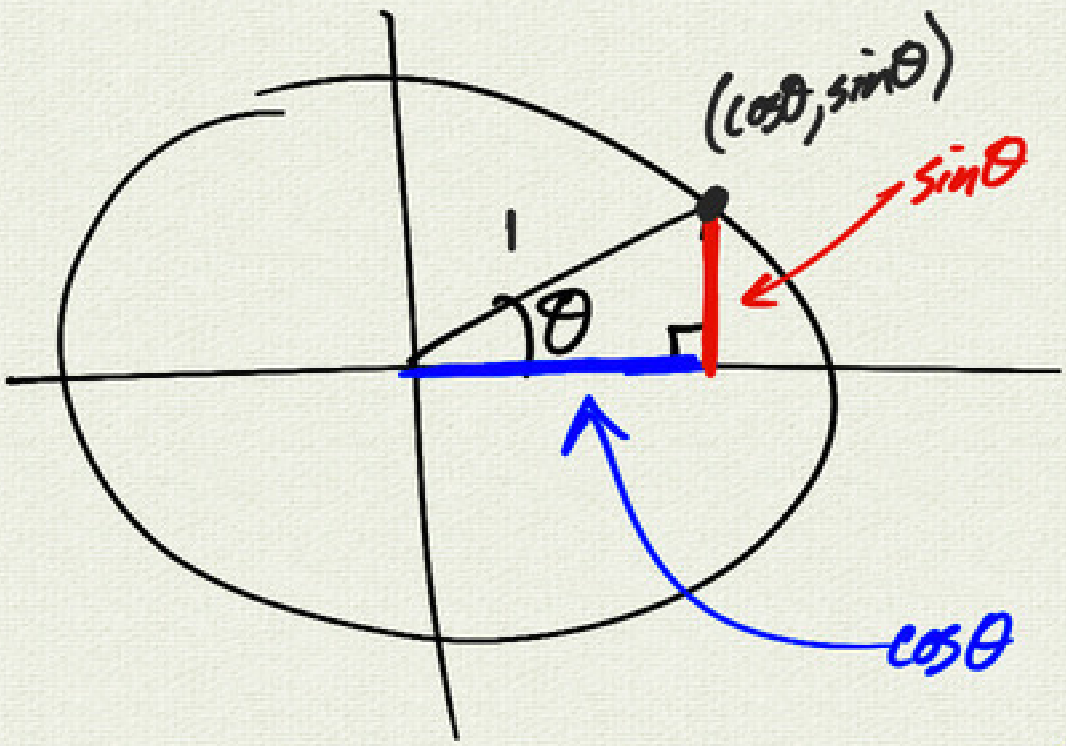
2.1 – 2.2 Trig Identities

identity = equation that is always true (for any x)

basic

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$



Pythagorean

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

$(\sin\theta)^2$

$$\sin^2\theta = 1 - \cos^2\theta$$

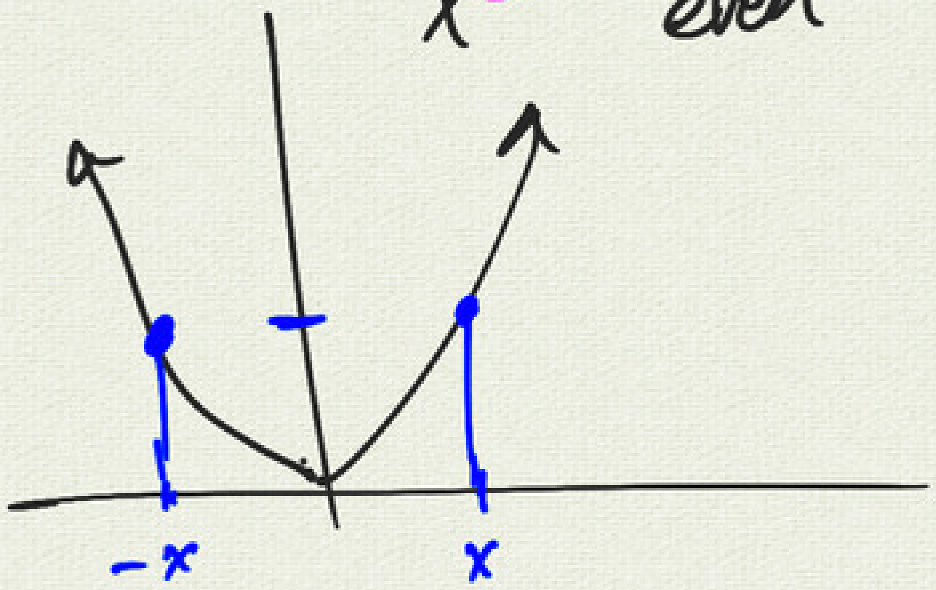
$$\boxed{\frac{\sin^2\theta}{\cos^2\theta}} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

