(39) $\cos^2 6x$

$(\theta = 6x)$

$\cos^2 6x = \boxed{\dfrac{1+\cos 12x}{2}}$

power reducing

$\cos 2\theta = \cos^2\theta - \underbrace{\sin^2\theta}_{1-\cos^2\theta}$

$\cos 2\theta = 2\cos^2\theta - 1$

$2\cos^2\theta = 1 + \cos 2\theta$

$\boxed{\cos^2\theta = \dfrac{1+\cos 2\theta}{2}}$ power reducing

$\cos\theta = \sqrt{\dfrac{1+\cos 2\theta}{2}}$ half-angle

$\cos\left(\dfrac{v}{2}\right) = \sqrt{\dfrac{1+\cos v}{2}}$

$\cos 15^\circ = \sqrt{\dfrac{1+\cos 30^\circ}{2}}$ $\left(\begin{matrix} v = 30^\circ \\ \theta = 15^\circ \end{matrix}\right)$

(59) $\sin 3x = \sin(2x + x)$

$= \sin 2x \cos x + \cos 2x \sin x$

$= (2\sin x\cos x)\cos x + (\cos^2 x - \sin^2 x)\sin x$

$= 2\sin x\cos^2 x + \cos^2 x \sin x - \sin^3 x$

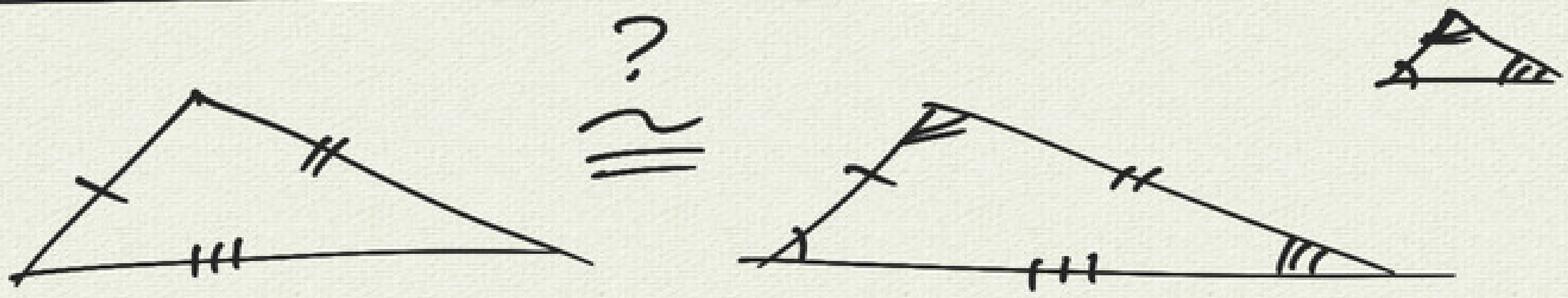
$= 3\sin x\cos^2 x - \sin^3 x$ ✓

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

double angle

2.4 Law of Sines/Cosines



SSS AAS

ASA

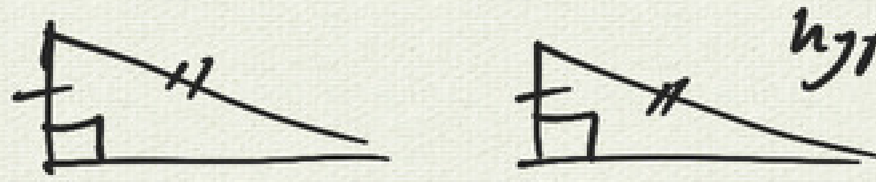
SAS

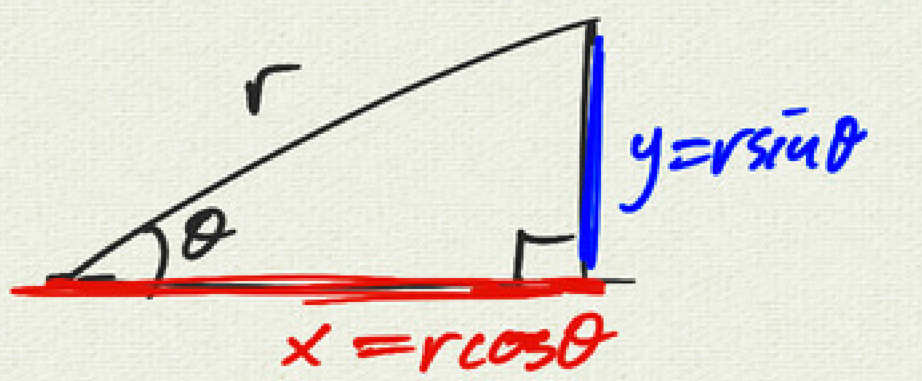
congruent $\longleftrightarrow$ everything equal

similar $\longleftrightarrow$ equal angles, proportional sides

ASS $\longleftarrow$ ambiguous (sometimes)

