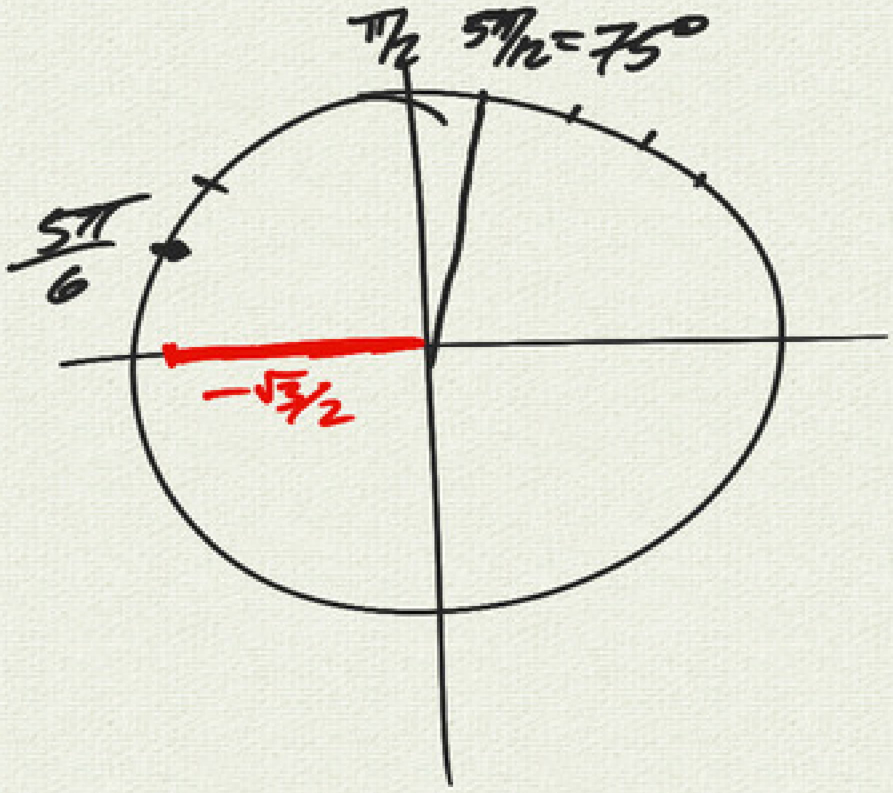
7.3

17 $\tan \frac{5\pi}{12}$



$$\sin\theta = \pm\sqrt{\frac{1-\cos 2\theta}{2}}$$

$$\cos\theta = \pm\sqrt{\frac{1+\cos 2\theta}{2}}$$

$$\theta = \frac{5\pi}{12} \Rightarrow 2\theta = \frac{5\pi}{6}$$

$$\sin\theta = \sqrt{\frac{1-(\cos 5\pi/6)}{2}}$$

$$= \sqrt{\frac{1-(\sqrt{3}/2)}{2}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

$$\cos\theta = \frac{\sqrt{2+\sqrt{3}}}{2}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}} = \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}}}$$

