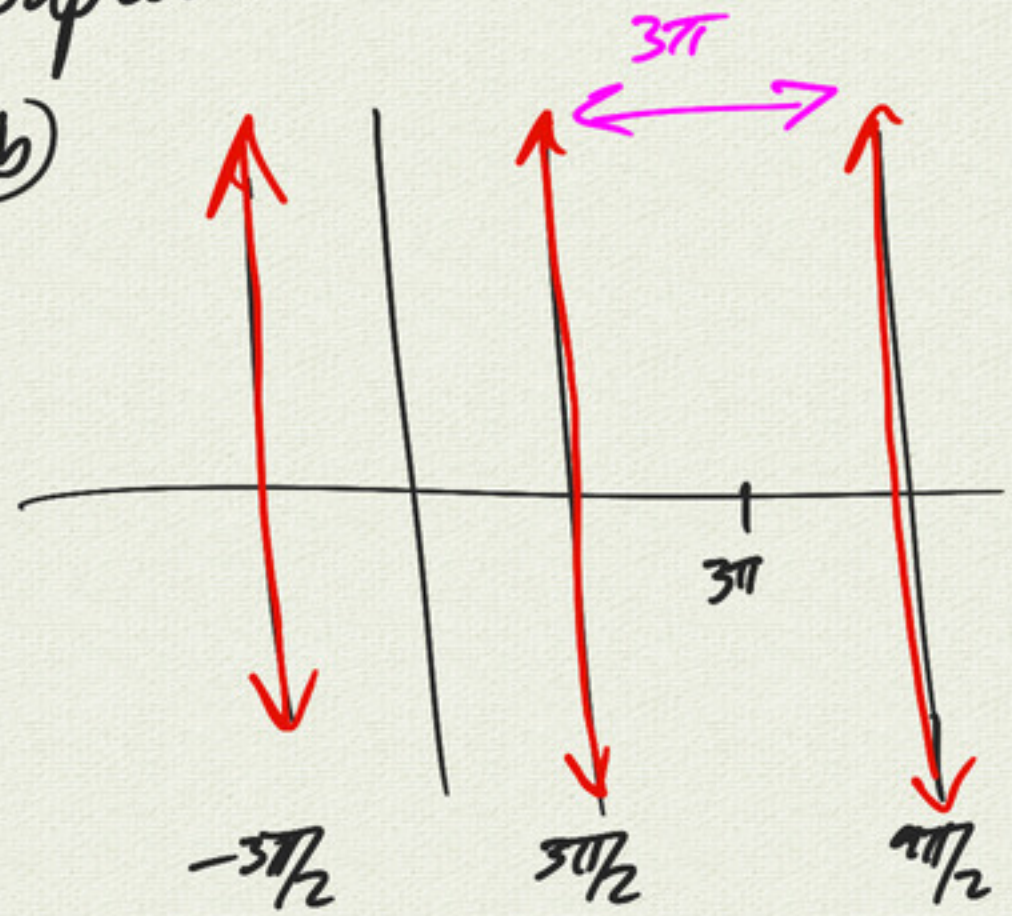
group work

(36)