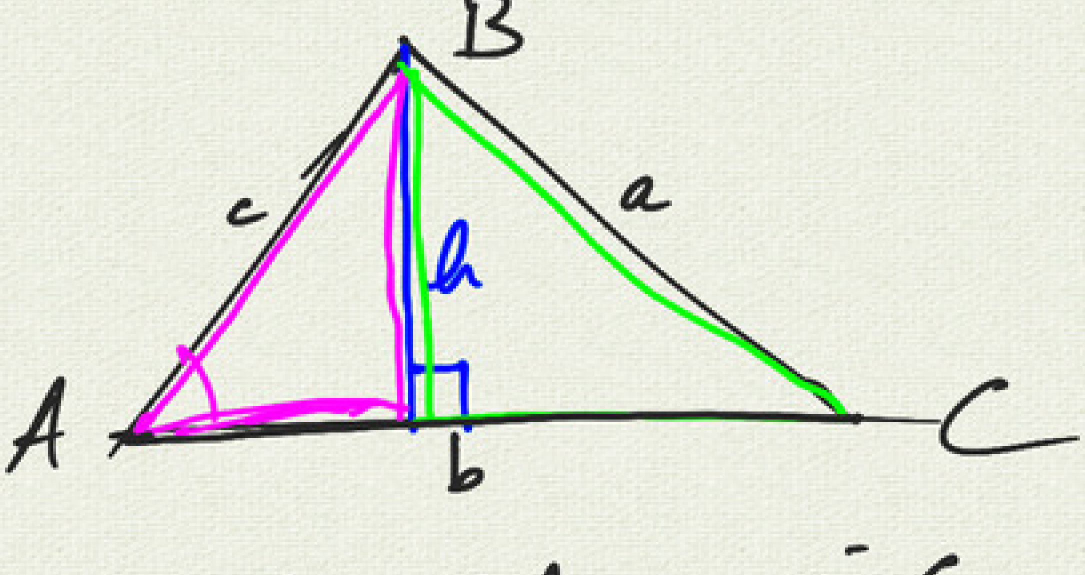
HL
hypotenuse/leg





$$\sin\theta = \frac{y}{r}$$

$$\Longrightarrow y = r\sin\theta$$



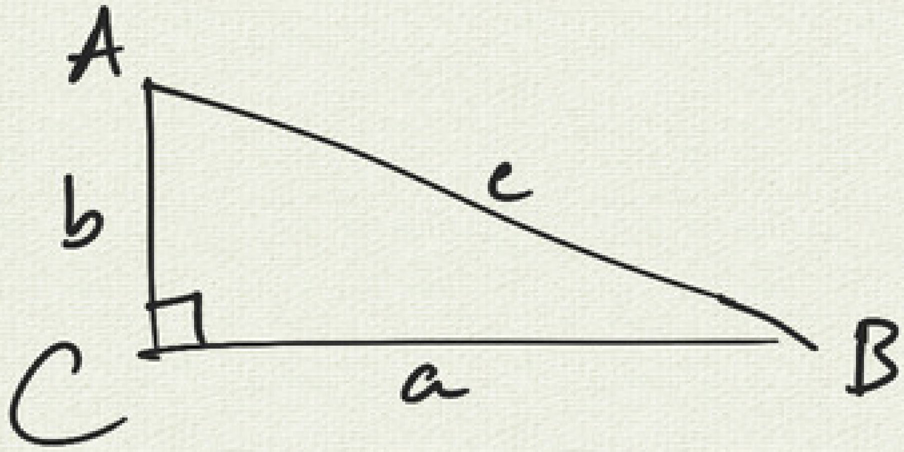
ABC $\longleftrightarrow$ α, β, γ

alpha beta gamma