$$\boxed{1 + \cot^2\theta = \csc^2\theta}$$

odd/even identities

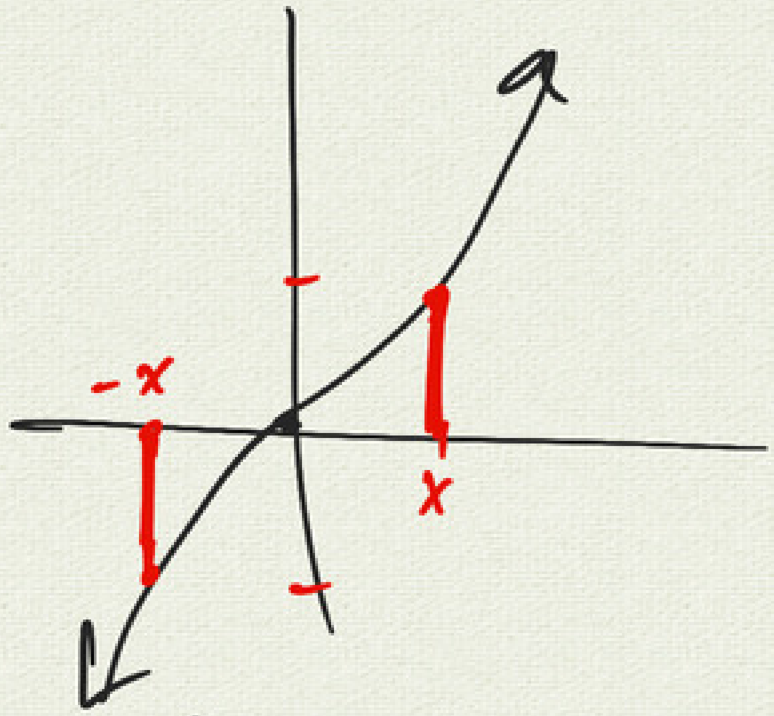
x^2 even



$f(-x) = f(x)$ even

$(-x)^2 = x^2$

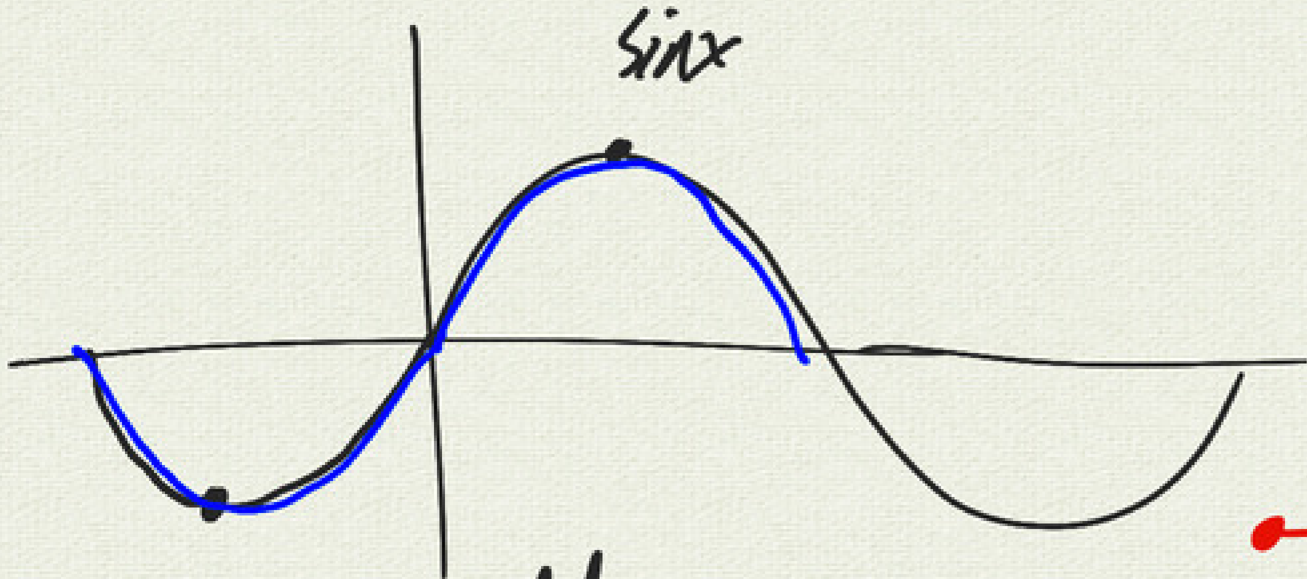
x^3 odd



$f(-x) = -f(x)$ odd

$(-x)^3 = -x^3$

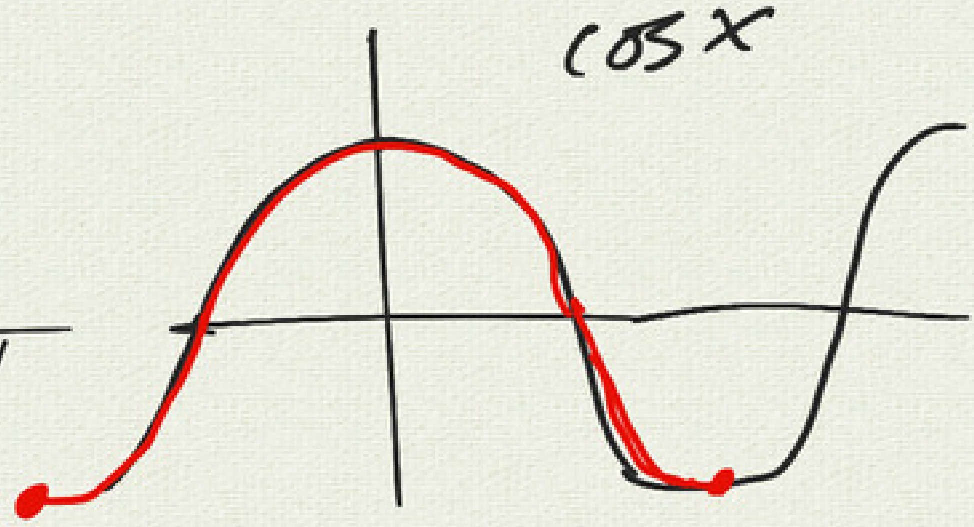
$\sin x$



odd

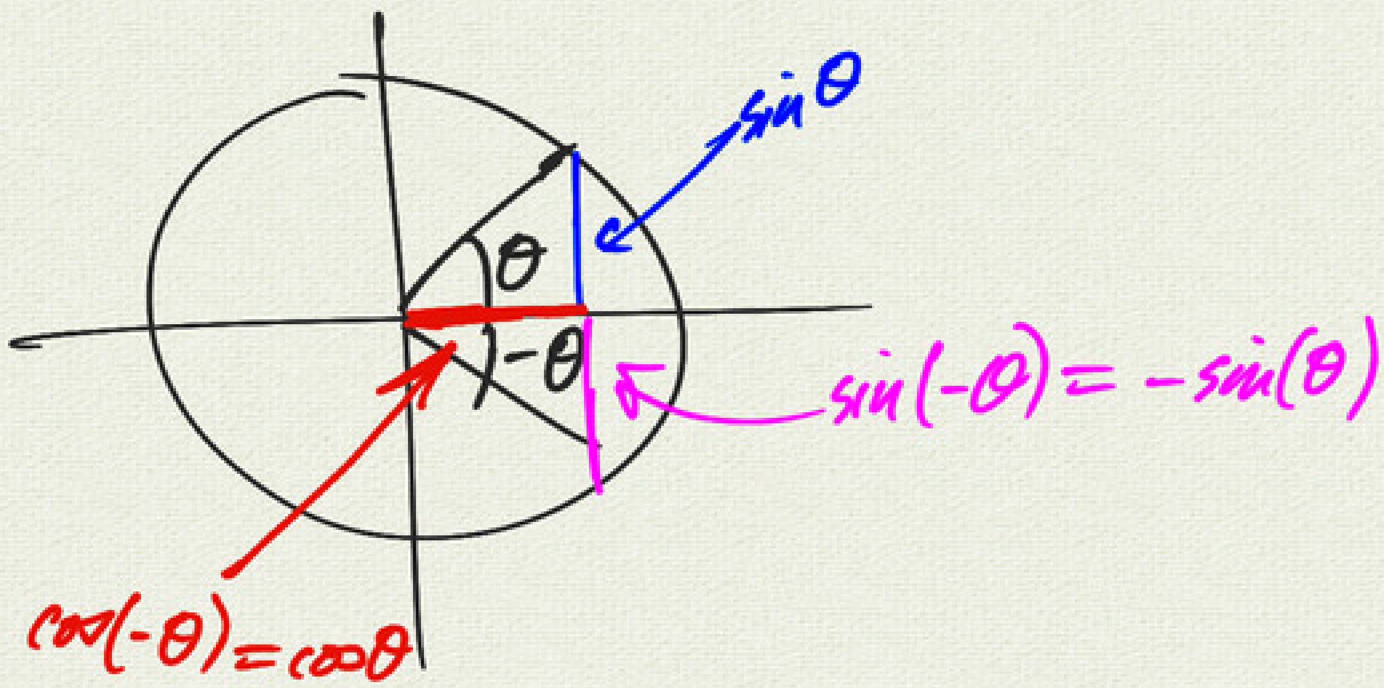
$\sin(-x) = -\sin x$

$\cos x$



even

$\cos(-x) = \cos x$



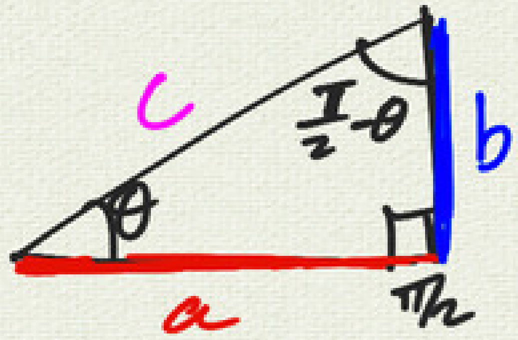
cofactor identities

$$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$$