domain

$x \neq \frac{3\pi}{2} + 3\pi n$

$(n \in \mathbb{Z})$

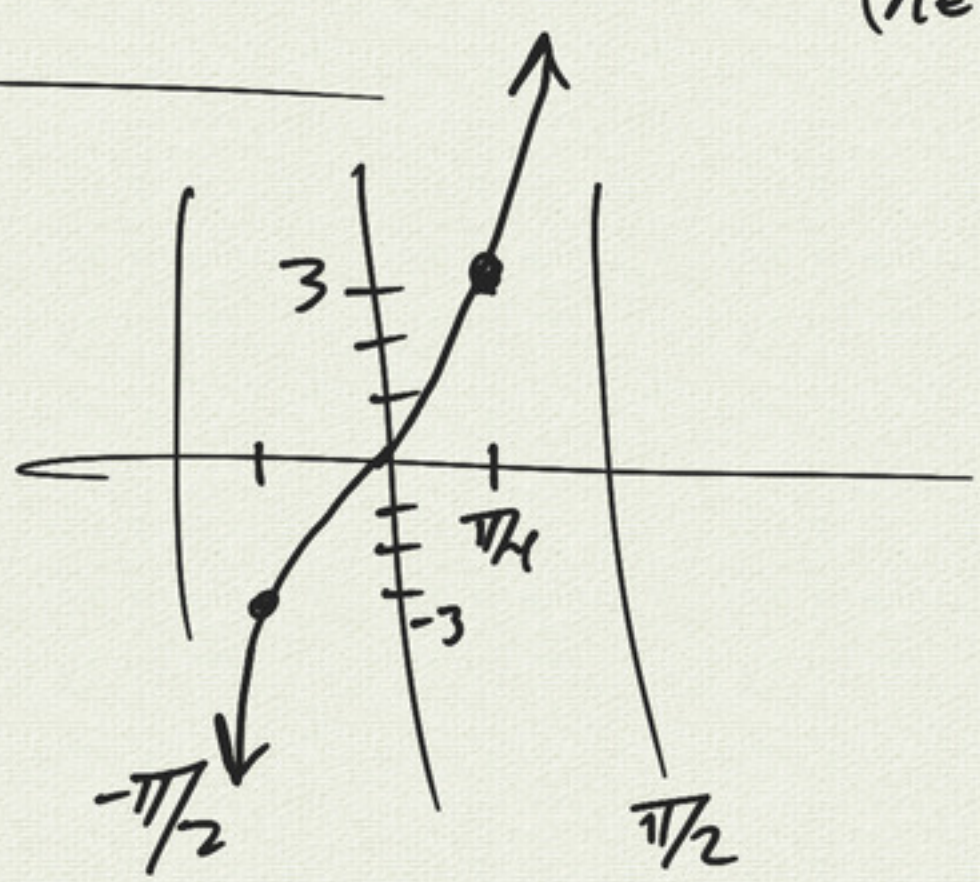
asymptotes

$x = \frac{3\pi}{2} + 3\pi n$

$(n \in \mathbb{Z})$

tangent

$y = 3\tan x$



2.1 Trig Identities

identity = equation that is always true (for any x)

basic

$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

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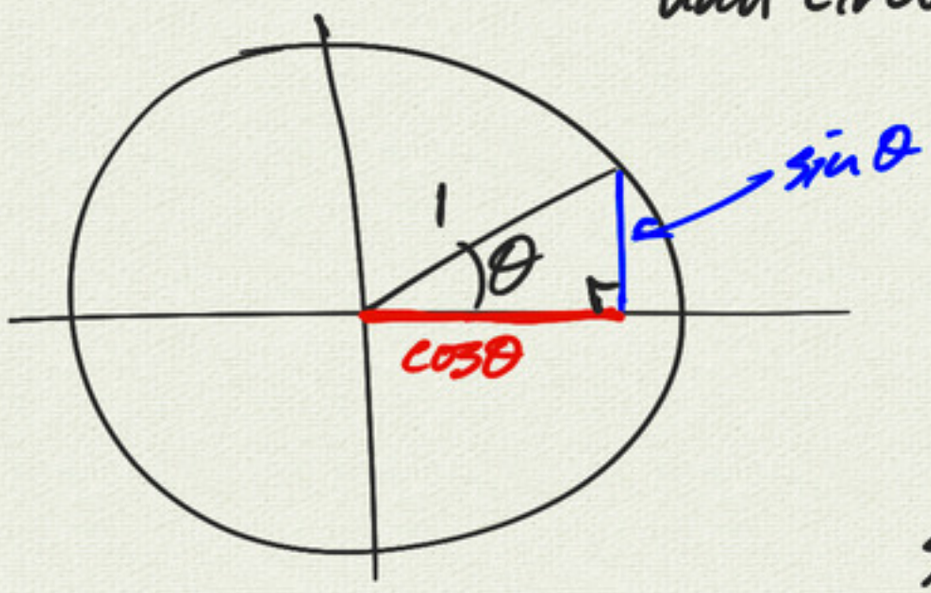
unit circle