$$= \sqrt{\frac{(2+\sqrt{3})^2}{4-3}}$$

$$= 2+\sqrt{3}$$

59 $\sin 3x = \sin(\underbrace{2x}_{u} + \underbrace{x}_{v})$

$$= \sin 2x \cos x + \cos 2x \sin x$$

$$= (2\sin x\cos x)\cos x + (\cos^2 x - \sin^2 x)\sin x$$

$$= 2\sin x\cos^2 x + \cos^2 x \sin x - \sin^3 x$$

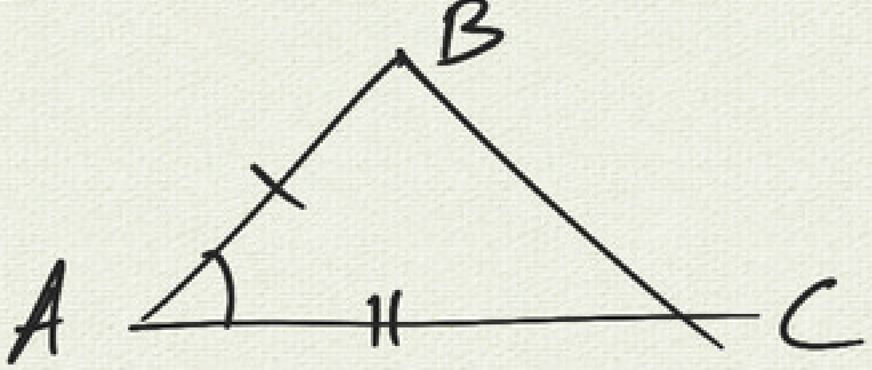
$$= 3\sin x\cos^2 x - \sin^3 x$$

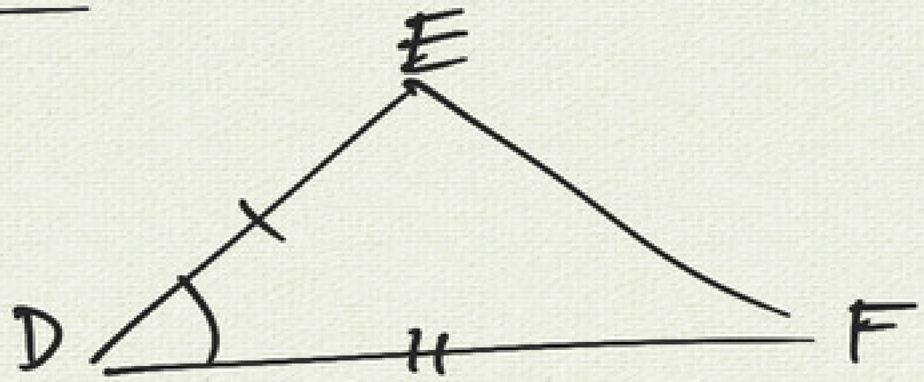
$\sin(u+v) = \sin u\cos v + \cos u \sin v$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

2.4 Law of Sines/Cosines



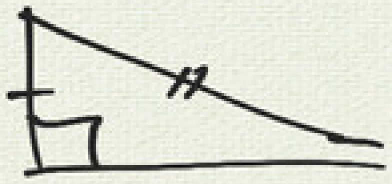


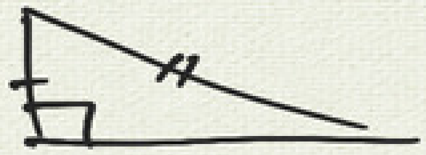
SAS

SSS

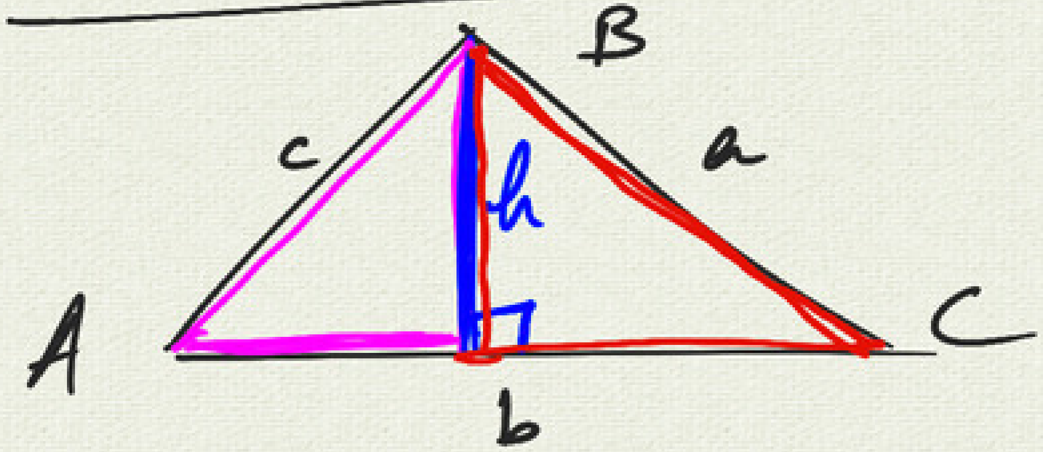
AAS

$\overline{\text{ASS}}$ ← ambiguous (sometimes)