$$h = c\sin A = a\sin C$$

$$\Longrightarrow \frac{\sin A}{a} = \frac{\sin C}{c} \quad \left(= \frac{\sin B}{b}\right)$$

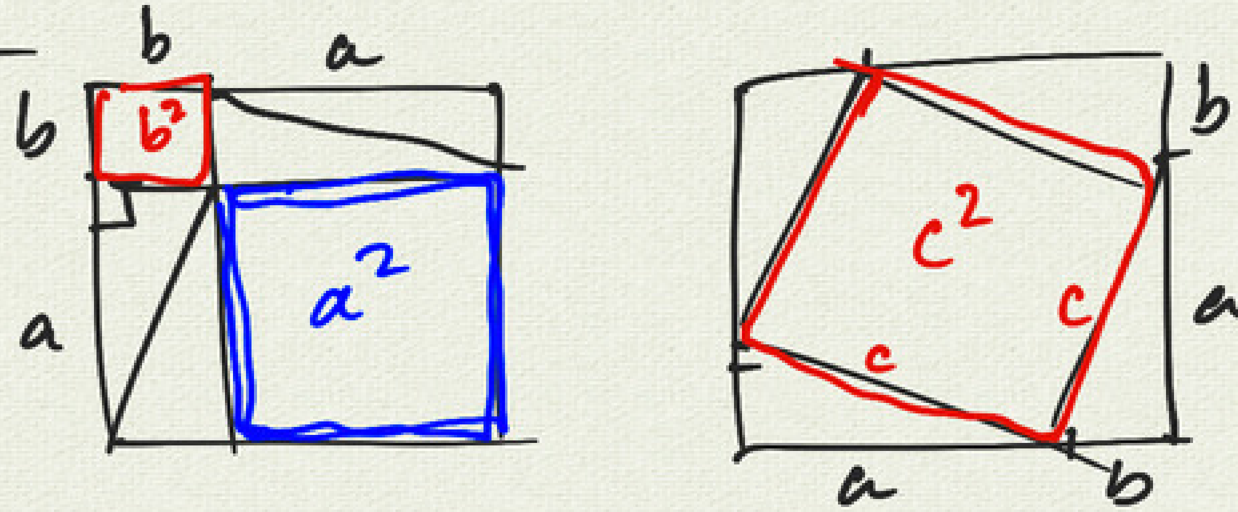
Law of Sines



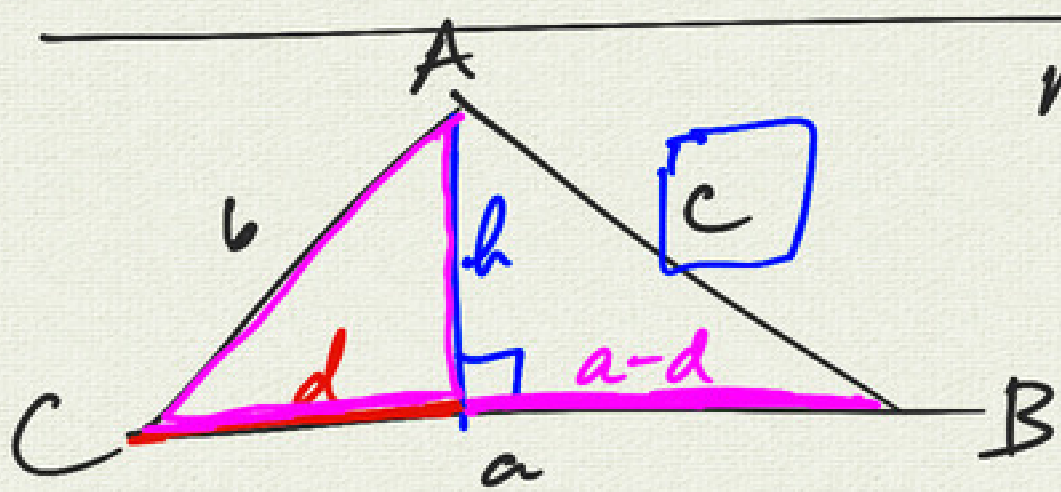
Pythagorean Theorem:

$$c^2 = a^2 + b^2$$

proof:



more general: C not right angle



$h = b\sin C$

$d = b\cos C$

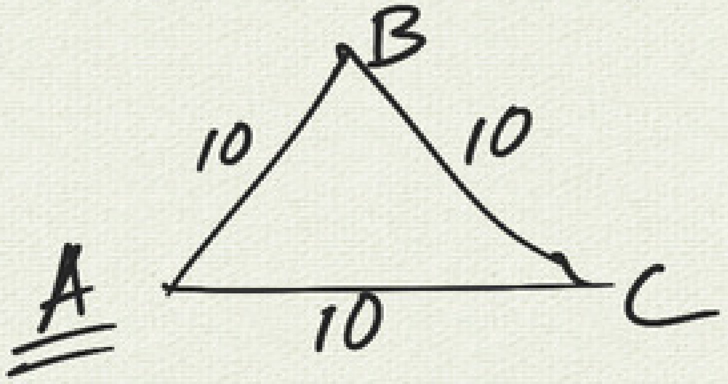
$$\begin{aligned}
c^2 &= h^2 + (a-d)^2 \\
&= (b\sin C)^2 + (a - b\cos C)^2 \\
&= (b^2\sin^2 C) + (a^2 - 2ab\cos C + b^2\cos^2 C) \\
&= b^2(\underbrace{\sin^2 C + \cos^2 C}_{1}) + a^2 - 2ab\cos C
\end{aligned}$$

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$$ Law of Cosines

Example:

$a = b = c = 10$

Solve triangle:



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$10^2 = 10^2 + 10^2 - 2\cdot 10\cdot 10\cdot \cos A$$

$$200\cos A = 100$$

$$\cos A = \frac{1}{2}$$

$$A = \pi/3$$

$B = ?$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

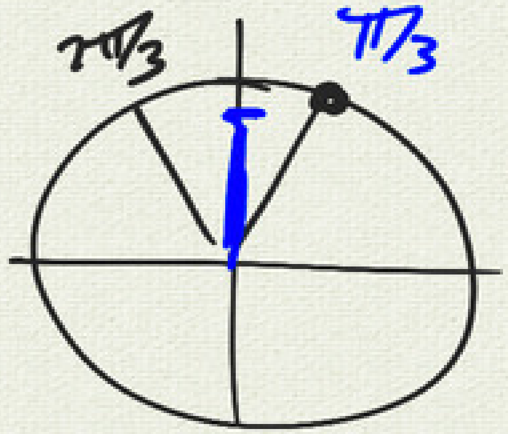
$$\frac{\sin B}{10} = \frac{\sin A}{10}$$