Pythagorean identity

$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

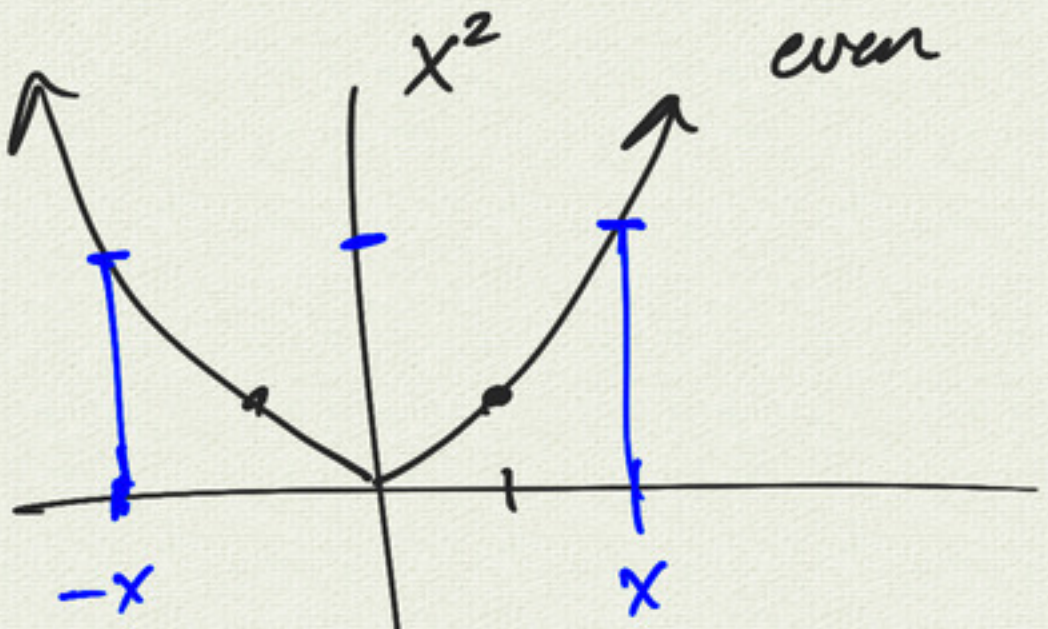
$$\sin^2\theta = 1 - \cos^2\theta$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

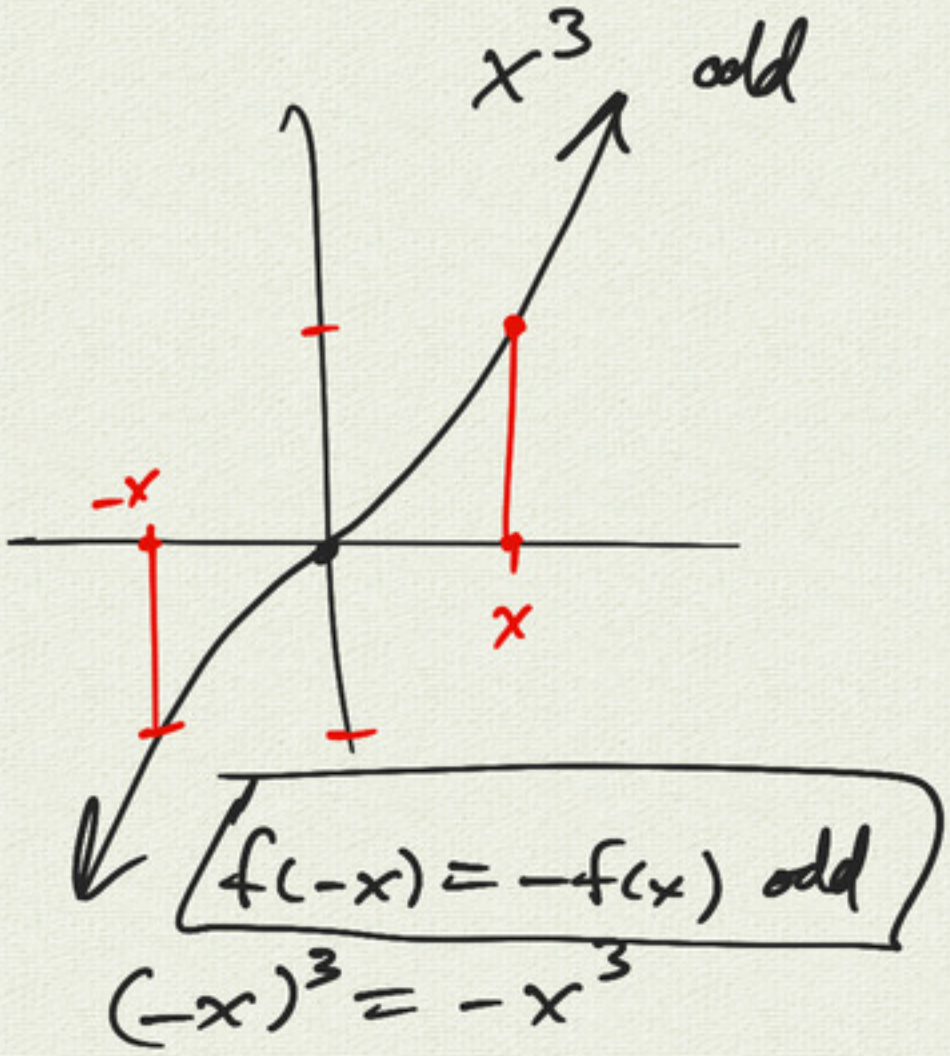
$$\boxed{1 + \cot^2\theta = \csc^2\theta}$$