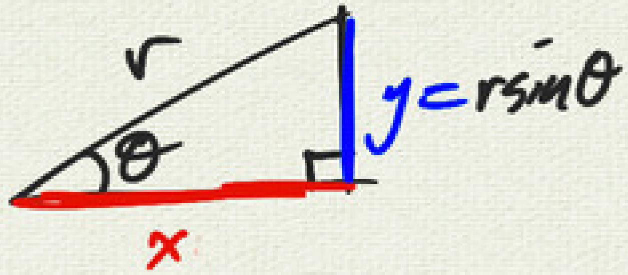
HL hypotenuse leg



$h = c\sin A = a\sin C$

$$\frac{\sin A}{a} = \frac{\sin C}{c} \quad \left(= \frac{\sin B}{b} \right)$$

Law of Sines



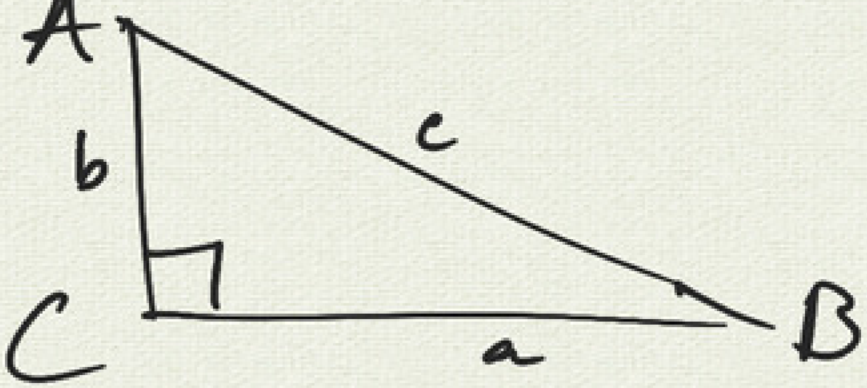
$\cos\theta = \dfrac{x}{r}$

$\Longrightarrow x = r\cos\theta$

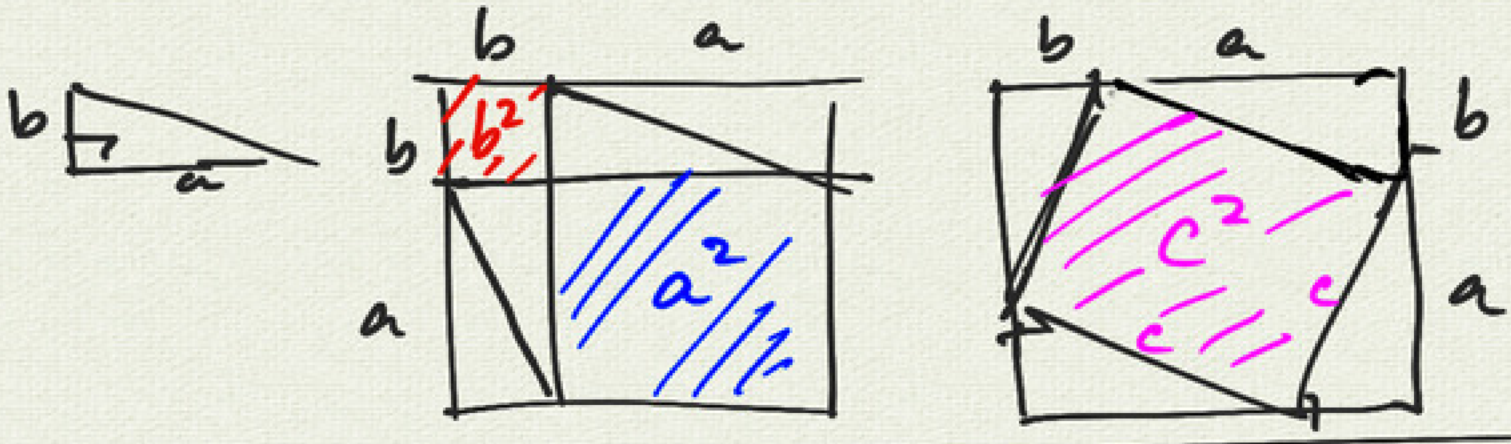
$A, B, C \Longrightarrow \alpha, \beta, \gamma$