$$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$$

$$\sec\left(\frac{\pi}{2}-\theta\right)=\csc\theta$$



$$\sin\left(\frac{\pi}{2}-\theta\right)=\frac{a}{c}=\cos\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right)=\frac{\sin\left(\frac{\pi}{2}-\theta\right)}{\cos\left(\frac{\pi}{2}-\theta\right)}=\frac{\cos\theta}{\sin\theta}=\cot\theta$$

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example $\boxed{7.1}$ #7

simplify

$$\tan x\sin x+\sec x\cos^2 x$$

$$=\frac{\sin x}{\cos x}\sin x+\frac{1}{\cos x}\cdot\cos^2 x$$

$$=\frac{\sin^2 x}{\cos x}+\cos x$$

$$=\frac{\boxed{\sin^2 x+\cos^2 x}}{\cos x}$$

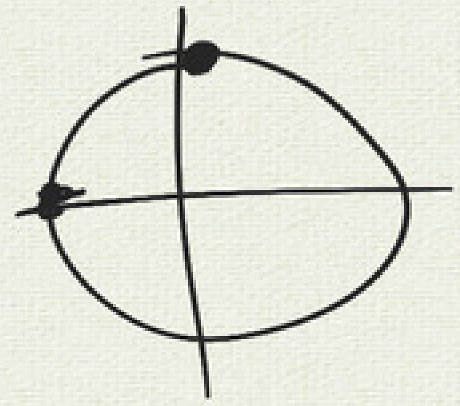
$$=\frac{1}{\cos x}$$

$$=\sec x \quad \checkmark$$

2.2 Sum/Difference identities

$$(u+v)^2 \overset{?}{=} u^2 + v^2 \qquad \text{NO!}$$

$$\sin(u+v) \neq \sin u + \sin v$$



$$\sin\left(\underbrace{\tfrac{\pi}{2}+\tfrac{\pi}{2}}_{\pi}\right) \overset{?}{=} \sin\tfrac{\pi}{2} + \sin\tfrac{\pi}{2}$$