odd/even identities



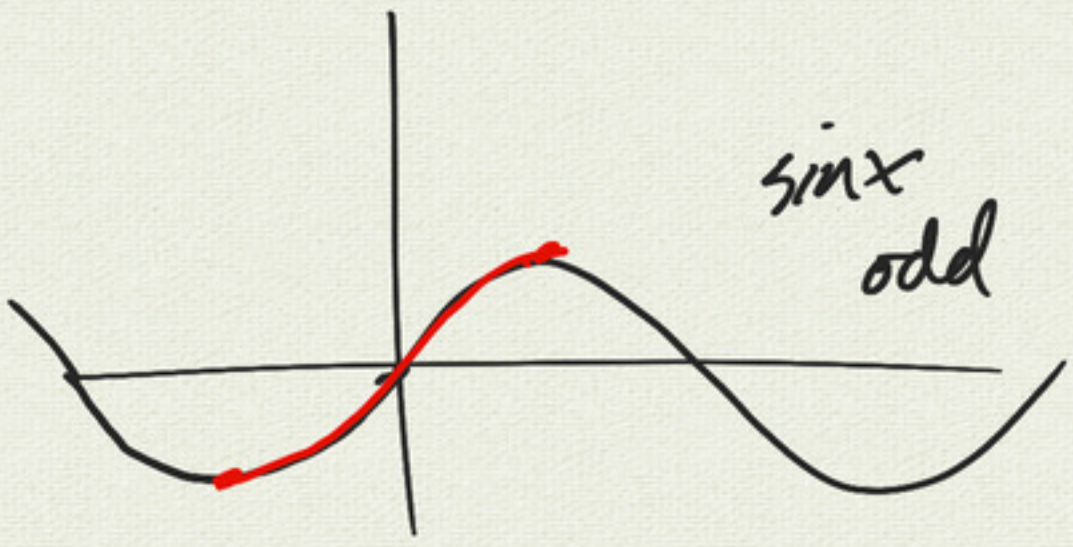
$f(-x) = f(x)$ even

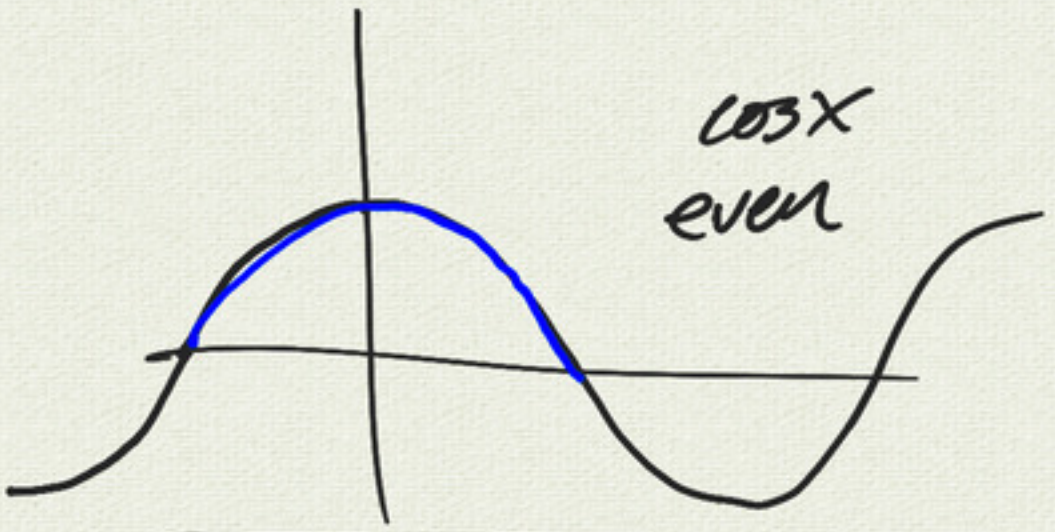
$(-x)^2 = x^2$



$f(-x) = -f(x)$ odd

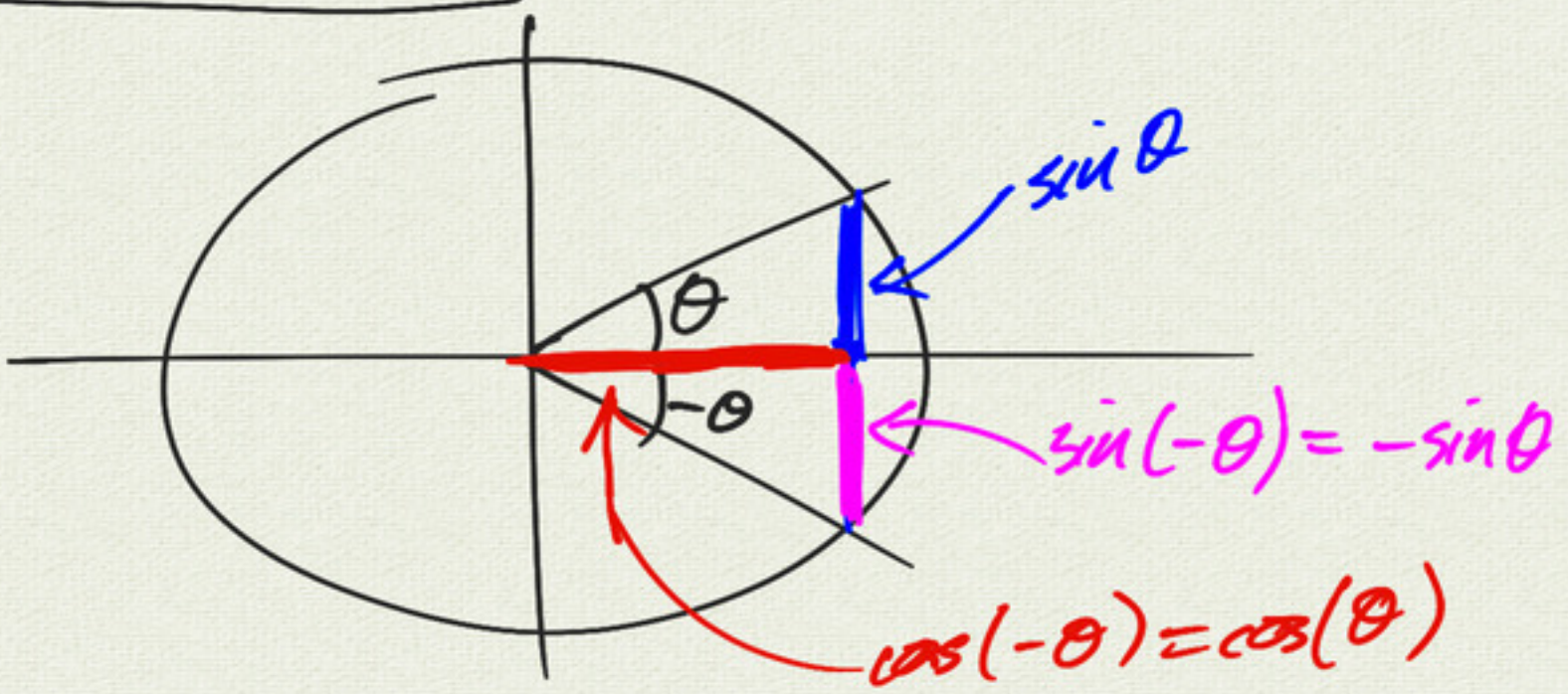
$(-x)^3 = -x^3$





$\sin(-x) = -\sin x$

$\cos(-x) = \cos(x)$



cofactor identities

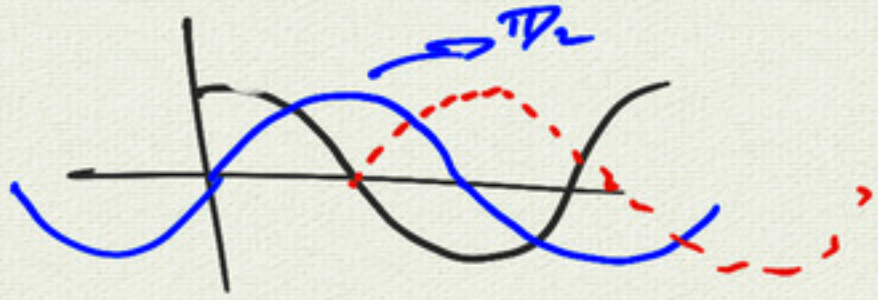
$$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$$