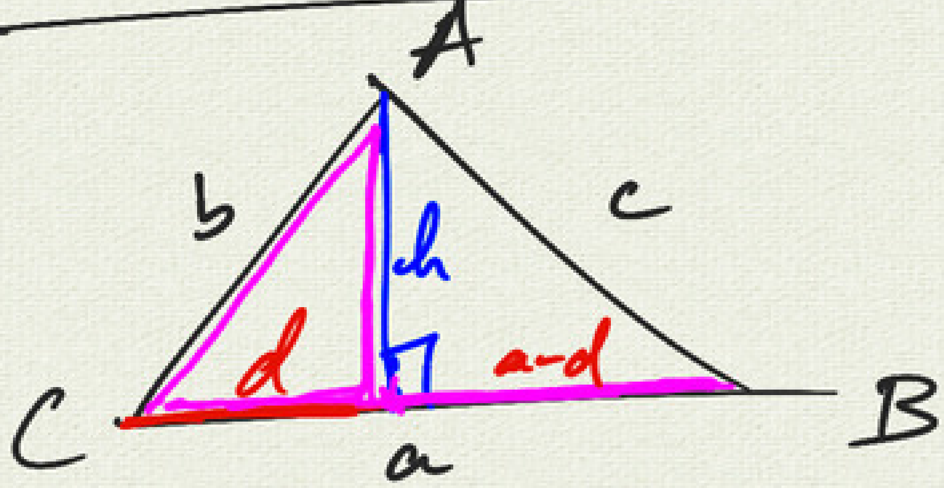
alpha beta gamma

Pythagorean Theorem



$$c^2 = a^2 + b^2$$





$$h = b\sin C$$

$$d = b\cos C$$

$$c^2 = h^2 + (a-d)^2$$

$$c^2 = (b\sin C)^2 + (a - b\cos C)^2$$

$$= b^2\sin^2 C + a^2 - 2ab\cos C + b^2\cos^2 C$$

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$$

Law of Cosines

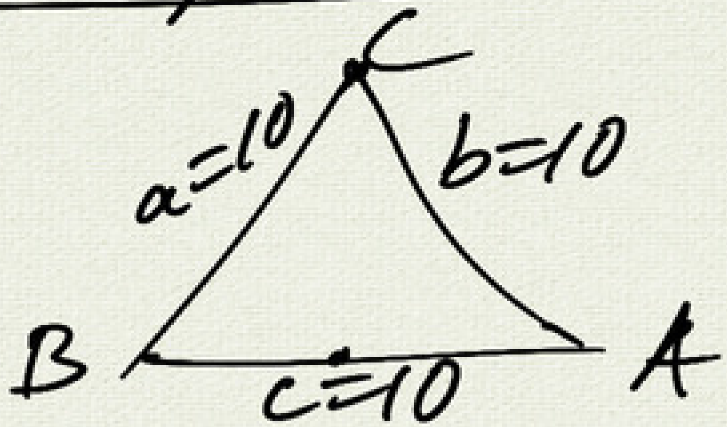
$$b^2(\underbrace{\sin^2 C + \cos^2 C}_{1})$$

$$= b^2$$

note: $C = \frac{\pi}{2} \Rightarrow c^2 = a^2 + b^2$

$\cos C = 0$

Example:



$a = b = c = 10$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

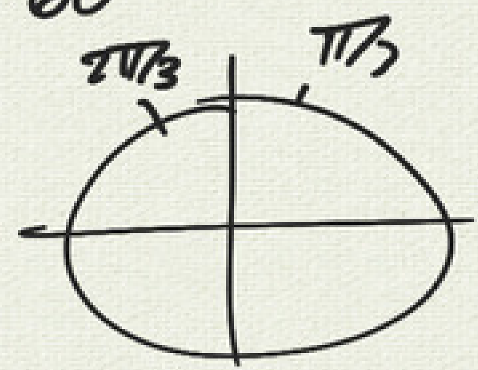
$$10^2 = 10^2 + 10^2 - 2 \cdot 10 \cdot 10 \cos C$$

$$100 = 200 - 200\cos C$$

$$\cos C = \frac{1}{2}$$

$$C = \pi/3 = 60^\circ$$

$$\frac{\sin A}{10} = \frac{\sin 60^\circ}{10} \left(= \frac{\sin C}{c}\right)$$



$$\Rightarrow \sin A = \frac{\sqrt{3}}{2}$$

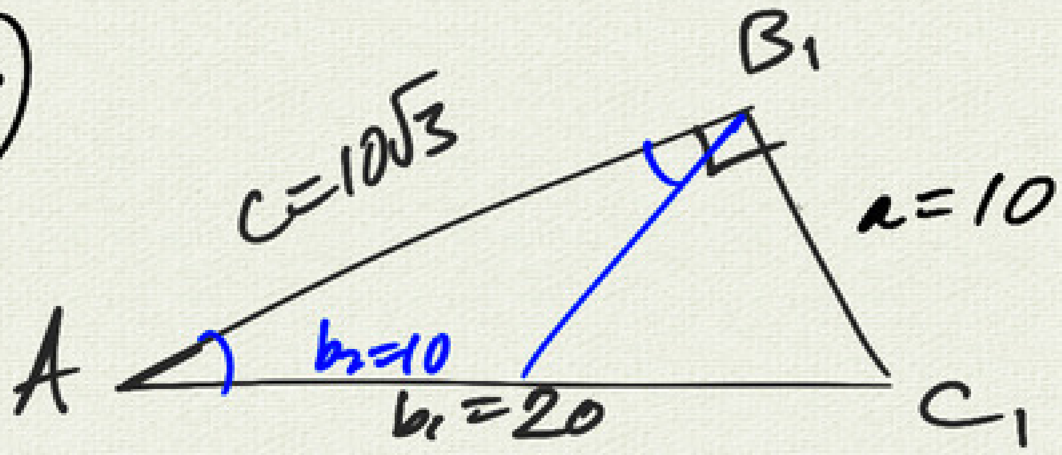
$$A = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$

no: $\frac{\pi}{3} + \frac{2\pi}{3} = \pi$

$$B = \pi - A - C \quad (180^\circ - A - C)$$

$$= \pi/3$$

Example (ASS)



$A = \frac{\pi}{6}$

$c = 10\sqrt{3}$

$a = 10$

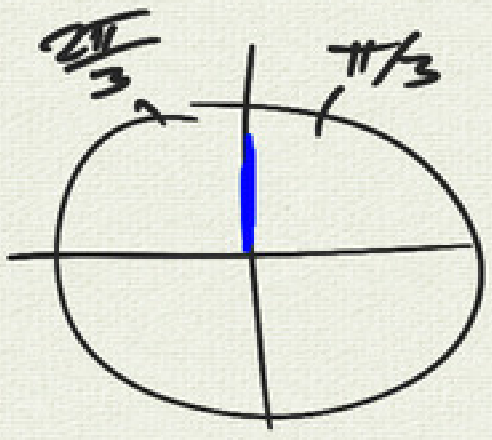
solve the Δ

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\sin C = \frac{c \sin A}{a}$$