$$= \frac{\sqrt{3}/2}{10}$$

$$\longrightarrow \quad \sin B = \frac{\sqrt{3}}{2}$$

$$B = \frac{\pi}{3} \text{ or } \cancel{\frac{2\pi}{3}} \longleftarrow A+B=\pi$$

180°



$A = 60°,\ B = 60° \longrightarrow C = 60°$

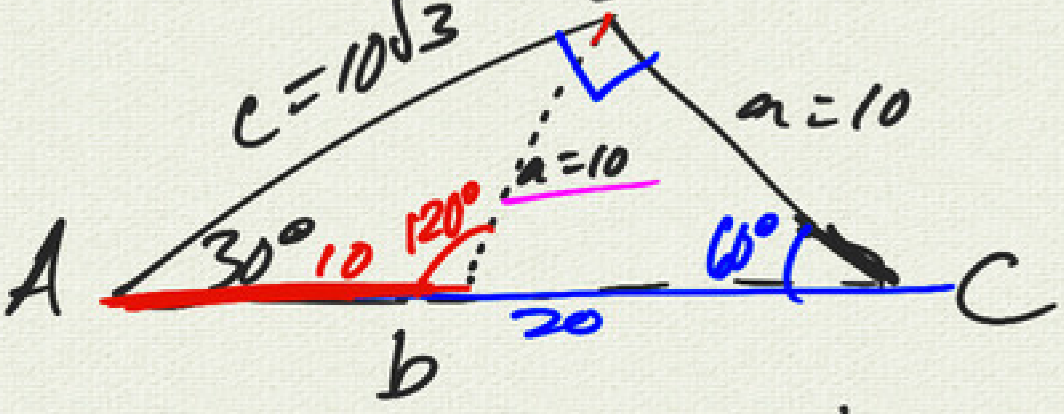
Example:

ASS

$A = 30^\circ$

$c = 10\sqrt{3}$

$a = 10$



$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin C}{10\sqrt{3}} = \frac{1/2}{10}$$

$$\sin C = \sqrt{3}/2$$

$$\Longrightarrow C = \frac{\pi}{3}, \frac{2\pi}{3}$$

$$\qquad 60^\circ \quad 120^\circ$$

$$C_1 = \frac{\pi}{3}\ (60^\circ)$$

$$B_1 = \frac{\pi}{2}\ (90^\circ)$$

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{b}{1} = \frac{10}{(1/2)}$$

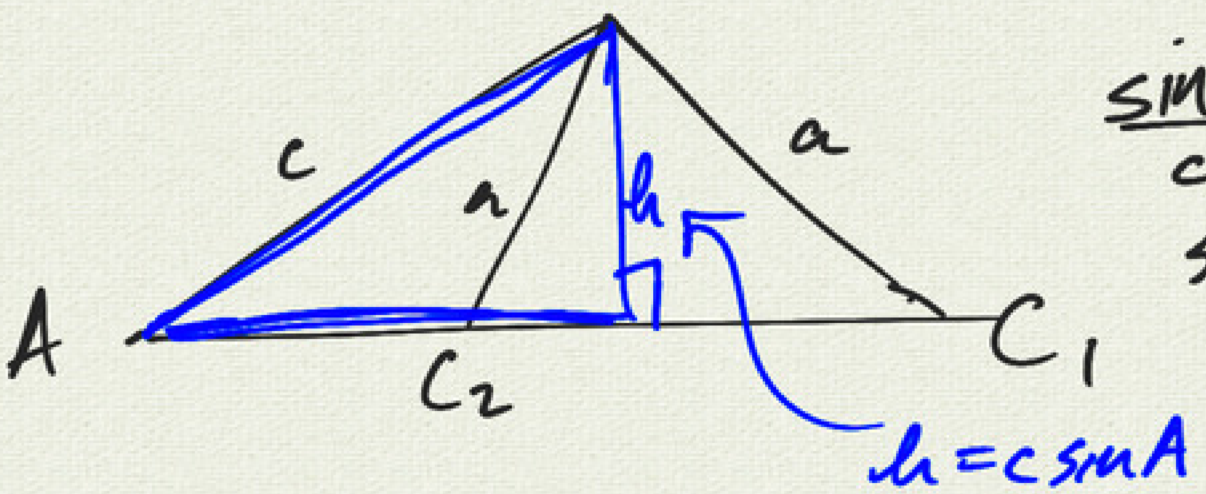
$$b_1 = 20$$

$$C_2 = \frac{2\pi}{3}\ (120^\circ)$$

$$B_2 = \frac{\pi}{6}\ (30^\circ)$$

$$\frac{b}{(1/2)} = \frac{10}{(1/2)}$$

$$b = 10$$



$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\sin C = \boxed{\frac{c \sin A}{a}}$$

$$= \frac{h}{a}$$

$\sin C = \dfrac{h}{a}$ $\quad = 1$ (single solution) $h = a$

< 1 2 possible solutions ($h < a$)

> 1 ($h > a$) no solutions