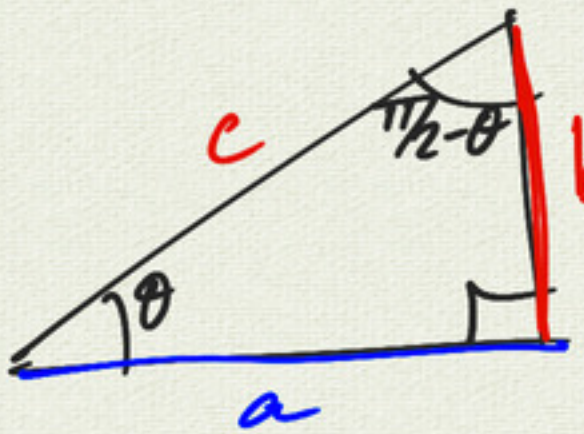
$$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$$

$$\sec\left(\frac{\pi}{2}-\theta\right)=\csc\theta$$

$$\sin\left(\frac{\pi}{2}-\theta\right)=-\sin\left(\theta-\frac{\pi}{2}\right)$$





$$\sin\theta=\frac{b}{c}=\cos\left(\frac{\pi}{2}-\theta\right)$$

$$\tan\left(\frac{\pi}{2}-\theta\right)=\frac{\sin\left(\frac{\pi}{2}-\theta\right)}{\cos\left(\frac{\pi}{2}-\theta\right)}=\frac{\cos\theta}{\sin\theta}=\cot\theta$$