$$= \frac{10\sqrt{3}\left(\frac{1}{2}\right)}{10}$$

$$\sin C = \frac{\sqrt{3}}{2}$$



$$C = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$

$C_1 = \frac{\pi}{3}$ $(=60°)$

$B_1 = \frac{\pi}{2}$ $(=90°)$

$b_1^2 = a^2 + c^2 - 2ac\underbrace{\cos B}_{0}$

$= 10^2 + (10\sqrt{3})^2$

$= 100 + 300$

$= 400$

$b_1 = 20$

$C_2 = \frac{2\pi}{3}$ $(=120°)$

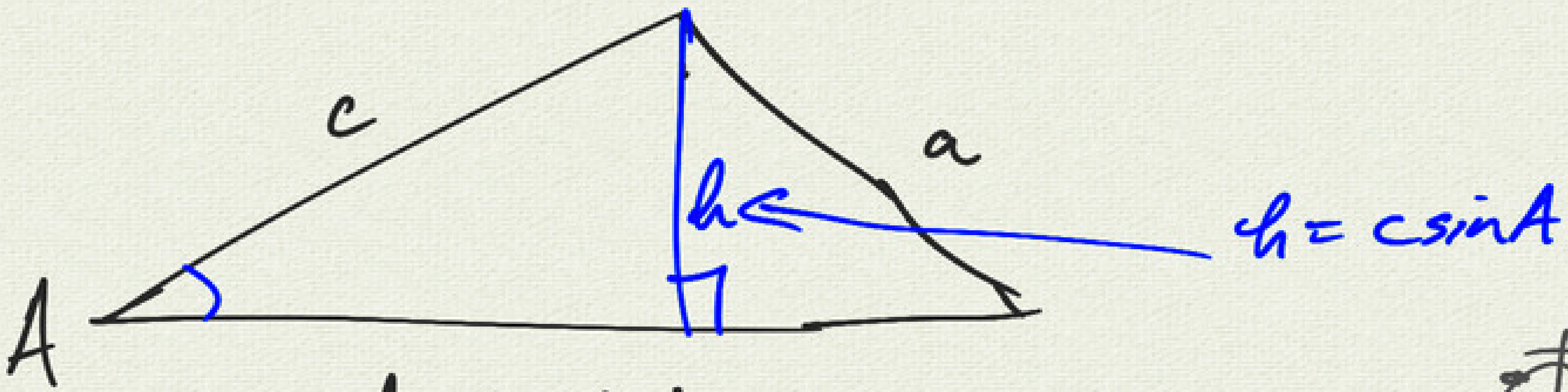
$B_2 = \frac{\pi}{6}$ $(=30°)$

$b_2^2 = a^2 + c^2 - 2ac\cos\left(\frac{\pi}{6}\right)$

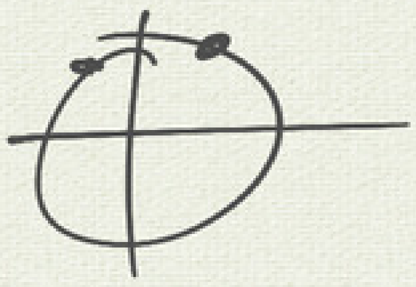
$= 100 + 300 - \underbrace{2 \cdot 10 \cdot 10\sqrt{3} \cdot \left(\frac{\sqrt{3}}{2}\right)}_{300}$

$= 100$

$b_2 = 10$



given A, a, c:

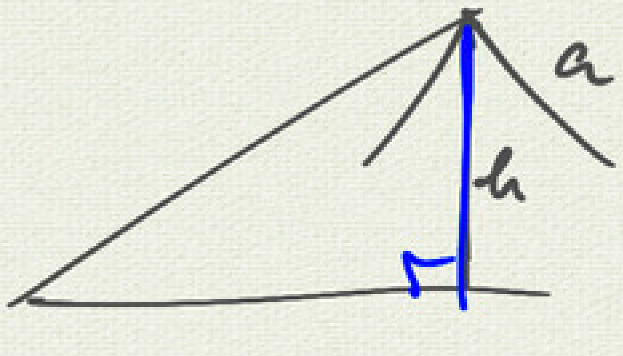


$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\sin C = \frac{\boxed{c \sin A}}{a} = \frac{h}{a} < 1$$

2 potential angles

$\frac{h}{a} = 1$ $\sin C = 1$ $C = \frac{\pi}{2}$ one possibility



$\Leftarrow$ $\frac{h}{a} > 1$ $\sin C > 1$ impossible