2.1 / 7.1

(29) verify

$\cos x - \cos^3 x = \cos x \sin^2 x$

begin end

$\cos x - \cos^3 x = \cos x (1 - \cos^2 x)$ (factor)

$= \cos x \sin^2 x$ ✓ (Pythagorean)

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best practice: # 29-33

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(31) verify:

$\dfrac{1+\sin^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x} + \dfrac{\sin^2 x}{\cos^2 x}$

$= 1 + 2\tan^2 x$ ⟸ target

$\dfrac{1+\sin^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x} + \dfrac{\sin^2 x}{\cos^2 x}$

$= \sec^2 x + \tan^2 x$

$= (1+\tan^2 x) + \tan^2 x$

$= 1 + 2\tan^2 x$ ✓

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

(39) verify

$$\frac{1+\sin x}{\cos x} = \frac{\cos x}{1+\sin(-x)}$$

$$\frac{1+\sin x}{\cos x}\cdot\frac{1-\sin x}{1-\sin x} = \frac{1-\sin^2 x}{\cos x(1-\sin x)}$$

$\cos^2 x$

difference of squares + Pythagorean

$$= \frac{\cos^2 x}{\cos x(1-\sin x)}$$

$$= \frac{\cos x}{1-\sin x}$$

$$= \frac{\cos x}{1+\sin(-x)}$$ ✓ (sin is odd)

(45) Simplify

$$\frac{1-\cos^2 x}{\tan^2 x} + 2\sin^2 x$$

$$= \frac{\sin^2 x}{(\sin^2 x/\cos^2 x)} + 2\sin^2 x$$

$$= \sin^2 x \cdot \frac{\cos^2 x}{\sin^2 x} + 2\sin^2 x$$

$$= \cos^2 x + 2\sin^2 x$$

$$= (\underbrace{\cos^2 x + \sin^2 x}_{1}) + \sin^2 x$$

$$= 1 + \sin^2 x$$

2.3 Multiple Angle Identities

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

sum/diff

$u = v$

$$\sin(2u) = \sin(u+u)$$
$$= \sin u \cos u + \cos u \sin u$$
$$\boxed{\sin 2u = 2 \sin u \cos u}$$

double angle formula

$$\cos(2u) = \cos(u+u)$$
$$= \cos u \cos u - \sin u \sin u$$
$$\boxed{\cos 2u = \cos^2 u - \sin^2 u}$$
$$= 1 - 2\sin^2 u$$
$$= 2\cos^2 u - 1$$

double angle

$\sin^2 u + \cos^2 u = 1$

$\sin^2 u = 1 - \cos^2 u$

$\cos^2 u = 1 - \sin^2 u$

$$\sin 30^\circ = 2 \sin 15^\circ \cos 15^\circ$$
$$\cos 30^\circ = \cos^2 15^\circ - \sin^2 15^\circ$$

} relation between trig values for 30° & 15°

$\cos 2u = 1 - 2\sin^2 u$ $\qquad$ $\cos 2u = 2\cos^2 u - 1$

$2\sin^2 u = 1 - \cos 2u$ $\qquad$ $2\cos^2 u = 1 + \cos 2u$

$\sin^2 u = \dfrac{1 - \cos 2u}{2}$ $\qquad$ $\cos^2 u = \dfrac{1 + \cos 2u}{2}$

power reducing

$\sin u = \pm \sqrt{\dfrac{1 - \cos 2u}{2}}$ $\qquad$ $\cos u = \pm \sqrt{\dfrac{1 + \cos 2u}{2}}$

half angle

how do you know ±? look at quadrant

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example:

$$\sin 15^\circ = \sqrt{\frac{1 - \cos 30^\circ}{2}}$$
$$= \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2} \cdot \frac{2}{2}}$$
$$= \sqrt{\frac{2 - \sqrt{3}}{4}}$$
$$\sin 15^\circ = \frac{\sqrt{2 - \sqrt{3}}}{2} \stackrel{?}{=} \frac{\sqrt{6} - \sqrt{2}}{4}$$

challenge: show these are equal