7.1 (7)

$$\tan x \sin x + \sec x \cos^2 x$$

$$= \frac{\sin x}{\cos x} \sin x + \frac{1}{\cos x} \cdot \cos^2 x$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x}$$

$$= \frac{1}{\cos x}$$

$$= \sec x$$

2.2 sum/difference identities

$(u+v)^2 \stackrel{?}{=} u^2+v^2$ NO

$\sin(u+v) \neq \sin u + \sin v$

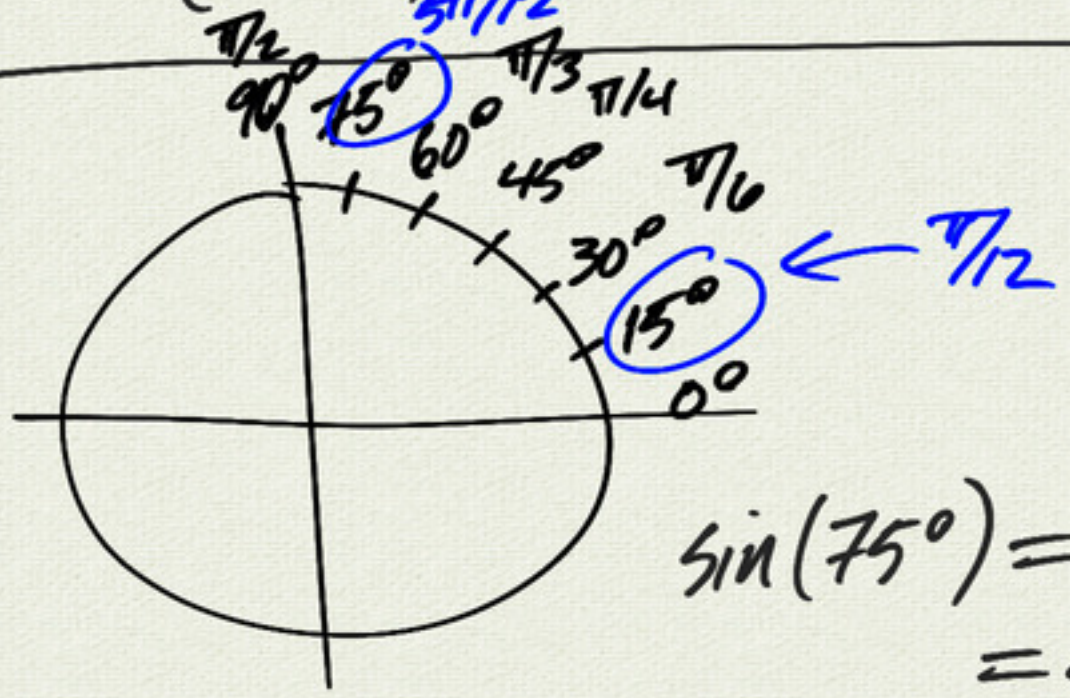
example:

$$\sin\left(\underbrace{\frac{\pi}{2}+\frac{\pi}{2}}_{\pi}\right) \stackrel{?}{=} \underbrace{\sin\frac{\pi}{2}}_{1} + \underbrace{\sin\frac{\pi}{2}}_{1}$$

$$\pi \stackrel{?}{=} 2$$

$$0 = 2$$

$$\boxed{\sin(u+v) = \sin u\cos v + \cos u\sin v}$$ sum formula



$\sin(75°) = \sin(45°+30°)$

$= \sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$

check:

$\frac{\pi}{4}+\frac{\pi}{6}$

$= \frac{3\pi}{12}+\frac{2\pi}{12}$

$= \frac{5\pi}{12}$

$\sin(u-v) = \sin(u+(-v))$

$= \sin u\cos(-v) + \cos u\sin(-v)$

$= \sin u\cos(v) - \cos u\sin v$

$$\boxed{\sin(u\pm v) = \sin u\cos v \pm \cos u\sin v}$$ sum formula

$\sin(15°) = \sin\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

$$\boxed{\cos(u+v) = \cos u \cos v - \sin u \sin v}$$

Sum formula for cos

$$\cos(u-v) = \cos(u+(-v))$$
$$= \cos u \cos(-v) - \sin u \sin(-v)$$
$$= \cos u \cos v + \sin u \sin v$$

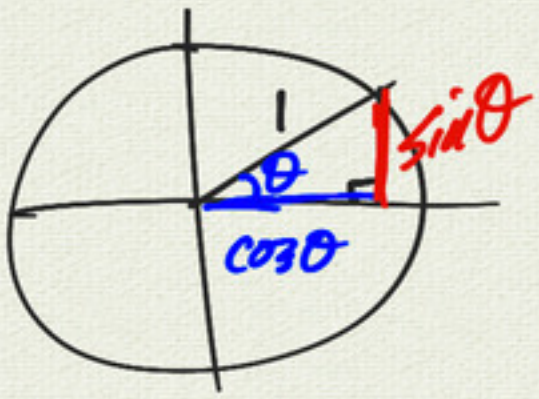
$$\boxed{\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v}$$

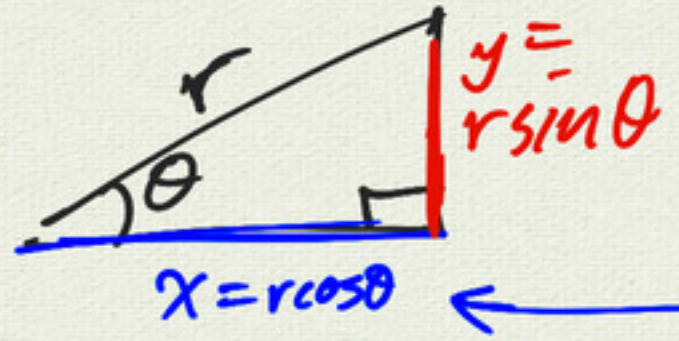
$$\tan(u+v) = \frac{\sin(u+v)}{\cos(u+v)}$$
$$= \frac{\sin u \cos v + \cos u \sin v}{\cos u \cos v - \sin u \sin v} \quad \frac{\frac{1}{\cos u \cos v}}{\frac{1}{\cos u \cos v}}$$
$$= \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\boxed{\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}}$$