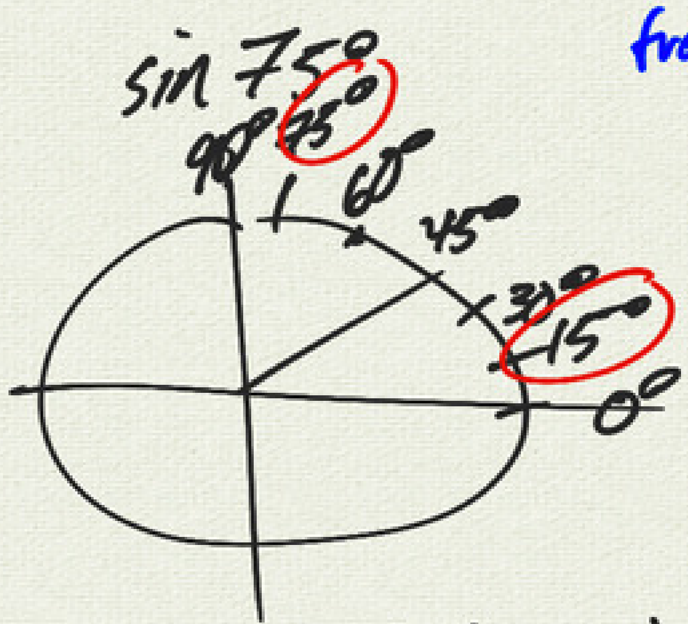
$$0 \overset{?}{=} 2$$

challenge: for what functions is it true that
$$f(x_1+x_2) = f(x_1) + f(x_2) \quad ?$$

sum identity: $\boxed{\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v}$

from odd/even

$\sin 75°$



$$\sin(75°) = \sin(30° + 45°)$$
$$= \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$$
$$= \sin\frac{\pi}{6}\cos\frac{\pi}{4} + \cos\frac{\pi}{6}\sin\frac{\pi}{4}$$
$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$
$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

$$\sin(u-v) = \sin(u+(-v))$$
$$= \sin u \cos(-v) + \cos u \sin(-v)$$
$$= \sin u \cos(v) - \cos u \sin v$$

$$\sin(15°) = \sin\left(\frac{\pi}{12}\right)$$
$$= \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$$
$$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}$$
$$= \frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\,\frac{1}{2}$$
$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\begin{aligned}\cos(u-v) &= \cos(u+(-v)) \\ &= \cos u \cos(-v) - \sin u \sin(-v) \\ &= \cos u \cos v + \sin u \sin v\end{aligned}$$

