1 = \frac{f(x_0)}{f'(x_0)}$$

$$\boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

Newton's Method

in this case: $f(x) = x^2 - 10$

$f'(x) = 2x$

$$\Longrightarrow \quad x_1 = x_0 - \frac{x_0^2 - 10}{2x_0}$$

$$= \frac{2x_0^2 - (x_0^2 - 10)}{2x_0}$$

$$= \frac{x_0^2 + 10}{2x_0}$$

$$x_1 = \frac{x_0 + {}^{10}/_{x_0}}{2}$$