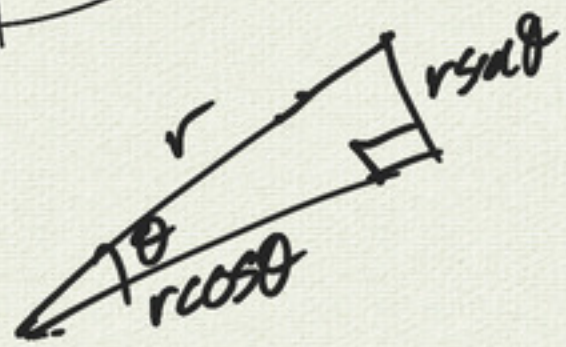
do not memorize

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

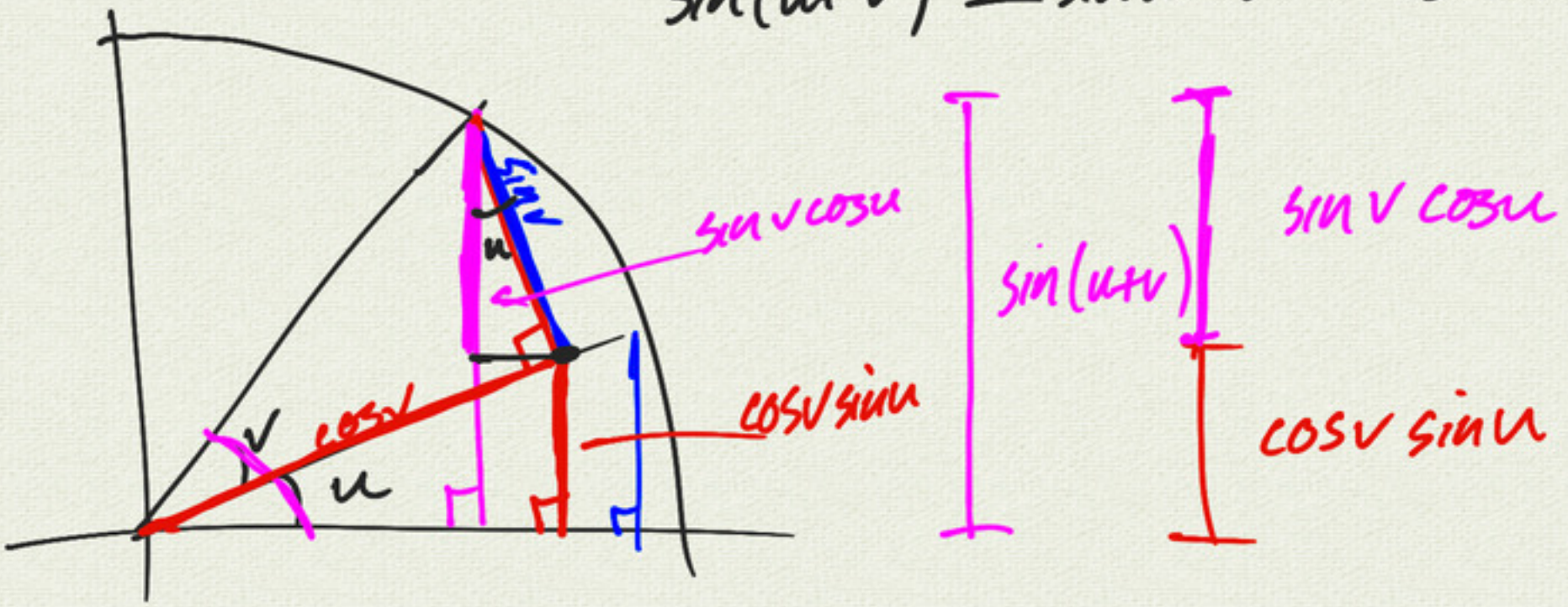




$$\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta$$



$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$



challenge: cosine?

challenge: $f(u+v) = f(u) + f(v)$

what functions f satisfy this identity?