$-\sin(-v)$

$$\boxed{\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v}$$

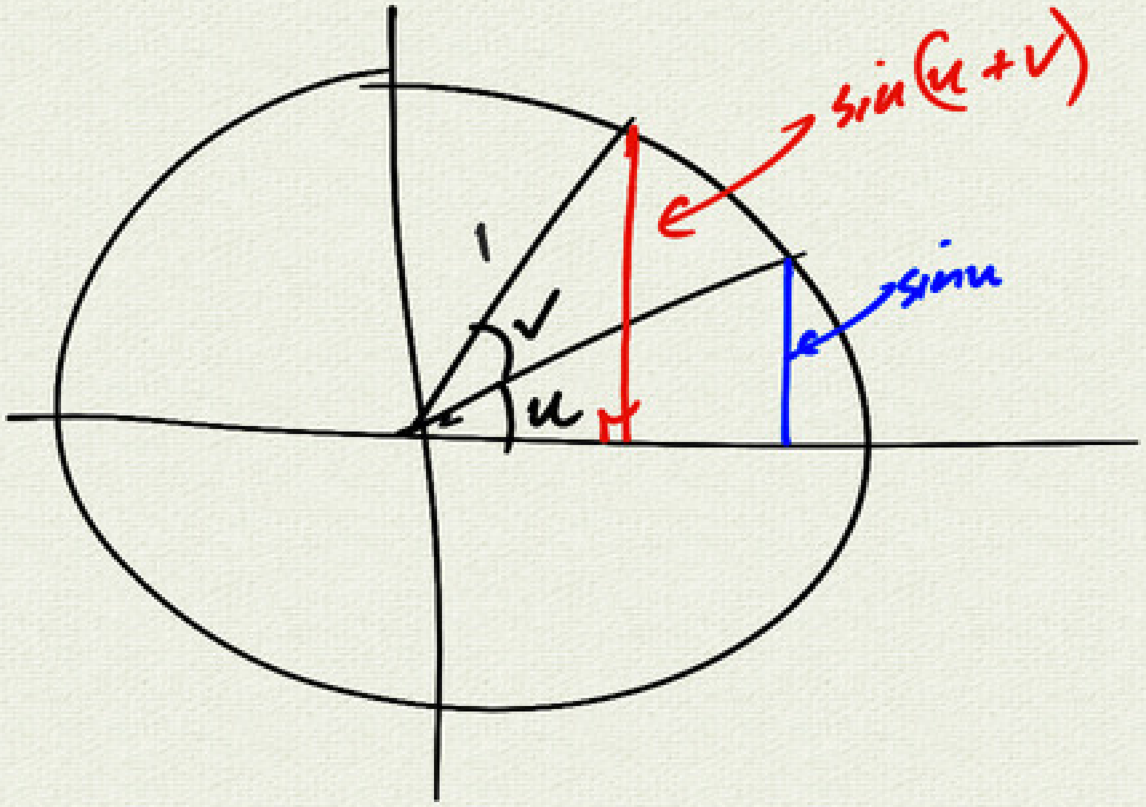
$$\tan(u+v) = \frac{\sin(u+v)}{\cos(u+v)} = \frac{\boxed{\sin u \cos v} + \cos u \sin v}{\cos u \cos v - \sin u \sin v} \cdot \frac{\frac{1}{\cos u \cos v}}{\frac{1}{\cos u \cos v}}$$

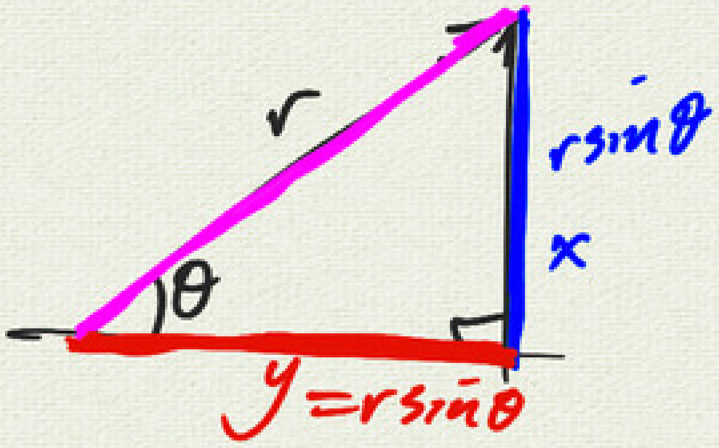
$$= \frac{\boxed{\tan u} + \tan v}{1 - \tan u \tan v}$$

$$\boxed{\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}}$$

don't memorize

$\sin(u+v) = \sin u \cos v + \cos u \sin v$

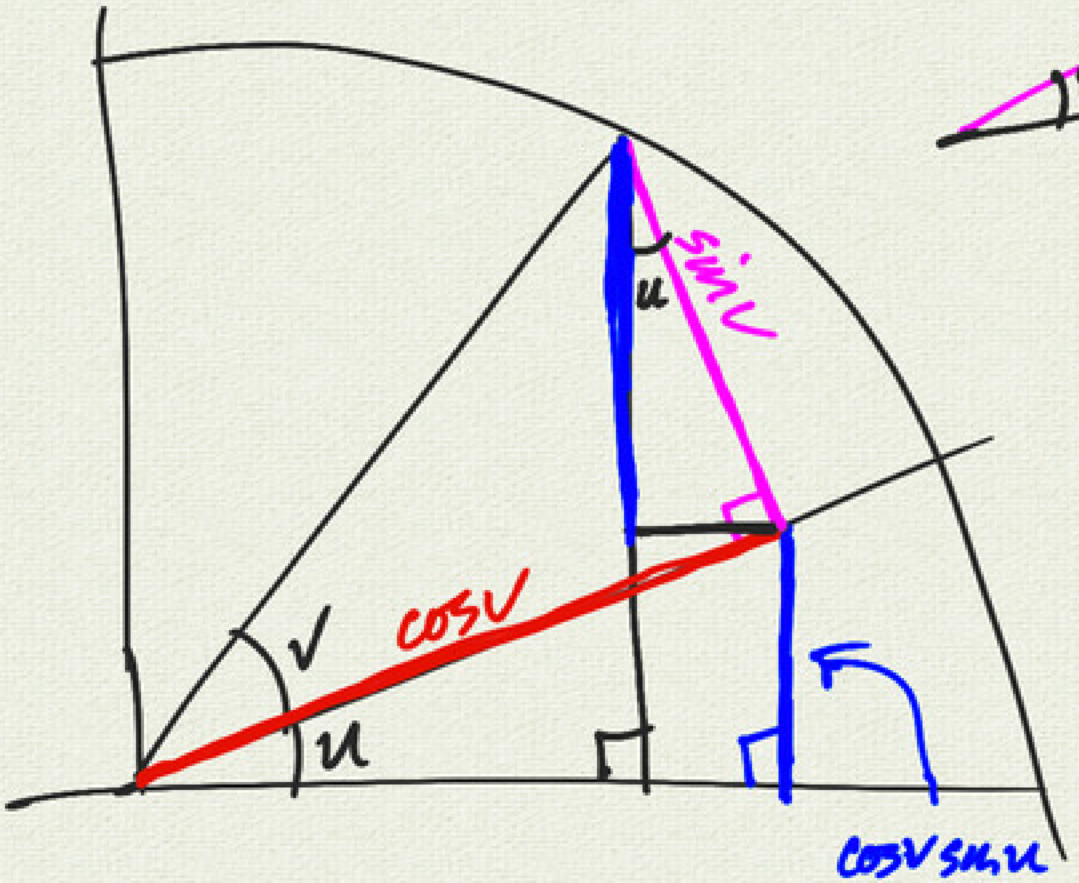


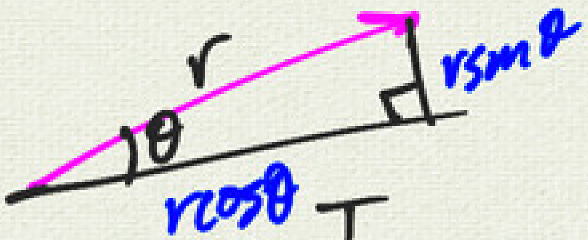


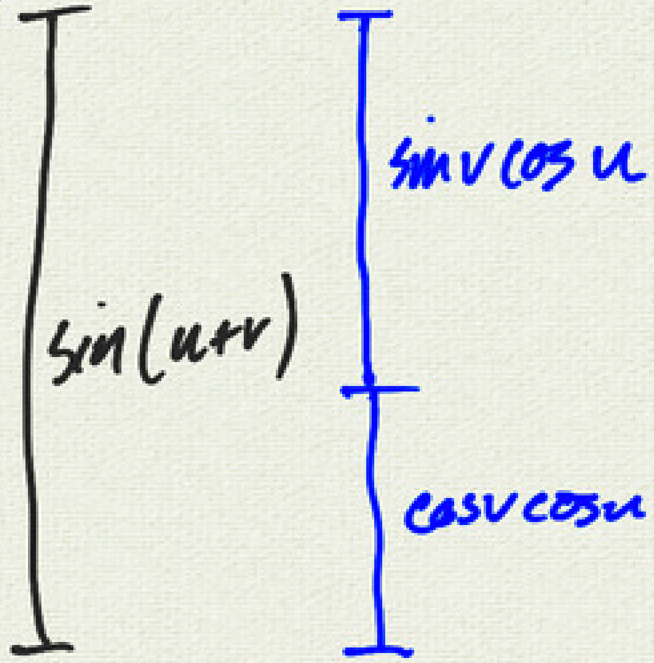
$y = ?$

$\cos\theta = \frac{y}{r}$

$y = r\cos\theta$







$\boxed{\sin(u+v) = \sin v \cos u + \cos v \sin u}$

challenge: derive $\cos(u+v)$ formula